THE THEORY AND DESIGN OF MAGNETIC AMPLIFIERS

E.H. FROST-SMITH

1938

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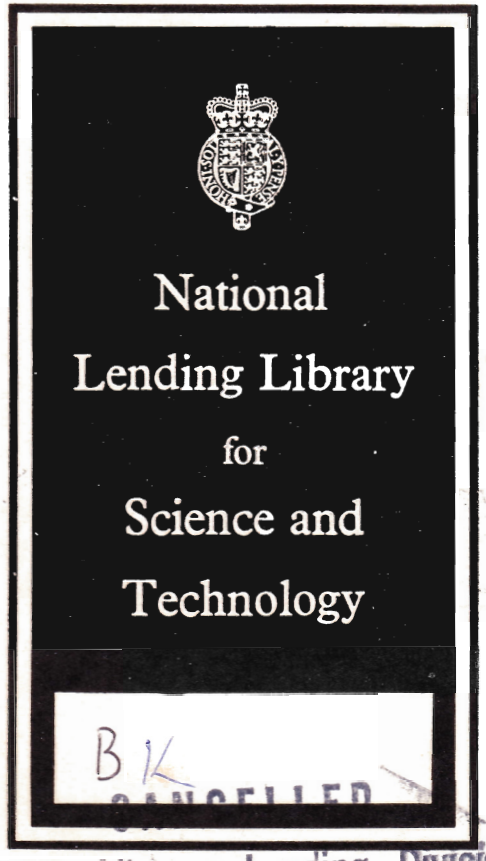

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Automation and Control Engineering Series

THE THEORY AND DESIGN OF MAGNETIC AMPLIFIERS

Automation and Control Engineering Series

EDITED BY JOHN F. COALES, O.B.E., M.A., M.I.E.E., F.Inst.P.

THE THEORY AND DESIGN OF MAGNETIC AMPLIFIERS

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FOREWORD

By JOHN F. COALES, O.B.E., M.A., M.I.E.E., F.INST.P.

THIS book is the first of a series on Automation and Control Engineering. The rapidity with which automatic controls have been applied in all forms of human activity has increased enormously during the last few years. Not only have automatic controls become essential in military applications such as radar and guided missiles, but there is hardly an industry which does not employ them at some stage of manufacture and computers and automatic accounting machinery are gradually finding their way into most offices. More and more engineers and technologists are meeting problems which can be considered as coming within the field of control engineering. This field is wide indeed, because in order to design the best possible control system, taking into account such considerations as economy of operation, reliability, ease of maintenance, development time and capital cost, it is not only necessary to understand the process that is to be controlled, but also to choose the most suitable components, whether they be mechanical, electrical, hydraulic or pneumatic. Thus, the old categories of engineer ; Civil, Mechanical, Electrical and Chemical, are becoming too narrow ; a new category is coming into being : the Control Engineer, who must have a good understanding, not only of the underlying principles of automatic control, but also of the fundamentals of all other branches of engineering.

The series of books, of which this is one, is planned to cover as comprehensively as possible this wide field in the most economical way for the Control Engineer. The underlying theory of automatic control and the design of control systems will be covered in some of the first volumes to be published. In addition, there will be the volumes dealing with special subjects which are of importance to a large number of control engineers, if not all. Some of these will deal with special areas of application of automatic controls in industry while others, like this first volume on Magnetic Amplifiers, will deal with special components or special techniques. These books on techniques and special components will be as comprehensive as possible from the point of the control engineer and will include such fundamentals as are necessary for a

proper understanding of the subject and those practical aspects which are likely to be of importance to him.

That magnetic amplifiers should be the subject of the first volume to be published needs no apology, since the last few years have seen their widespread introduction into control systems. They have certain important characteristics which cannot be achieved in any other way, and they should therefore always be considered as a possible means of amplification when power of more than a few milliwatts is required. Thus every control engineer should know something about their behaviour and capabilities, and many will have to design them for specific purposes. This volume, which successfully links up the theory of all types of magnetic amplifiers, also gives design methods to meet specific requirements and so should provide the control engineer with what he wants to know.

J. F. C.

AUTHOR'S PREFACE

A KNOWLEDGE of the theory and design of transductors and magnetic amplifiers has become a valuable asset to the practising electrical engineer. The increasing number of applications for these devices in recent years has come about, partly with the advent of new projects such as nuclear power plants, guided missiles, etc., but primarily due to the universal requirements for simple and reliable components in electrical equipment which is necessarily becoming very much more complex.

Magnetic amplifiers used in industry also play a valuable part in reducing the servicing requirements for industrial instruments, where it is often not permissible to use electronic devices due to the high maintenance and replacement costs usually associated with the latter.

The aim of this book is to give an account of the theory of magnetic amplifiers and to link up the theory with the design in a way which, it is hoped, will be of value, not only to the professional engineer, but also to the university student.

Many of the theoretical aspects are supplemented by short numerical examples and, in addition, a number of complete designs is carried out in Chapter 13. Some typical problems to which magnetic amplifiers have been applied successfully are discussed in Chapter 14.

Much of the work covered by this book was originally carried out by the author in the Electrical Engineering Department of University College, London, under the guidance of Professor H. M. Barlow, and later at the laboratories of Messrs. Elliott Brothers (London) Ltd., under Mr. J. F. Coales, to both of whom the author expresses gratitude. Thanks are also due to friends and colleagues at Elliott Brothers, the de Havilland Propellor Co. and the Royal Aircraft Establishment for the benefit derived from many useful discussions. In particular the author would like to thank Mr. J. F. Coales and Mr. J. N. Hutchinson for their critical and helpful reading of the manuscript.

Lastly, appreciation and thanks are expressed to the Directors of Messrs. Elliott Brothers for their support and their permission to publish the results of work carried out in the Company's Laboratories.

E. H. FROST-SMITH.

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LIST OF PRINCIPAL SYMBOLS

Symbol	*Definition*
A_c	Sectional area occupied by control windings assuming perfect winding.
A_a	Ditto for a.c. windings.
A_a', A_c'	Area normally occupied by imperfect windings.
a	Core area embraced by one a.c. winding.
$\mathbf{B}$	Flux density.
$\mathbf{B}_z$	Remanent flux density.
$\mathbf{B}_s$	Saturation flux density.
$\mathbf{B}_t$	Peak value of alternating flux density in transductor core.
$\mathbf{B}_q$	Flux density in unsaturated core at start of conduction.
$\mathbf{B}_Q$	Flux density in unsaturated core at end of conduction.
b_a	Mean length of one a.c. turn.
b_c	Mean length of one control turn.
C_L	Load capacitance.
C_r	Self capacitance of load rectifier.
D	Operator, d/dt.
E, e	Instantaneous voltages*
e'	r.m.s. value of voltage.
$\hat{E}$	Peak voltages.
E_c	Applied control voltage.
E_a	Instantaneous value of a.c. supply voltage.*
$\bar{E}_{as}$	Mean supply voltage which just fails to saturate transductor cores in the absence of a control voltage.
$\bar{e}_{ts}$	Mean transductor voltage corresponding to a supply voltage $\bar{E}_{as}$.
e_t	Voltage across transductor a.c. windings.*
e_{tA}, e_{tB}	Component of e_t across either winding A or B.*
e_a	Output voltage in a.c. circuit.*
e_f	Voltage across self-exciting windings.*
e_L	Voltage applied to load.*
e_{r1}, e_{r2}	Reverse voltages on rectifiers.*
e_d	Voltage across pick-off windings.*
F	M.M.F., ampere turns.*
$\hat{F}$	Peak value of alternating m.m.f.
F_A, F_B	M.M.F. magnetizing cores A or B.*
F_a	M.M.F. due to a.c. coils.*
F_c	M.M.F. due to control coils.*
F_d	M.M.F. due to pick-off coils.*
F_s	M.M.F. necessary to saturate core.
f	Supply frequency $= \omega/2\pi$.
f_c	Signal frequency.
G	General symbol for steady-state gain.
G_v	Steady-state voltage gain.
G_i	Steady-state current gain.
G_m	Steady-state ampere turns ratio.
G_p	Steady-state useful power gain.

List of Principal Symbols

Symbol	*Definition*
G_p'	Steady-state circuit power gain.
G_{vi}	Steady-state (output voltage)/(control current) ratio.
g	Steady-state gain of a device with feedback (open-loop gain).
g_v, g_v'	$\frac{2}{\pi}\frac{\omega M_{ca}}{R_c}, \frac{1}{\pi}\frac{\omega M_{ca}}{R_c}$ = Voltage gain of transductor with complete self-excitation or auto-excitation.
g_i, g_i'	$\frac{2}{\pi}\frac{\omega M_{ca}}{R_a}, \frac{1}{\pi}\frac{\omega M_{ca}}{R_a}$ = Current gain of transductor with complete self-excitation or auto-excitation.
g_m, g_m'	$\frac{2}{\pi}\frac{\omega L_{aw}}{R_a}, \frac{1}{\pi}\frac{\omega L_{aw}}{R_a}$ = M.M.F. ratio of transductor with complete self-excitation or auto-excitation.
$\mathbf{H}$	Magnetizing field.*
$\mathbf{H}_z$	Coercive force.
$\mathbf{H}_c$	Control field.*
H_v	Overall voltage feedback ratio.
h	Feedback ratio.
h_v	$\frac{R_c}{R_a}\frac{N_a - N_f}{N_c}$.
h_i	$\frac{N_a - N_f}{N_c}$.
h_m	$\frac{N_a - N_f}{N_a}$.
h_m'	M.M.F. feedback ratio for auto-excited transductor.
i	Instantaneous current.*
i_a	Current in transductor a.c. circuit.*
i_c	Current in transductor control circuit.*
i_L	Load current.*
i_f	Current in self-exciting circuit.*
i_d	Current in pick-off circuit and windings.*
$\bar{i}_{as}$	Current in a.c. circuit (i_a) when both cores are fully saturated.*
$\bar{i}_{a0}$	Current in a.c. circuit when both cores are unsaturated.*
$\bar{i}_{a\theta}$	Current in a.c. circuit for a firing angle θ.*
$\bar{i}_{cs}$	Direct control current which will just saturate the core when acting alone.
i_k	Bias current.*
K	Supply voltage constant, equal to: $\frac{\text{Optimum mean supply voltage}}{\bar{E}_{as}}$.
L	Inductance.
L_{aw}	Inductance of one a.c. winding on an unsaturated core.
L_{cw}	Inductance of one control winding on an unsaturated core.
L_c	Inductance in external control circuit.
L_L	Load inductance.
l	Length of magnetic path in the core (mean value).
M_A, M_B	Mutual inductance between two coils on core A or B.
M_{ca}, M_{ac}	Mutual inductance between one control winding and one a.c. winding when coupled by an unsaturated core.
M_{cd}, M_{dc}	Ditto for control winding and pick-off winding.
m	Control ratio $= \bar{i}_{as}/\bar{i}_{a0}$.
N	Load transformer turns ratio.
N_a	Number of turns on each a.c. winding.

List of Principal Symbols

Symbol	*Definition*
N_b	Number of turns on each feedback winding for auto-excited transductor.
N_c	Number of turns on each control winding.
N_d	Number of turns on each pick-off winding.
N_f	Number of turns on each self-exciting winding.
N_k	Number of turns on each bias winding.
N_L	Number of turns on each half of a centre tapped load.
n	Number in a series (of half-cycles).
P	Mean power output.
q	Form factor.
R_a	Total a.c. circuit resistance consisting of a.c. windings, load, forward resistance of rectifiers, etc.
R_c	Control circuit resistance.
R_L	Load resistance.
R_s	Ballast resistance used for push-pull operation with voltage summing.
R_E	Effective value of R_a for one transductor of a push-pull pair when the adjacent transductor is cut off.
$\bar{R}_a$	$\bar{e}_a/\bar{i}_a$ (steady-state).
$\bar{R}_{aL}$	$\bar{e}_a/\bar{i}_L$ (steady-state).
r_{aw}	Resistance of one a.c. coil.
r_{ag}	Internal resistance of a.c. supply.
r_{ar}	Forward resistance of self- or auto-exciting rectifier element.
r_b	Feedback resistance placed across auto-exciting rectifier element.
r_{cf}	Resistance of one control winding.
r_{dw}	Resistance of one pick-off winding.
r_c	Resistance of inductor L_c in control circuit.
r_x	Internal resistance of transductor output circuit consisting of a.c. windings, load and rectifiers, etc.
S	Reluctance of magnetic circuit, $l/\mu a$.
s_a, s_c	Cross sectional area of wire in a.c. windings or control windings.
T	Period of one cycle = 1/f.
t	Time.
t_q	Time at which conduction starts.
t_Q	Time at which conduction ends.
$t_{q1\ldots n}$	Value of t_q referred to 1st...nth half-cycle.
$t_{Q1\ldots n}$	Value of t_Q referred to 1st...nth half-cycle.
u	Fraction expressing proportion of supply voltage across a.c. windings of even harmonic transductor when one core has saturated.
W	Energy stored in a magnetic core.
X_{aw}	Reactance of one a.c. winding.
$\overline{Y}(D)$	Frequency dependent part of a system transfer function.
$\overline{Y}_a(D)$	Frequency dependent part of transfer function defined by $\bar{e}_a/\bar{i}_a$.
$\overline{Y}_{aL}(D)$	Frequency dependent part of transfer function defined by $\bar{e}_a/\bar{i}_L$.
$\overline{Y}_v(D)$	Frequency dependent part of transfer function defined by $\bar{e}_a/E_c$.
$y(D)$	Frequency dependent part of the transfer function for a device without feedback (open loop).
Z_L	Load impedance.
Z_a	Total impedance in a.c. circuit assuming both cores saturated.

List of Principal Symbols

Symbol	*Definition*
β	Duration of conducting period.
$\beta_{1\ldots n}$	Value of β in 1st...nth half-cycle.
γ	$\dfrac{N_c^2 R_a}{N_a^2 R_c}$.
γ'	$\dfrac{N_c^2}{(N_a - N_f)^2} \cdot \dfrac{R_a}{R_c}$.
ζ	Damping factor for a 2nd order oscillating system.
η	Efficiency $= \dfrac{\text{r.m.s. power in load}}{\text{total r.m.s. power supplied to system}}$.
η_m	Mean power efficiency $= \dfrac{\text{mean power output}}{\text{total r.m.s. power supplied}}$.
θ	Particular value of ωt.
θ_i, θ_o	Input and output quantities for a servo system.
κ	Coefficient of magnetic coupling.
λ	Equal to unity for a high control circuit impedance, and equal to $2/\pi$ for a low control circuit impedance.
μ	Permeability.
ρ	Specific resistance of copper.
ϕ	Flux.
$\hat{\phi}$	Peak flux.
Φ_s	Saturation flux.
ϕ_A, ϕ_B	Flux in core A or B.
ϕ_o	Mean flux level $= \dfrac{\phi_A + \phi_B}{2}$.
ϕ_c	Variable component of mean flux.
Φ_o	Constant component of mean flux, equal to flux which will just fail to cause saturation in the presence of an a.c. supply. $\phi_o = \phi_c + \Phi_o$.
ϕ_t	Instantaneous value of a.c. flux components.
ϕ_q, ϕ_Q	Flux in unsaturated core at start or finish of conduction.
$\phi_{q1\ldots n}$	Value of ϕ_q in the 1st...nth half-cycle.
$\phi_{Q1\ldots n}$	Value of ϕ_Q in the 1st...nth half-cycle.
ω	Angular supply frequency 2πf.
Ω	Angular signal frequency $2\pi f_c$.
τ	Time constant.
τ_L	Load time constant.
τ_t	Transductor flux time constant.
τ_L'	$R_L C_L$.
Ψ	Function denoting magnetization curve.
ψ_1	Rate of flux change in core approaching saturation.
ψ_2	Rate of flux change in core departing from saturation.
ψ_3	Rate of flux change in unsaturated core during conduction.

* A bar, thus $^-$, placed over any of these symbols indicates that the rectified mean value of the symbol is being considered.

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CHAPTER 1

INTRODUCTION

In most electromagnetic devices an iron core is employed because of the relative ease with which it can be magnetized. Thus, in a power transformer the magnetizing current is much smaller when an iron core is used than it would be if the transformer coils were air coupled. In general, it is desirable that the magnetization characteristic of the core should be linear ; that is to say, the magnetic induction should be proportional to the magnetizing field. When dealing with magnetic amplifiers, however, although ease of magnetization is an important feature, non-linearity is essential, and in general the more marked the non-linearity the more useful the core becomes for this purpose.

Although the problems associated with magnetic amplifiers are therefore non-linear ones, a number of elementary and fundamental points associated with linear theory inevitably arise. This chapter contains a brief revision of some of these elementary principles relevant to the more specialized aspects of magnetic amplifier and transductor theory.

1.1 Ferro-magnetism [1(a, b)]

When considering magnetic materials it is convenient to regard a magnetic field **H** as producing a magnetic induction **B** in the material. The field may be due to a permanent magnet or to an electric current flowing in a coil. In either case the magnetic induction is related to the field strength by the equation

$$\mathbf{B} = \mu\mathbf{H} \qquad . \quad . \quad . \quad (1.1)$$

This equation defines μ, the permeability of the material, which is not constant but varies with **B**.

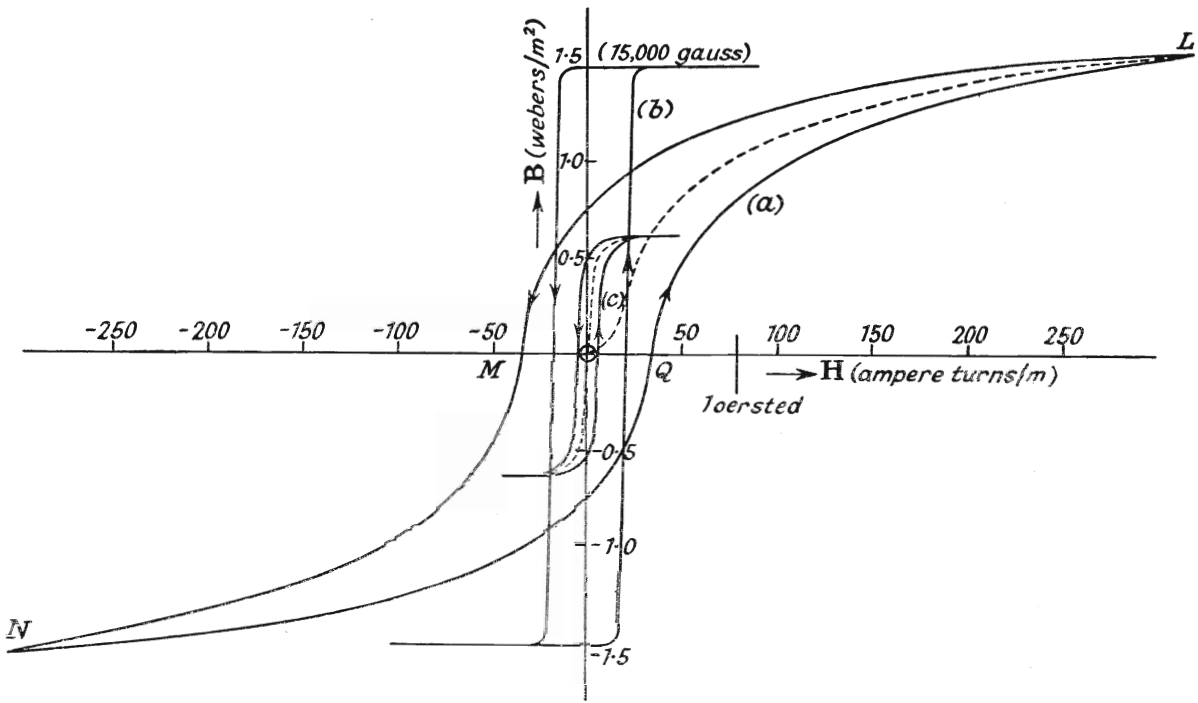


Figure 1.1. Hysteresis loops for some ferro-magnetic materials.
(*a*) 4% Silicon Iron (Stalloy).
(*b*) 50% Nickel Iron (Deltamax).
(*c*) 78% Nickel Iron (Mumetal).

If an unmagnetized ferro-magnetic material is subjected to a gradually increasing field, the induction increases along a curve such as OL (*Figure* 1.1*a*). If the field is then gradually decreased, the curve relating **B** and **H** is not given by OL, but by another curve LMN as shown. On increasing the value of **H** from N to L once more, the value of **B** increases along the curve NQL, thereby tracing out a closed loop $LMNQL$. It may be seen that the way in which **B** varies with **H** depends not only upon the value of the applied field, but also upon the previous changes which have taken place. This phenomenon is known as hysteresis and the curve is called a hysteresis loop.

The curve OL is known as the magnetization curve or normal induction curve, and the permeability is usually defined by means of this curve.

The nature of the relationship between **B** and **H** depends largely on the magnetic material, and typical hysteresis loops for a number of magnetic materials are shown in *Figure* 1.1.

If the range of the symmetrically varying field **H** is decreased, a family of minor loops may be obtained as shown in *Figure* 1.2*a*, and if the tips of these loops are joined, the resulting curve is very nearly coincident with the magnetization curve OL in *Figure* 1.1*a*.

A unidirectional field superimposed on an alternating field will result in a hysteresis loop which is unsymmetrical, and for various values of the d.c. field the resulting hysteresis loops will take the form shown in *Figure* 1.2*b*.

Since the behaviour of the material is not only dependent upon **B** and **H**, but also upon previous variations of these quantities, it is evidently very difficult to predict from the major loop the way in which **B** will vary with **H** under different conditions.

The hysteresis effect always involves a dissipation of energy, and it may be shown that the energy loss for a complete cycle is proportional to the area of the closed hysteresis loop.

The magnetic induction for $\mathbf{H} = 0$ on the largest possible closed loop for the material is known as the remanent induction $\mathbf{B}_r$ and the negative field required to reduce **B** to zero is the coercive force $\mathbf{H}_c$. A small coercive force is associated with a small energy loss due to hysteresis, and our present interest is confined to materials having these properties. Such materials are described as magnetically soft.

In these cases the area of the hysteresis loop is comparatively

small, and it is usually permissible to represent the characteristic by a single valued non-linear relationship such as that shown in *Figure* 1.2*c*. Often, particularly in the case of the square loop

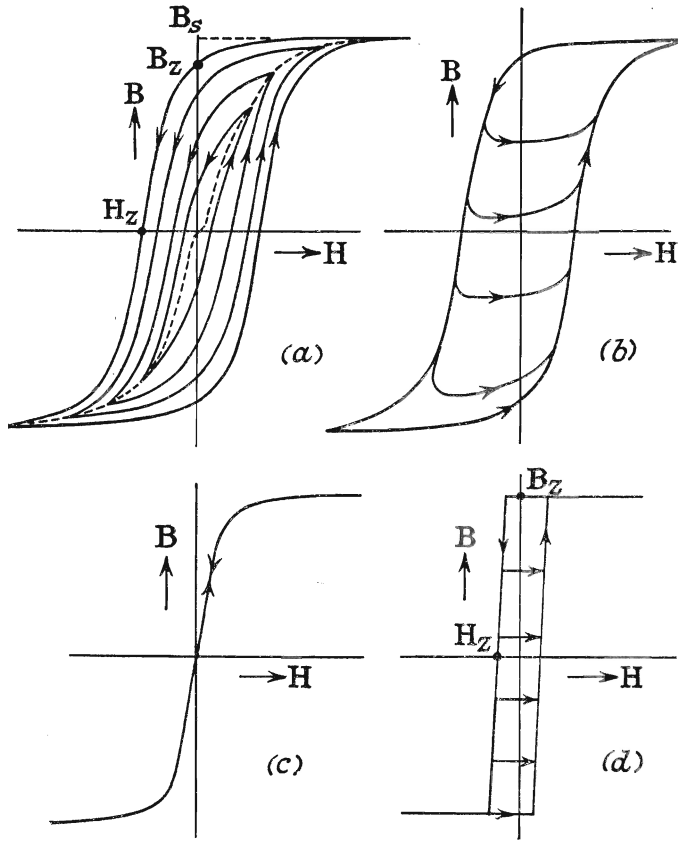


Figure 1.2. Magnetization characteristics.
(*a*) Symmetrical hysteresis loops for various amplitudes of applied field.
(*b*) Unsymmetrical hysteresis loops for various amplitudes of applied field.
(*c*) and (*d*) Assumed shapes of magnetization curve.

materials, the magnetization characteristic is assumed to be a closed rectangular loop as shown in *Figure* 1.2*d*.

In all cases the magnetic induction **B** increases with **H** up to a limit, after which very large increases in **H** produce only a relatively small increase in **B**. This limiting value of **B** is called the saturation induction, denoted by $\mathbf{B}_s$ (*Figure* 1.2*a*), which

varies from one material to another, but is a constant for a given material at a constant temperature. The region in which the slope changes rapidly is known as the " knee " of the **B**/**H** curve, and for some magnetic materials the slope changes by a factor of 100 or more over this region. Provided values of **B** are considered below the " knee," it is a reasonable approximation to assume that the permeability $\frac{\mathbf{B}}{\mathbf{H}}$ and the slope $d\mathbf{B}/d\mathbf{H}$ are constant and equal.

The total magnetic flux passing through a uniform area, a, normal to the direction of the field is

$$\phi = \mathbf{B} \, . \, a \, . \qquad . \quad . \quad . \quad (1.2)$$

The magnetic induction **B** is sometimes referred to as the flux density. In C.G.S. units **B** is measured in gauss and **H** in oersteds. In the rationalized M.K.S. system of units, **B** is measured in webers per square metre and **H** in ampere-turns per metre. The unit of magnetic flux, the weber, is defined such that the e.m.f. e, induced in a coil of N turns linked by a flux of ϕ webers is given by

$$e = N\frac{d\phi}{dt} \text{ volts} \qquad . \quad . \quad . \quad (1.3)$$

Also the magnetic field **H** produced by a current of i amperes flowing in a coil of N turns is given by

$$\mathbf{H} = \frac{Ni}{l} \text{ ampere turns per metre} \qquad . \quad . \quad . \quad (1.4)$$

where l is the mean length of the magnetic path in metres

$$1 \text{ weber/m}^2 = 10^4 \text{ gauss}$$

$$1 \text{ ampere turn/m} = 4\pi \times 10^{-3} \text{ oersted.}$$

The M.K.S. unit of permeability $= \frac{10^7}{4\pi} \times$ the C.G.S. unit of permeability, i.e. the permeability in M.K.S. units $= \frac{4\pi}{10^7} \times$ the permeability in C.G.S. units. The rationalized M.K.S. system of units is employed throughout this book.

Equations (1.1), (1.2) and (1.4) may be combined to give the following result

$$\phi = \frac{(Ni)}{(l/a\mu)}.$$

The quantity Ni is the magneto-motive force (m.m.f.) F and $l/a\mu$ the reluctance S thus

$$\phi = \frac{F}{S} * \qquad . \quad . \quad . \quad (1.5)$$

When using rationalized M.K.S. units, m.m.f. and ampere-turns are synonymous.

The relation between flux and m.m.f. will be of the same form as that between **B** and **H**, differing only by a scale factor depending upon the values of l and a. The ϕ–F curve therefore applies to a particular core, while the **B**–**H** curve is independent of dimensions and depends only upon the core material.

If a core is subjected to a varying magnetic field, then, in addition to the energy required to overcome hysteresis, there are further losses due to eddy currents. These arise due to induced e.m.f.s which are set up in the core when the flux linking the core changes. The eddy currents may be regarded as secondary currents flowing in the core itself, and drawing power from the a.c. supply in exactly the same way as the secondary current in a transformer. The total current required to produce a given flux is therefore not only dependent upon that required to magnetize the material and supply hysteresis losses, but also the current required to overcome the opposing field set up by eddy currents ; these are minimized in practice by making the core of thin laminations, thereby increasing the electrical resistance.

1.2 Inductance

In the circuit shown in *Figure* 1.3 an e.m.f. E volts is applied to a circuit with resistance R_a ohms in series with a coil (a)

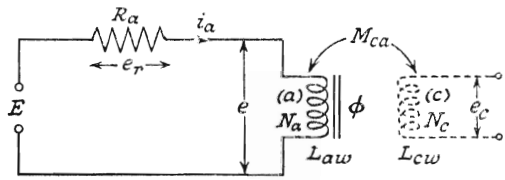


Figure 1.3. Circuit containing inductance and resistance.

* It is often convenient to think of a magnetic circuit as similar to an electrical circuit where the reluctance corresponds to the resistance and the quantities m.m.f. and flux correspond to the equivalent electrical quantities e.m.f. and current. Equation 1.5 may therefore be considered analogous to ohm's law. Thus a flux ϕ is produced in a magnetic " conductor " of reluctance S by applying an m.m.f. F.

having N_a turns. If the instantaneous current in the coil is i_a amperes, the m.m.f. $N_a i_a$ gives rise to a magnetic flux ϕ.

If this flux varies due to a variation of the currents, an e.m.f. is induced in the coil, according to equation 1.3, which will oppose the change of current. This property is known as self-inductance and its magnitude is defined as the e.m.f. induced in the coil due to unit rate-of-change of current in the coil.

If the flux ϕ varies, an e.m.f. of magnitude

$$e = N_a \frac{d\phi}{dt} \text{ volts}$$

is induced in the coil.

Similarly a voltage $e_r = R_a i_a$ volts appears across the resistance, so that the total effective e.m.f. in the circuit is equal to

$$R_a i_a + N_a \frac{d\phi}{dt}$$

which may be equated to the applied e.m.f. E, thus

$$E = R_a i_a + N_a \frac{d\phi}{dt} \quad . \quad . \quad . \quad (1.6)$$

$$= R_a i_a + N_a \frac{d\phi}{di_a} \cdot \frac{di_a}{dt} \quad . \quad . \quad . \quad (1.7)$$

Since $F = N_a i_a$

$$E = R_a i_a + N_a^2 \frac{d\phi}{dF} \cdot \frac{di_a}{dt} \quad . \quad . \quad . \quad (1.8)$$

Equation (1.8) may be written

$$E = R_a i_a + L_{aw} \frac{di_a}{dt} \quad . \quad . \quad . \quad (1.9)$$

where the quantity L_{aw} is the self-inductance of the coil measured in henrys, and is given by

$$L_{aw} = N_a^2 \frac{d\phi}{dF}.$$

If the magnetization curve is linear between the origin and the knee, then when operating below saturation we have :

$$\frac{d\phi}{dF} = \frac{\phi}{F} = \frac{1}{S}.$$

Therefore

$$L_{aw} = \frac{N_a^2}{S} \quad . \quad . \quad . \quad (1.10)$$

If a proportion κ of the flux produced by the coil (a) also links

another coil (c) having N_c turns, then the e.m.f. induced in (c) due to this coupling will be

$$N_c \frac{d(\kappa\phi)}{dt}$$
$$= \kappa N_c N_a \frac{d\phi}{dF} \cdot \frac{di_a}{dt}.$$

The constant κ is called the coefficient of coupling. If all the flux produced by one coil links the other then $\kappa = 1$. Usually there is some leakage flux, but for coils wound on high permeability closed iron cores κ is usually very nearly equal to unity.

The e.m.f. induced in the coil (c) may be written $M_{ca} \frac{di_a}{dt}$, where M_{ca} is the mutual inductance between the two coils and is equal to the e.m.f. induced in (c) due to unit rate-of-change of current in (a).

By writing down the expressions for M_{ca}, M_{ac}, L_{aw} and L_{cw} it is easily seen that :

$$M_{ca} = M_{ac} = \kappa \sqrt{L_{aw} L_{cw}}.$$

If the coupling coefficient is unity and the magnetization curve is linear below the knee then under these conditions of operation

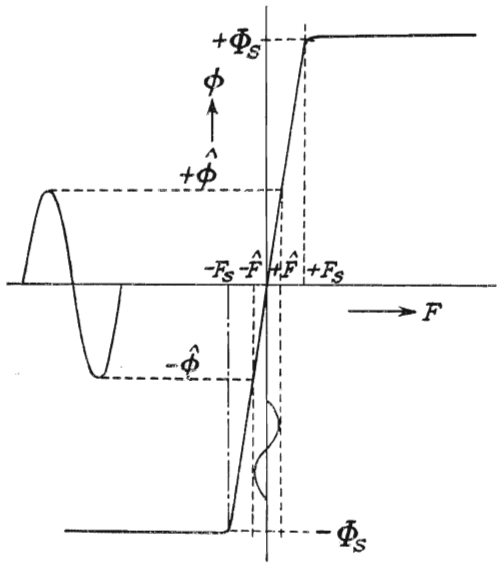


Figure 1.4. Flux and m.m.f. relations for core in *Figure* 1.2.

$$M_{ca} = \frac{N_c N_a}{S} \text{ (cf. equation 1.10) . . (1.11)}$$

Assuming the slope of the ϕ–F magnetization curve is constant for all values of m.m.f. then the self and mutual inductances do not vary with current and the circuits are linear. This will be reasonably true for variations of flux below the knee of the curve (*Figure* 1.4).

In these circumstances equation 1.9 is usually easy to solve

provided the applied voltage E is a known function of time. If the applied voltage varies sinusoidally with time at a frequency f cycles per second and has a peak value $\hat{E}$, then $E = \hat{E} \sin \omega t$ (where $\omega = 2\pi f$). The solution of 1.9 then reveals that the current i_a consists of two components, a transient component, which depends upon the initial condition of the circuit and which ultimately dies away, and a steady-state component. When the transients have disappeared, the current, and therefore the m.m.f. F, varies sinusoidally at the same frequency as the supply voltage but lags behind the voltage by a constant phase angle θ so that

$$i_a = \frac{\hat{E}}{\sqrt{R_a^2 + (\omega L_{aw})^2}} \sin(\omega t - \theta)$$

where $$\theta = \tan^{-1} \frac{\omega L_{aw}}{R_a}.$$

The flux in the core will be proportional to the m.m.f. and will vary between positive and negative peaks $\pm \hat{\phi}$ for variations of m.m.f. between $\pm \hat{F}$ about the origin, as in *Figure* 1.4.

The above results are true provided the coil (c) is open circuited. If, however, a resistance R_c is connected across the terminals of coil (c) then the induced e.m.f. will cause a current i_c to flow which will, in turn, induce an e.m.f. in coil (a) due to the mutual coupling between coils (a) and (c). The circuit equations will therefore be modified and equation 1.9 must be rewritten to take account of the mutually induced e.m.f.s. Thus for circuit (a)

$$E = R_a i_a + L_{aw} \frac{di_a}{dt} + M_{ca} \frac{di_c}{dt} \quad . \quad . \quad . \quad (1.12)$$

For circuit (c) the applied e.m.f. is zero and therefore

$$0 = R_c i_c + L_{cw} \frac{di_c}{dt} + M_{ac} \frac{di_a}{dt} \quad . \quad . \quad . \quad (1.13)$$

In particular, if R_c is zero

$$\frac{di_c}{dt} = -\frac{M_{ac}}{L_{cw}} \frac{di_a}{dt}.$$

Substituting in (1.12)

$$E = R_a i_a + \left(L_{aw} - \frac{M_{ca}^2}{L_{cw}}\right) \frac{di_a}{dt}.$$

The last term in this equation is the induced e.m.f. due to flux

changes in the core. Provided the coupling coefficient is unity this induced e.m.f. is equal to zero and thus, if coil (*c*) is short-circuited, the voltage e across the coil (*a*) is zero and the core flux does not change. This is because as the current i_a varies so also does i_c vary, in such a manner as to oppose flux changes. In this case the full applied voltage is developed across the load, thus

$$E = R_a i_a.$$

1.3 Measurement of Currents and Voltages

The solutions of equations 1.9 and 1.12 give the instantaneous values of i_a at any arbitrarily chosen time. The value measured, however, will depend upon the type of measuring instrument employed. For electrical instruments the deflection is usually proportional either to the mean value of the quantity being measured or to the mean value of its square.

The mean value of a periodic function of time such as $i(t)$ is defined as

$$\frac{1}{T}\int_0^T i(t)dt \qquad . \quad . \quad . \quad (1.14)$$

where T is the period of one cycle and the frequency $f = \frac{1}{T}$ cycles per second.

For any periodically varying quantity this corresponds to the magnitude of the d.c. component. Thus for an alternating quantity containing no such component, but only the frequency f or integral multiples of f, the mean value is zero.

The quantity $i(t)'$ defines the mean square value of $i(t)$ and is given by :

$$\left[i(t)'\right]^2 = \frac{1}{T}\int_0^T \left[i(t)\right]^2 dt \qquad . \quad . \quad . \quad (1.15)$$

and this is proportional to the energy expended in the circuit over one cycle. From energy considerations the most realistic definition for the effective current is the root mean square (r.m.s.) value.

However, many electromagnetic devices, such as polarized relays, motor fields and magnetic amplifiers, respond to mean values. The mean value of an alternating quantity with no direct component is zero, but if it is rectified then the mean value of the resulting output is often more relevant than the r.m.s. value.

Let the *mean rectified* value of a periodic function $i(t)$ be the quantity measured after *full-wave* rectification on a mean d.c. measuring instrument and be denoted by $\bar{i}(t)$. This may be defined by integrating $i(t)$ over one cycle, always considering its value to be positive, and dividing by the period.

Thus :
$$\bar{i}(t) = \frac{1}{T}\int_0^T |i(t)|dt \quad . \quad . \quad . \quad (1.16)$$

where $i(t)$ is now placed within the modulus sign to indicate rectification.

For the particular case of the wave-function with no d.c. component, $I(t)$ say, since the *true-mean* value (equation 1.14) is zero over one complete cycle, it follows that positive and negative areas under the current—(or voltage)—time curve must be equal and opposite. Hence

$$\int_0^T |I(t)|dt = 2 \times \text{(positive area)}$$

and thus from (1.16), $\bar{I}(t) = \frac{2}{T} \times \text{(positive area)} \quad . \quad . \quad . \quad (1.17)$

In the special case of a sine wave the positive area extends over the period from $t = 0$ to $t = T/2$. Therefore if $I(t) = \hat{I} \sin \omega t$

$$\bar{I}(t) = \frac{2\hat{I}}{T}\int_0^{T/2} \sin \omega t \, dt = \frac{2}{\pi}\hat{I} \quad . \quad . \quad . \quad (1.18)$$

In future for brevity the term mean will be applied to quantities defined by equation 1.16 and will therefore imply the d.c. component after full-wave rectification.

1.4 Addition of Mean Values

When dealing with linear a.c. circuit theory the resultant r.m.s. value of a number of voltages is obtained by applying the rules of normal vectorial addition to these voltages. Thus in the circuit of *Figure* 1.3, assuming linear operation and sinusoidal quantities,

$$(E')^2 = (e')^2 + (e'_r)^2 \quad . \quad . \quad . \quad (1.19)$$

where e' and e'_r, the r.m.s. voltages (defined according to equation 1.15), are 90° out of phase since the resistance of the coil is assumed zero.

An important restriction limits the usefulness of this type of summation. The rules governing vectorial addition are only true for sinusoidal quantities. If we concentrate on measured values, however, we find that it is still possible to obtain rather similar and very useful relationships for non-sinusoidal periodic functions, even in non-linear circuits.

Even if the circuit in *Figure* 1.3 is non-linear, the following relation may be expressed between instantaneous values

$$E = e_r + e \qquad . \quad . \quad . \quad (1.20)$$

In addition, if e and e_r are always of the same sign then we may also write

$$\begin{aligned} |e| + |e_r| &= |e + e_r| \\ &= |E| \end{aligned} \qquad . \quad . \quad . \quad (1.21)$$

Integrating between 0 and T and dividing by T

$$\frac{1}{T}\int_0^T |e|dt + \frac{1}{T}\int_0^T |e_r|dt = \frac{1}{T}\int_0^T |E|dt$$

and it follows from equation 1.16 that

$$\bar{e}_r + \bar{e} = \bar{E} \qquad . \quad . \quad . \quad (1.22)$$

Mean values can be measured comparatively easily and therefore, provided the instantaneous values of both e and e_r always have the same sign (which may admit of one or both being zero), the above equation may be applied to any circuit and it shows that the measured mean values when added give the measured mean value of the resultant. This is particularly useful when dealing with magnetic amplifiers, as will be seen later, because although the waveforms are markedly non-linear this relationship is often applicable.

1.5 Mean Voltage and Flux Changes

When a voltage is applied to the terminals of a coil we have seen that the resulting current variations cause a flux change which induces an e.m.f. in the coil equal and opposite to the applied e.m.f. The total flux change in the core during a given time interval is proportional to the area under the curve of applied voltage against time. This follows directly from equation 1.3 by integration, thus

$$\frac{1}{N_a}\int_{t_m}^{t_n} e\,dt = \phi_n - \phi_m.$$

Consider the waveforms of voltage and flux shown in *Figure* 1.5. If no d.c. component is present in the voltage e, then the positive and negative areas X and Y under the voltage-time curve must be equal. Thus the total increase of flux $2\hat{\phi}$ must be equal to the

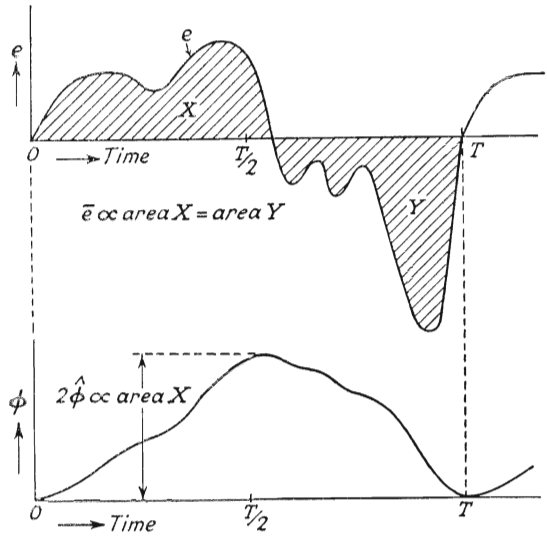


Figure 1.5. Diagram showing relationship between flux changes and voltage-time areas.

total decrease of flux. Now assuming that there is only one positive area per cycle (i.e. that the voltage changes sign only twice per cycle) this positive area will correspond to the total increase of flux. Similarly the equal negative area will correspond to the total decrease of flux. Thus the peak to peak flux $2\hat{\phi}$ will be proportional to the area under the voltage time-curve for the positive part of the cycle, i.e.

flux swing, $$2\hat{\phi} = \frac{1}{N_a} \times \text{(positive area)}.$$

Also from (1.17) we have :

$$\bar{e} = \frac{2}{T} . \text{(positive area)}$$

and therefore $$2\hat{\phi} = \frac{T}{2N_a} \bar{e}$$

or since $$T = \frac{1}{f}$$

$$2\hat{\phi} = \frac{\bar{e}}{2fN_a} \quad . \quad . \quad . \quad (1.23)$$

This expression for the flux change is valid for all alternating voltages provided that there is no d.c. component and that the sign changes only twice per cycle.

Since most practical cases meet these requirements, the above relationship between mean voltage and flux change is a very important one.

It is equally important to note the converse of the above remarks, viz., if, over the complete cycle, there is no resultant change of flux linking the coil (a) then there can be no d.c. components present in the voltage e.

If the flux at any instant t is denoted by $\phi(t)$, then if $\phi(T) - \phi(0) = 0$ we have

$$\int_0^T e\,dt = 0 \quad . \quad . \quad . \quad (1.24)$$

i.e. there is no d.c. component in the voltage e.

1.6 Energy Storage in a Magnetic Core

The energy required to establish a magnetic field is normally regarded as stored in the field, the energy being released when the magnitude of the field returns to zero.

The magnetic flux ϕ is a function of the field, and thus a given flux level in a core is associated with a certain stored energy.

This may be considered in relation to the coil in *Figure* 1.3 as follows :

the e.m.f. applied to the coil is $e = N_a \frac{d\phi}{dt}$

and the current in the coil is $i_a = F/N_a$.

Therefore the instantaneous power $= e\,i_a = F\frac{d\phi}{dt}$.

Also $F = S\phi$

so that $e\,i_a = S\phi\frac{d\phi}{dt}$

Therefore provided S is constant the energy stored over a time interval t_m to t_n

$$= \int_{t_m}^{t_n} e\, i_a dt = \frac{S}{2}(\phi^2{}_n - \phi^2{}_m) \text{ joules} \quad . \quad . \quad . \quad (1.25)$$

Since $S = \dfrac{l}{a\mu}$ this shows that for given values of ϕ_n and ϕ_m the energy storage in a core of given dimensions is inversely proportional to the permeability and is zero in the case of infinite permeability assuming linearity and zero loop area.

1.7 Summary of Useful Relationships

(1) True mean value (d.c. component) of any periodic quantity $i(t)$ of fundamental frequency $f = \dfrac{1}{T}$.	$\dfrac{1}{T}\int_0^T i(t)dt$
(2) In particular, true mean value of any periodic quantity $I(t)$ containing no d.c. component.	$\dfrac{1}{T}\int_0^T I(t)dt = 0$
(3) Mean (rectified) value of $i(t)$ . . .	$\bar{i}(t) = \dfrac{1}{T}\int_0^T \lvert i(t)\rvert dt$
(4) In particular, mean (rectified) value of $I(t)$	$\bar{I}(t) = \dfrac{1}{T}\int_0^T \lvert I(t)\rvert dt$ $= \dfrac{2}{T}$ (positive area)
(5) If the sum of the instantaneous voltages e and e_r is always equal to E, then provided e and e_r never have opposite signs	$\bar{e} + \bar{e}_r = \bar{E}$
(6) For a coil of N_a turns, to which is applied an alternating voltage e, of fundamental frequency f, the mean value of e is related to the peak to peak flux swing $2\hat{\phi}$ thus :—	$2\hat{\phi} = \dfrac{\bar{e}}{2fN_a}$. Provided there is no d.c. component in e and the sign of e only changes twice per cycle.

CHAPTER 2

THE TRANSDUCTOR

2.1 Amplifying Devices

In addition to the familiar applications to radio, television and public address systems, other requirements for reliable and effective amplifying devices have been steadily increasing for many years. For example, the manual control of industrial plant by human operators is now giving way to automatic systems which in general ensure more effective and accurate control. Such systems almost invariably involve the amplification of small mechanical or electrical signals from devices such as strain gauges, thermocouples and photo-cells.

In the Defence Services, automatic control or servo systems are vitally important and in many applications such as automatic pilots, guided rockets, control of gun turrets, some kind of amplifying device is essential. Again, the solution of important and highly complex mathematical problems has been greatly facilitated by the use of analogue computers, the basic elements of which are amplifiers ; a large computer may contain several hundred of these.

In view of all these requirements and many others it is not surprising that a considerable amount of research has been devoted to the development of amplifying techniques.

In general an amplifier is a device which controls some output quantity at a comparatively high power level according to the variations of a signal at a lower power level. The characteristics generally required are that the output/signal ratio should always be constant within the range of operation and that this ratio, the gain, should be adequate. This of course implies that the amplifier shall respond immediately to changes of input and that it shall not generate parasitic signals within itself. Time delays, zero drift

and non-linearity are always present in some degree, however, and these effects combine to make practical amplifiers fall short of the ideal. Characteristics such as these, which determine whether a certain type of amplifier is capable of a given task, may be termed primary characteristics. Other considerations of nearly equal importance, however, are reliability, cost, and ease of maintenance, and very often the engineer finds it necessary to make a compromise between the economics and the basic requirements.

2.2 Transductor Amplifiers

Transductors * and magnetic amplifiers are low impedance devices, in which amplification is achieved by utilizing the non-linear properties of ferro-magnetic materials. The transductor is the basic amplifying element in a magnetic amplifier just as the valve is the amplifying element in an electronic amplifier. It is an amplifier to which the input signal is a voltage applied to a coil linking an iron-cored system. Other coils linking the system carry the load current which is usually supplied from a low impedance source of alternating e.m.f. Variation of the signal current changes the flux levels in the core and modifies the operating conditions in the load circuit so that the load current is modulated according to the variations of the signal. By the use of special techniques to be described later, the power required to polarize the core can be made very small compared with the power controlled in the load, so that the device may be used as a power amplifier. Transductors are usually considered to be complementary to valves. They are normally employed in magnetic amplifiers, but are sometimes used with valves either as low impedance output stages or as a.c.–d.c. converters to precede electronic amplifiers where low zero drift is required (see Chapter 12). In many respects the properties of transductors closely resemble those of rotary amplifiers such as the metadyne and amplidyne. The time constant of magnetic amplifiers is usually much longer than that of valves, seldom being less than 1 millisecond and often as high as 1 second.

Transductors and magnetic amplifiers are now frequently used in industrial and service equipment. Their electrical characteristics are probably not unique and in general an electronic

* Transductors should not be confused with transducers. The latter is a general term for a device which converts one form of energy into another, such as a microphone which converts acoustic to electrical energy.

valve, relay or rotary amplifier may provide an alternative. Often, however, the magnetic amplifier is preferred on account of its greater robustness and reliability and lower heat dissipation.

The basic circuit elements employed in magnetic amplifiers are non-linear inductors and rectifiers, and since the mathematical relations for such circuits are non-linear they nearly always defy solution. A rigorous approach to the problem has therefore generally been abandoned in favour of less rigorous methods, based on specific assumptions regarding the circuit or the magnetic core. Although such treatments give only approximate results, nevertheless the salient features are usually brought out sufficiently to enable the engineer to give a reliable physical interpretation of the facts.

2.3 Basic Operation of the Transductor

In Chapter 1 we considered the behaviour of the circuit in *Figure* 1.3 for variations of m.m.f. and flux confined to the high permeability (linear) region of the core characteristic. When the circuit was excited by a sinusoidal e.m.f. it was shown that the alternating current i_a produced an alternating m.m.f. which varied symmetrically about the zero line as shown in *Figure* 1.4.

If by any means an additional direct m.m.f. is applied to the core the total m.m.f. will no longer vary symmetrically but will be displaced by a constant amount depending upon the magnitude and direction of the direct m.m.f. If this is sufficiently large, then the total ampere-turns will produce a flux which saturates the core during a part of the cycle of alternating current. This will cause a considerable reduction in the circuit inductance during this part of the cycle, thereby leading to a non-linear mode of operation which will probably be incapable of a general solution.

At times when the inductance ($\propto d\phi/dF$) is large, the voltage developed across the coil will be a large fraction of the supply voltage and only a small e.m.f. will be available to drive current through the circuit resistance ; the current i_a will therefore be small.

When the inductance is small, however, most of the supply voltage will appear across the resistor and the current i_a will be correspondingly large provided the resistance is small compared with the unsaturated reactance. The precise nature of the current-time variation will be fully considered later. Meanwhile, suffice it to say that if the " knee " of the magnetization curve is sharp,

a sudden decrease in the value of the inductance will occur every other half-cycle. This will modify the instantaneous values of the current i_a, which will no longer remain sinusoidal but will tend to assume two distinct ranges of values within one cycle, viz., those corresponding to a high inductance and those corresponding to a low inductance coil.

The relative duration of these two conditions will depend upon the magnitude of the applied d.c. field, which will therefore govern the mean value of the alternating current.

A principle of this kind is employed in various types of flux-gate magnetometer [2(a)], in which a polarizing field **H** is measured by the effect it produces in an electrical circuit containing an iron-cored coil (see Chapter 12).

In a transductor the polarizing field is produced by applying the signal voltage to an additional winding (*c*) (*Figure* 1.3) linking the core. The load current will then correspond to that flowing in R_a. The signal winding is identified by the suffix *c* and will in future be called the control winding. It is connected to the signal source. The other windings and associated circuit are denoted by the suffix *a* ; they will be called the a.c. windings. The resistance R_a is in series with the latter windings and a source of alternating e.m.f. E_a of frequency f.

With the above arrangement, however, the two coils are mutually coupled and an alternating voltage of frequency f will be induced in the signal circuit. Thus interaction will occur between the two circuits, so disturbing the amplifying properties. If, on the other hand, the coefficient of coupling is arranged to be small, then a large signal current would be required to polarize the core effectively, and the power level at the input would be unreasonably high. This would be contrary to the requirement that the device should behave as a power amplifier.

When using the above arrangement, saturation only occurs once every cycle and this therefore forms the basis of a half-wave transductor element. When used in conjunction with rectifiers it can be made to give a useful gain (Chapter 9) ; otherwise the circuit is of little value.

The coupling effects described above can largely be overcome by the use of twin windings in each circuit, those in the load circuit being in series opposition (see *Figure* 2.1). This arrangement forms the full-wave transductor and eliminates most of the interaction. Provided the mutual inductances M_A and M_B are

equal then, at such times, no a.c. components can be induced in the control circuit. When a control voltage E_c is applied the desired polarizing effect is still achieved in each core, and it will be shown that under these conditions the only voltages induced in the control circuit are even harmonics of the supply frequency.

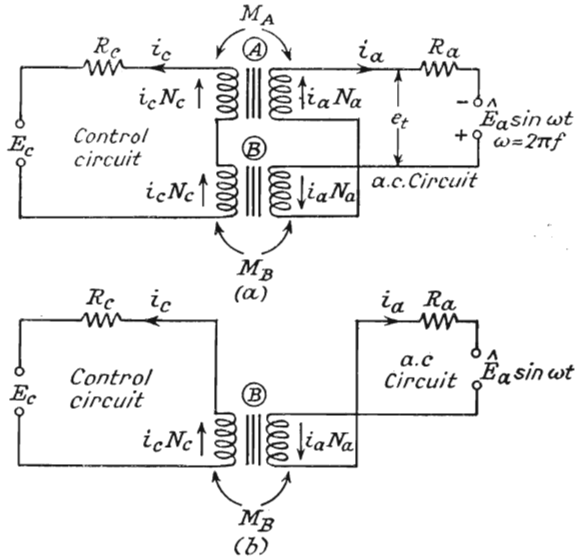


Figure 2.1*a*. Circuit for simple transductor (Positive Half-Cycle).

Figure 2.1*b*. Equivalent circuit of simple transductor after saturation of core *A*.

In practice there are a number of ways of arranging the windings and various core systems exist. All achieve the same cancelling effect, however, and the reason for employing a particular method is often little more than one of convenience. The various methods of achieving this effect and their respective merits will be discussed in the succeeding chapters. A simple physical interpretation of transductor action is now given for the circuit of *Figure* 2.1*a*. The relevant waveforms are shown in *Figure* 2.2. They imply a particular set of conditions, which will emerge as we proceed. The magnetization curves of the two similar cores are assumed to be of the form shown in *Figure* 1.4, and the supply voltage E_a in the a.c. circuit to have been adjusted so that for no signal input ($E_c = 0$) the alternating fluxes ϕ_A and ϕ_B in cores (*A*) and (*B*) have a peak value which just fails to saturate either core.

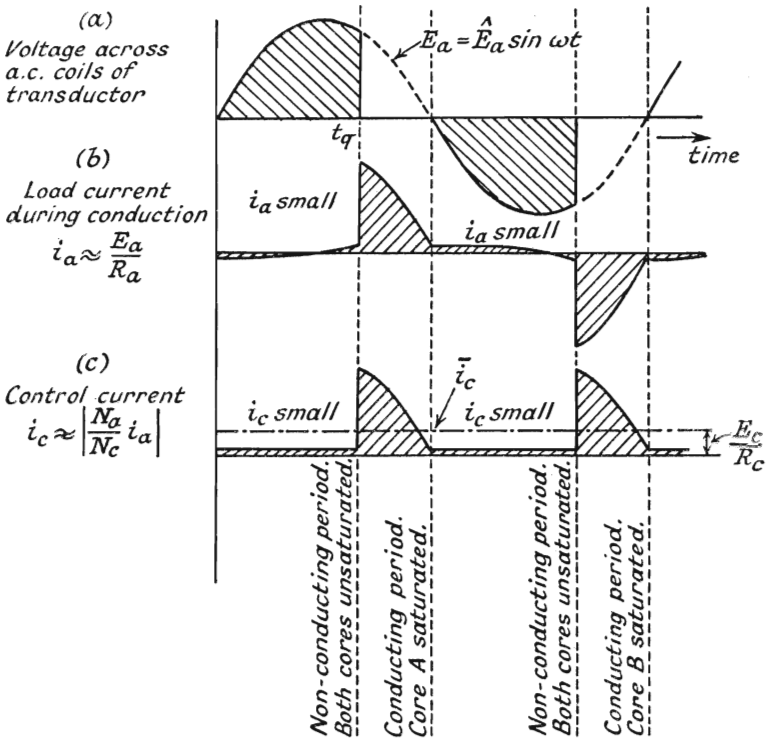


Figure 2.2. Waveforms of current and voltage for full-wave transductor with low control circuit impedance. (Curves applicable to case where $N_c = N_a$.)

If a direct control voltage E_c is applied, a control current must flow and this will produce equal m.m.f.s $i_c N_c$ in each core.

The total m.m.f. magnetizing a core is

$$F = \Sigma \text{ (ampere-turns)}$$

For core A (*Figure* 2.1*a*) $F_A = i_c N_c + i_a N_a$ . . . . (2.1)

and for core B $F_B = i_c N_c - i_a N_a$ . . . . (2.2)

With the addition of the m.m.f. due to the control current there comes a time during the positive half-cycle when the total m.m.f. in core A has increased sufficiently to saturate it. The instant of saturation t_q is called the firing time.

Before saturation, the total reactance in the a.c. circuit is always large compared with the resistance R_a. Thus the current through the resistance is very small and lags behind the applied voltage by nearly 90°.

Now during unsaturated periods the two-core m.m.f.s F_A and F_B are both less than the small value F_s. It follows from equations (2.1) and (2.2) that the control current i_c must also be small. Furthermore during these periods all voltages induced from the a.c. circuit must cancel and hence the instantaneous control current depends only upon the control voltage. During unsaturated periods the control current is considerably less than E_c/R_c, due to the switching action caused by saturation and the large unsaturated inductance of the control windings. The full significance of this will become apparent shortly.

When core A saturates, the self-inductances and mutual inductance of the coils on core A become negligible. The control circuit and load circuit are now effectively coupled by the large mutual inductance M_B, since the opposing effect of M_A has now been removed.

When this occurs, the circuit may be considered modified to that shown in *Figure* 2.1*b*. Voltages are mutually induced in both circuits due to the coupling through core B. The control current and load current in general increase suddenly and assume a new range of values consistent with the changed circuit conditions. This is termed the conducting period.

Immediately upon saturation of core A the coils on core B behave like a transformer with a short-circuited coil N_c (assuming R_c is very small). This is because any change of current in the a.c. circuit is accompanied by an equal and opposite change in the control circuit in such a direction as to prevent any further flux changes in core B (see section 1.2). Thus no voltage appears across either the coil N_a or across N_c. The whole supply voltage appears across the circuit resistance R_a and the load current i_a will therefore be limited only by this resistance so that

$$i_a = \frac{\hat{E}_a}{R_a} \sin \omega t \qquad . \quad . \quad . \quad (2.3)$$

Since the total impedance during saturation is much smaller than the unsaturated reactance of the a.c. windings, the current i_a is much greater during the conducting period than when both cores are unsaturated.

Now core B has not saturated and from the previous paragraphs no further flux change can occur in this core. But since the load current has increased to a large value the control current must also have increased, in such a way that the total m.m.f. in core B

remains almost constant at a value less than F_s. This small value represents the difference between the large opposing m.m.f.s i_aN_a and i_cN_c. It follows that the control ampere-turns will be approximately equal to the load ampere-turns.

Thus we see that although core B is unsaturated during conduction it is not able to limit the load current which suddenly increases at the beginning of the conducting period, while a similar increase in the control current takes place to ensure no flux change. This in fact follows directly from Lenz' Law.*

The conducting period lasts until the m.m.f. in core A becomes sufficiently small to desaturate this core.

Since $F_A = i_cN_c + i_aN_a$

and $$i_aN_a \doteq i_cN_c$$

then $$F_A \doteq 2i_aN_a = \frac{2\hat{E}_aN_a}{R_a} \sin \omega t \text{ (from equation 2.3).}$$

Therefore core A will become unsaturated when $\sin \omega t$ approaches zero, which is at the end of the positive half-cycle of supply voltage.

Although it is now evident that the control current is not constant during the cycle, the control voltage is however. We are assuming steady-state behaviour with consecutive cycles similar, and so there can be no net flux change in the cores over one cycle.

It follows from equation 1.24 that the mean level of the voltages induced in the control circuit is zero even though their instantaneous values evidently are not. Therefore the mean voltage in the control circuit can only be due to the control voltage, and the constant current E_c/R_c must be the mean control current $\bar{i}_c$, but not the instantaneous control current. As we have seen, this current is nearly zero during unsaturated periods but corresponds with part of a sine wave during conduction (*Figure* 2.2c). The time of saturation t_q must therefore adjust itself via the core fluxes until the mean value of the control current is equal to E_c/R_c. This is considered further in section 3.13.

During the negative half-cycle of the alternating current, core B saturates. Equation 2.1 now applies *numerically* to core B and equation 2.2 numerically to core A.

* When an e.m.f. is induced in a conductor due to a change of flux linking the conductor the direction of the resulting induced current is such as to oppose the change of flux. In the present case the flux change is caused by i_a, the induced current being i_c.

Therefore the m.m.f. in the unsaturated core is always equal to

$$i_c N_c - |i_a N_a| \quad . \quad . \quad . \quad (2.4)$$

When core B saturates, exactly the same sequence of events occurs excepting that coupling takes place through M_A.

If the permeability of the cores is very large indeed and losses are neglected, then the condition that a core is unsaturated must imply a very small m.m.f. therein so that, by 2.4,

$$i_c N_c \doteq |i_a N_a| \text{ during saturation} \quad . \quad . \quad . \quad (2.5)$$

To summarize, during the period of one cycle, when neither core is saturated both the load current and the control current are relatively small and practically the whole supply voltage is developed across the transductor. When any one core saturates nearly the whole supply voltage is developed across the load ; simultaneously the load and control currents suddenly increase so that :

$$i_a = \frac{\hat{E}_a}{R_a} \sin \omega t$$

and

$$i_c N_c \doteq |i_a N_a|.$$

Referring to *Figure* 2.2 it will be seen that if the firing angle moves to the left, the mean load current is increased. Maximum current is delivered to the load when the firing angle occurs at the beginning of each half-cycle. This is discussed more fully in the following chapters.

Perhaps the most significant feature brought out by the above consideration is the way in which current is drawn from the control source.

It is seen that the control current is not steady but pulsating at twice the supply frequency.

By integrating both sides of 2.5 between 0 and T and dividing by $\frac{1}{T}$

$$N_c \times \frac{1}{T}\int_0^T |i_c| dt = \bar{i}_c N_c = N_a \times \frac{1}{T}\int_0^T |i_a| dt$$

therefore,

$$\bar{i}_a = \frac{N_c}{N_a}\bar{i}_c = \frac{N_c}{N_a}\frac{E_c}{R_c} \quad . \quad . \quad . \quad (2.6)$$

This gives the current gain as

$$\frac{\bar{i}_a}{\bar{i}_c} = \frac{N_c}{N_a} \text{ (neglecting the magnetizing current).} \quad . \quad (2.7)$$

In practice the permeability of the core is not infinite and the m.m.f. in the unsaturated core is small but finite. Therefore the rectified load ampere-turns $|i_a N_a|$ do not quite balance the control ampere-turns.

However, the difference is very small and in the circuit of *Figure* 2.1 the load current is determined almost exclusively by the control current and the turns ratio. Thus, although the a.c. supply is derived from a low impedance source the load current is constrained by the transductor to be independent of all parameters, other than the control current, e.g. load resistance and a.c. supply voltage. This, of course, is a desirable feature since it implies that the output current is effectively constant even though the supply voltage, for example, may be varying.

On the other hand it is apparent that the control voltage must be large enough to supply a mean control current, E_c/R_c, which will balance the load ampere-turns and not merely one which will produce a polarizing flux in the cores. For these reasons the voltage, current and power gains of this circuit are generally low.

In Chapter 4 it will be shown that the system may be considered in the same light as an amplifier having external negative feedback. The negative feedback of current in this case gives rise to the characteristic features of linearity, high output impedance and low gain.

2.4 Characteristics of the Transductor

The three variables which are usually considered to define the steady-state operation of transductors are the mean transductor voltage across the a.c. windings $\bar{e}_t$, the mean current $\bar{\imath}_a$ in the a.c. windings and external circuit and the mean control current $\bar{\imath}_c$ (see *Figure* 2.1*a*).

A family of curves relating these three variables is normally plotted for the no-load condition (in which case the a.c. circuit resistance is very small, consisting only of the internal resistance r_a of the windings). These curves are of the form shown in *Figures* 2.3 *a* and *b* in which the mean transductor voltage $\bar{e}_t$ is plotted against the mean load current $\bar{\imath}_a$ for various values of mean control current $\bar{\imath}_c$. *Figure* 2.3*a* is an idealized characteristic based on ideal cores, and *Figure* 2.3*b* is a practical characteristic.

It was shown in section 1.5 that, for the waveforms with which we are concerned, the mean voltage across an a.c. coil is proportional to the total flux swing in the corresponding core. Further-

more, the flux cannot change by an amount greater than $2\Phi_s$ since $\pm \Phi_s$ correspond to the positive and negative saturation values.

Thus considering the curve for zero control current ($i_c = 0$ in *Figure* 2.3*a*), when the mean transductor voltage $\bar{e}_t$ reaches a certain value, $\bar{e}_{ts}$, the flux is varying over its full range and so $\bar{e}_{ts}$ is

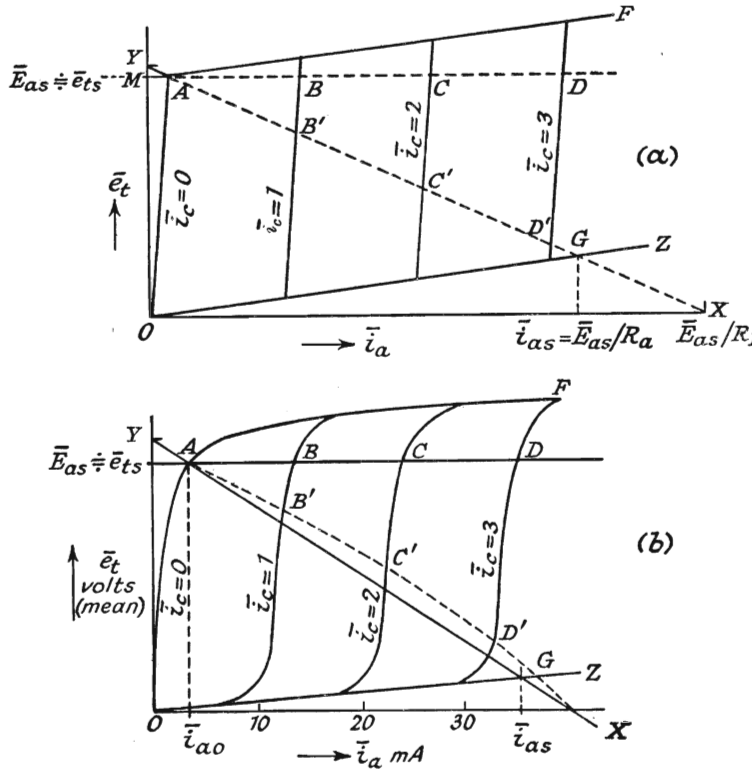


Figure 2.3*a*. Idealized voltage-current characteristics.

Figure 2.3*b*. Voltage current curves for practical transductor having turns ratio $\frac{N_c}{N_a} = 10$.

the maximum voltage which can be supported across the transductor. If the supply voltage is derived from a low impedance source and is increased beyond this critical value, then, since the voltage supported by flux changes can increase no further, the excess voltage will be supported solely by the resistance of the windings and the current will increase rapidly with respect to voltage along the line AF (*Figure* 2.3*a*).

In the practical case of *Figure* 2.3*b* the value of $\bar{e}_{ts}$ must be chosen on the basis of the shape of the characteristic in relation to the particular application (Chapter 13).

The characteristics for different values of control current are also shown. These are displaced to the right by an amount depending upon the magnitude of the control current. As discussed in section 2.3, the mean load current is very nearly constant over the working range and is determined only by the control current.

The useful range of the transductor lies between the line *ABCD* and the line *OZ*, the slope of which corresponds to the resistance of the windings in the a.c. circuit. The horizontal extent of the range is limited by the temperature rise in the a.c. windings while the vertical height of *ABCD* is governed as we have shown by the maximum voltage which can be applied to the windings without causing saturation.

The line passing through *ABCD* defines the circuit when operating with a constant transductor voltage and control currents equal to 0, 1, 2 and 3 milliamps respectively.

If a load is present in series with the a.c. windings a load line can be drawn for a fixed supply voltage. The load line *YX* (*Figure* 2.3*a*) corresponds to a supply voltage *OY*. In practice *OY* is approximately equal to *OM* and therefore the appropriate supply voltage $\bar{E}_{as}$ is approximately given by $\bar{e}_{ts}$. The transductor voltage cannot now remain constant as the current $\bar{\imath}_a$ increases, due to the voltage drop in the load R_L. In the practical case (*Figure* 2.3*b*) the conditions are defined by the dotted line *AB'C'D'*, which corresponds to the load line.

For zero control current practically the whole supply voltage appears across the transductor, and the load current $\bar{\imath}_a$ under these conditions is ideally very small. As the control current is increased, the mean load current also increases and therefore the voltage developed across the load rises while that across the transductor falls, until eventually the whole supply voltage is available to drive current through the circuit resistance (point *G* on *Figures* 2.3 *a* and *b*). The mean load current cannot therefore increase any further and this limiting value, denoted by $\bar{\imath}_{as}$, is given by

$$\bar{\imath}_{as} = \frac{\bar{E}_{as}}{R_a} \qquad . \quad . \quad . \quad (2.8)$$

where $R_a = R_L + r_x$ and r_x is the winding resistance.

For a fixed supply voltage and a fixed circuit resistance the

straight line XY defines the range of the transductor, where the point Y corresponds to the value of the supply voltage and the point X is the maximum possible current assuming zero winding resistance. The slope of XY is equal to $-R_L$ and it is shown in Chapter 3 that, in practice, this line is very nearly coincident with the load line and may be taken as being so for practical purposes.

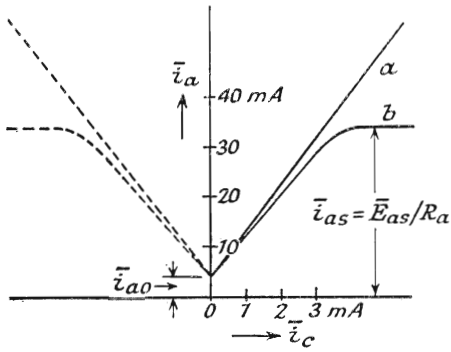


Figure 2.4. Control current characteristic for two load conditions of *Figure* 2.3*b*.
a. No load ($ABCD$).
b. Load corresponding to line $AB'C'D'$.

The characteristic for a given load may be plotted as in *Figure* 2.4, in which the mean load current $\bar{i}_a$ is plotted against the mean control current $\bar{i}_c$ for the two load conditions of *Figure* 2.3. The slope of this curve is the current gain, which has been shown to be $\dfrac{N_c}{N_a}$, and the maximum value of the load current is $\bar{E}_{as}/R_a$ while the minimum value is $\bar{i}_{a0}$.

2.5 Control Ratio

In order to obtain an adequate range of control, the maximum load current should be large and the minimum should be small. If the load resistance is constant, then $\bar{i}_{as}$ may be increased by increasing the supply voltage. If the mean supply voltage is increased beyond that which can be supported by the transductor, however, then the minimum current $\bar{i}_{a0}$ would also suffer a considerable increase, due to saturation of the transductor. This would be undesirable for three reasons. Firstly, the control range would be decreased due to the large increase of the standing current. Secondly, the transductor would be outside the operating range for small values of $\bar{i}_c$. Thirdly, a large initial load current would lead to problems of zero drift.

The control ratio m is often used as a criterion of the range of control and this is defined by

$$m = \frac{\bar{i}_{as}}{\bar{i}_{a0}} = \frac{\bar{E}_{as}/R_a}{\bar{E}_{as}/2X_{aw}} \text{ approximately}$$

where $2X_{aw}$ is the reactance of the a.c. windings in the unsaturated state. Thus the control ratio is approximately given by

$$m = \frac{2X_{aw}}{R_a} \quad . \quad . \quad . \quad (2.9)$$

provided that iron losses are negligible and $X_{aw} \gg R_a$.

2.6 Supply Voltage

The maximum permissible supply voltage has already been shown to be approximately equal to $\bar{e}_{ts}$ and it may therefore be determined from equation 1.23.

Since the total flux swing is $2\Phi_s$, and in this case there are two coils in series,

$$\bar{E}_{as} = \bar{e}_{ts} = 4fN_a \times 2\Phi_s = 8fN_a a\mathbf{B}_s \quad . \quad . \quad . \quad (2.10)$$
(see equation 1.2)

In practice saturation does not occur sharply and the appropriate value for $\mathbf{B}_s$, which determines the supply voltage, is not definite. Some fraction of the ultimate saturation flux density is usually chosen for design purposes. This fraction varies somewhat from one magnetic material to another but it is usually about 0·7 of the ultimate saturation density.

2.7 Power Output

Transductor power amplification and output are defined in terms of rectified mean values of currents or voltages. This is primarily because interest is generally centred on electromagnetic loads. Thus, considering a smooth direct current i_{dc} flowing in any coil, the power dissipated is $R(i_{dc})^2$. If one now considers a rectified alternating current flowing in a similar coil, the same electromagnetic effect is produced if the mean value of the rectified current is equal to i_{dc} and therefore the equivalent expression for the useful "power" output is given by $R(\bar{i})^2$. Strictly speaking it is agreed that this does not specify the actual power dissipated and possibly a preferable term would be mean volt-amperes. Nevertheless, the term power or mean power is now generally employed in transductor terminology to indicate the quantity $R(\bar{i})^2$ and it will be used here. It should also be noted that the input/output relationships are only linear when plotted in terms of mean values.

The transductor delivers full power to the load when $\bar{i}_a = \bar{i}_{as}$ and the maximum power which can be dissipated in the load

resistance R_L is therefore $\frac{\bar{E}_{as}^2}{R_a^2} R_L$. If the load resistance were the only resistance in the circuit, then R_a would be equal to R_L and the output efficiency would be 100%. However, the windings also dissipate some of the power and the efficiency of the a.c. circuit is some factor less than unity (section 2.12).

For a given transductor, maximum power output is delivered when the load is matched to the internal resistance of the transductor, in which case $R_L = \frac{R_a}{2}$ and the maximum output is $\frac{\bar{E}_{as}^2}{2R_a}$. In general the output may be written in the form

$$P = \frac{\bar{E}_{as}^2}{R_a} \eta \qquad . \quad . \quad . \quad (2.11)$$

where η is equal to R_L/R_a and $\frac{\bar{E}_{as}^2}{R_a}$ is the total mean power dissipated in the a.c. circuit. By combining this with the expression for the supply voltage (equation 2.10) and writing $\Phi_s = a\mathbf{B}_s$

$$P = \frac{64 f^2 \mathbf{B}_s^2 a^2 N_a^2}{R_a} \eta \qquad . \quad . \quad . \quad (2.12)$$

This expression will be used in the following section to relate the power output to the power amplification.

2.8 Power Amplification

If the mean load current is equal to $\bar{i}_a$ the mean power gain of a transductor is defined by

$$G_p = \left[\frac{\bar{i}_a}{\bar{i}_c}\right]^2 \frac{R_L}{2r_{cw}} * \qquad . \quad . \quad . \quad (2.13)$$

where r_{cw} is the resistance of each control winding.

The above expression may be termed the useful power gain as opposed to the circuit power gain, which involves the total circuit resistance in the input and output and is equal to

$$G_p' = \left[\frac{\bar{i}_a}{\bar{i}_c}\right]^2 \frac{R_a}{R_c}.$$

Combining 2.13 with the expression for the current gain (equation 2.7)

$$G_p = \frac{N_c^2}{N_a^2} \frac{R_L}{2r_{cw}} \qquad . \quad . \quad . \quad (2.14)$$

* In some cases where the load is reactive and fed through a bridge rectifier, the mean load current $\bar{i}_L$ is not equal to the current in the a.c. coils $\bar{i}_a$. Examples of this occur in Chapter 6.

and also $$G_p = G'_p \frac{R_c}{2r_{cw}} \cdot \eta \qquad . \quad . \quad . \quad (2.15)$$

Let us now determine how the useful gain varies with the other parameters. Firstly, the ratio $\frac{N_c^2}{2r_{cw}}$ (equation 2.14) is primarily a function of core dimensions. This may be shown as follows. It will be assumed that the control winding area available on the core after allowing a space factor for imperfect winding is A_c, the mean length of one turn b_c and the wire section s_c. The wire may be assumed to lie evenly in the area A_c as shown in *Figure* 2.5, and therefore $s_c = \frac{A_c}{N_c} \cdot \frac{\pi}{4}$

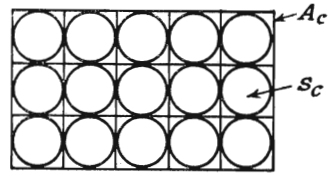


Figure 2.5. Section of an ideal winding. Total area occupied by wire $= \frac{\pi}{4} A_c$.

and the total length of wire on two coils is

$$2N_c b_c.$$

Now $2r_{cw} = \dfrac{\text{specific resistance} \times \text{total length of wire}}{\text{area of wire section}}$

therefore $$2r_{cw} = \frac{8\rho}{\pi} \cdot \frac{b_c}{A_c} \cdot N_c^2 \qquad . \quad . \quad . \quad (2.16)$$

Combining 2.14 and 2.16

$$G_p = \frac{\pi}{8\rho} \cdot \frac{A_c}{b_c} \cdot \frac{R_L}{N_a^2} \qquad . \quad . \quad . \quad (2.17)$$

or alternatively $$G_p = \frac{\pi}{8\rho} \cdot \frac{A_c}{b_c} \cdot \frac{R_a}{N_a^2} \cdot \eta \qquad . \quad . \quad . \quad (2.18)$$

As in equation 2.16 we have for the two a.c. windings

$$2r_{aw} = \frac{8\rho}{\pi} \frac{b_a}{A_a} \cdot N_a^2.$$

Thus from (2.17) we have

$$G_p = \frac{A_c b_a}{A_a b_c} \cdot \frac{R_L}{r_a}.$$

This shows that, to a first order, the useful power amplification is independent of the number of turns on either winding for a given winding area.

The quantity $\frac{R_a}{N_a^2}$ may be eliminated from equation 2.18 by combining it with the expression for the output (equation 2.12) giving

$$G_p = \left[\frac{8\pi}{\rho} \cdot \frac{A_c a^2}{b_c} \cdot f^2 \mathbf{B}_s^2 \eta^2\right] \cdot \frac{1}{P} \quad . \quad . \quad . \quad (2.19)$$

Thus for a given core and supply frequency the power gain is limited by the power output. The relation between gain and output is approximately a rectangular hyperbola, and for reasonable values of power output the power gain of the simple transductor is fairly small. Some idea of the magnitudes can be obtained by putting numerical values into the above expression. Taking

$$\begin{aligned} A_c &= 2 \times 10^{-4}\text{m}^2 \quad (2 \text{ sq. cm.}) \\ a &= 2 \times 10^{-4}\text{m}^2 \quad (2 \text{ sq. cm.}) \\ b_c &= 25 \times 10^{-2}\text{m} \quad (25 \text{ cm.}) \\ f &= 50 \text{ c.p.s.} \\ \mathbf{B}_s &= 0{\cdot}6 \text{ weber/m}^2 \quad (6{,}000 \text{ gauss}) \\ \eta &= 0{\cdot}8 \\ \rho &= 1{\cdot}7 \times 10^{-8} \text{ ohm.m.} \end{aligned}$$

which gives $G_p = 27/P$.

The size of such a unit would probably be that of a 4-in. cube and for an output of 1 watt at 50 cycles per second the power gain would be only about 27. It is clear, therefore, that the power gain is very limited in the case of the simple transductor so far described.

2.9 Self-Excited Transductor

In section 2.3, it was seen that during those parts of a cycle which contribute effectively to the mean load current, that is when the instantaneous value of the load current i_a is large, an opposing current flows in the control circuit such as to maintain ampere-turn balance between the control and a.c. circuits. This apparently induced current must be drawn from the d.c. source. Thus, although the current required to polarize the cores is only a very small steady current from the input, the actual mean current which must be supplied, in addition, to maintain this ampere-turn balance is very many times greater.

Considerable increase in gain is achieved by using self-excitation, which provides means whereby the rectified load current is made to flow through an additional winding similarly disposed to the control windings, thus providing a form of positive feedback to the input. A typical self-excited transductor circuit is shown in *Figure* 2.6. The full mechanism of self-excitation will be dis-

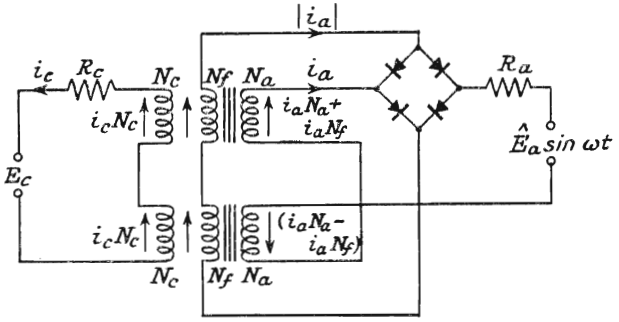


Figure 2.6. Circuit of self-excited transductor.

cussed later, but there are one or two points worth noting at this stage.

Since the control current was called upon to supply a large component necessary to balance the load ampere-turns, a part or all of this can now be supplied by the rectified load current itself, leaving only a small component to be supplied by the control source. The control voltage necessary to obtain a given output is therefore greatly reduced. The total ampere-turns necessary to balance the load ampere-turns are $\bar{i}_a N_a$ and assuming the core permeability to be infinite we showed that these were supplied by the control current thus

$$\bar{i}_c N_c = \bar{i}_a N_a$$

If, however, there are N_f turns on the self-exciting winding in series with the load, then the control m.m.f., now aided by the self-excitation, is reduced to

$$\bar{i}_c N_c = \bar{i}_a N_a - \bar{i}_a N_f \quad . \quad . \quad . \quad (2.20)$$

The current gain for infinite permeability becomes

$$\frac{\delta \bar{i}_a}{\delta \bar{i}_c} = \frac{N_c}{N_a - N_f} \quad . \quad . \quad . \quad (2.21)$$

which tends to infinity as $N_f \rightarrow N_a$. This condition corresponds to 100% self-excitation. In practice the permeability is not

infinite and the gain of a transductor with complete self-excitation is finite. The characteristics of the transductor using 100% self-excitation are very different from those of the transductor without self-excitation. The gain is large and dependent upon the core properties, non-linearities are considerably more marked and the device no longer has constant current characteristics. Furthermore the gain for large negative control currents is very small and the transductor is therefore polarity sensitive. By using the type of circuit shown in *Figure* 2.6 power gains of 10^4 and 10^5 can be obtained in one stage of amplification.

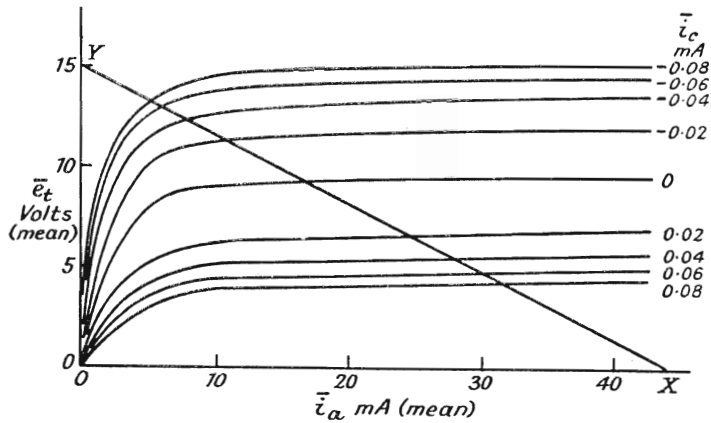


Figure 2.7. Voltage-current characteristics for fully self-excited transductor on no load (load-line XY).

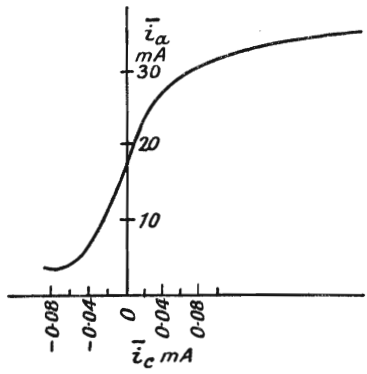


Figure 2.8. Control current characteristic for particular case of load-line XY (*Figure* 2.7).

A typical family of curves equivalent to those in *Figure* 2.3 is given in *Figure* 2.7 for the fully self-excited transductor. The line XY corresponds to a load of 330 ohms, and reading off from the points of intersection, the average current gain from 10 mA. to 30 mA. is approximately 200. Thus for a control circuit having the same resistance as the load circuit the circuit power gain is of the order of 40,000. The control

current characteristic for this load condition is plotted in *Figure* 2.8.

2.10 A.C. Magnetization Curve

Returning to the transductor characteristics in *Figure* 2.3*b* it has been shown that all the curves follow the same general shape over the working range. The particular curve plotted for zero control current is the a.c. magnetization curve. This is similar to the d.c. magnetization curve (**B**/**H** curve) excepting that it also brings out the effect of iron losses which tend to reduce the slope in the unsaturated region.

The core characteristics required by the transductor designer are a sharply saturating magnetization curve, a high saturation flux density and low iron losses. An additional circuit requirement is that the total a.c. circuit impedance for zero control current should be large compared with the impedance at maximum output. The unsaturated impedance is given by the slope of the line joining the point *A* (*Figure* 2.3) to the origin, while at maximum output the impedance is approximately that of the load and a.c. windings. It has already been shown that if the ratio of $\frac{\text{unsaturated reactance}}{\text{resistance}}$ is large the control ratio is high, and it will be shown in Chapter 4 that this ratio has considerable bearing upon the gain of transductors with a high degree of self-excitation.

The general form of the a.c. magnetization curve gives a good representation of the quality of the material in these respects and since it is a comparatively easy curve to obtain experimentally it is often used for the assessment of transductor cores.

If the curve is plotted in the form shown in *Figure* 2.3*b* then it will be applicable to a particular core only. If, however, the ordinate is plotted as a flux density swing from equation 1.23 and the abscissa as the mean m.m.f. corresponding to the mean current $\bar{\imath}_a$, the curve is then generalized and will apply to a particular material at a certain frequency; it may then be used for the comparison of various materials. This is shown in *Figure* 2.9 where a.c. magnetization curves are plotted at 400 c/s for toroidal cores of Mumetal (*a*), H.C.R. (*b*) and a grain-oriented silicon-iron, Crystalloy (*c*).

From these curves it is seen that Mumetal has the steepest slope in the unsaturated region but a lower saturation flux density

than either H.C.R. or Crystalloy. For this reason its use is generally confined to sensitive stages of low output and high gain. It is also clear that H.C.R. has better magnetic properties than Crystalloy. Its disadvantage lies in the cost which, by virtue of the nickel content, is necessarily higher than that of silicon-iron.

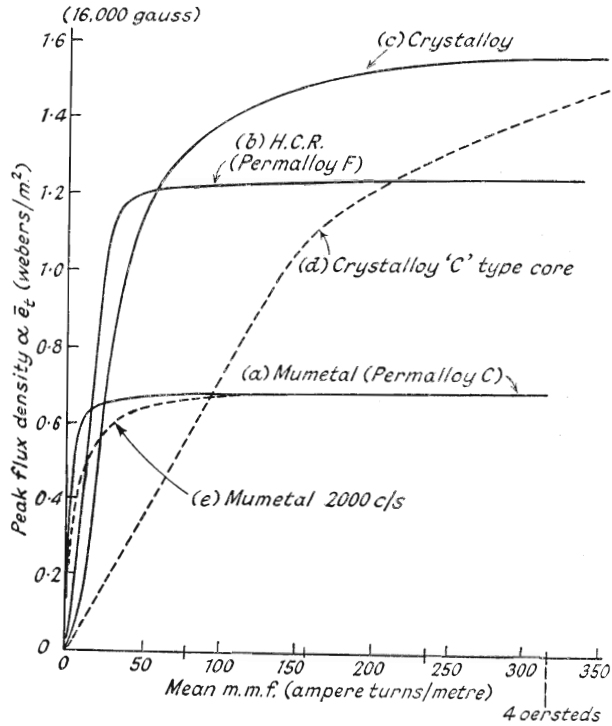


Figure 2.9. A.C. magnetization curves for various soft magnetic materials at 400 c/s and 2000 c/s.

In addition different types of core give rise to variations in shape of the a.c. magnetization curve, the best results usually being given by toroids. *Figure* 2.9*d* shows the curve obtained for a cut type *C* core. This may be compared with curve (*c*) for a continuous tape wound toroid showing the effect of the air gap at the joint of the *C* core. Finally the effect of increasing frequency is to introduce iron losses which reduce the slope in the high permeability region and tend to increase the slope in the low-permeability region. A curve for Mumetal at 2,000 c/s is shown dotted in *Figure* 2.9*e*.

2.11 Form Factor and Waveform of Load Current

The waveform of the load current shown in *Figure* 2.2*b* is seen to be part of a chopped sine wave. At maximum output the time of saturation t_q occurs at points corresponding to $\omega t = 0, \pi, 2\pi$, etc. As the control current is reduced, however, t_q moves to the right and so decreases the mean load current. Thus the waveform of the load current is one which varies over the working range from a sine wave at maximum output down to a very small pulse at the lower end of the characteristic. The form factor of a wave is the ratio of the r.m.s. value to the mean value of the quantity defined by the wave. For a sine wave the form factor is equal to 1.11, but in the present instance the form factor is not constant but a function of the output.

When considering the load current in relation to a specific application, interest is centred primarily on the mean value rather than the r.m.s. value (section 2.7). Nevertheless, when the load current flows in the a.c. windings it causes them to heat up and the heat generated will be proportional to the square of the r.m.s. current. If the form factor is known it enables the designer to estimate the heat generated for a given mean current output.

When the transductor is completely saturated firing occurs at $t_q = 0$, the waveform is sinusoidal and the form factor of the current is 1.11. But when self-excitation is used the linearity falls off considerably before complete saturation and hence the form factor must be known for the particular point at which the maximum useful output occurs in order that the maximum heating effect likely to be encountered may be estimated.

As an example let us consider the form factor when maximum linear output occurs half-way up the control characteristic.

At maximum possible output we have :

The mean load current $= \bar{\imath}_{as}$

and the power generated in a resistance r will be

$$r(\bar{\imath}_{as})^2 \times (1.11)^2.$$

Each half-cycle of the sinusoidal current i_{as} can be considered as two equal parts, the first lasting from $\omega t = 0$ to $\omega t = 90°$ and the second from $\omega t = 90°$ to $\omega t = 180°$. If current only flows for the latter period then exactly half the power will be dissipated in r, so that if i' is the r.m.s. load current corresponding to a firing angle of 90° the power will be

$$r(i')^2 = \tfrac{1}{2}\, r\, (\bar{\imath}_{as})^2 \,.\, (1.11)^2$$

giving $$i' = \frac{\bar{i}_{as}}{\sqrt{2}} \quad (1.11).$$

The mean current when t_q corresponds to 90° is equal to $\frac{1}{2}\bar{i}_{as}$ and therefore the form factor is

$$\frac{i'}{\frac{1}{2}\bar{i}_{as}} = 1.11\sqrt{2} = 1.57.$$

In practice for a firing angle of 90°, the form factor would probably be about 1·4 due to practical limitations such as indefinite saturation, iron losses, etc.

Under ideal conditions it may easily be shown by integration that for a firing angle θ the form factor is equal to

$$q = \frac{[\frac{1}{2}\pi(\pi - \theta) + \frac{1}{4}\pi \sin 2\theta]^{\frac{1}{2}}}{1 + \cos\theta} \quad . \quad . \quad . \quad (2.22)$$

and the firing angle θ for a mean current $\bar{i}_{a\theta}$ is given by

$$\theta = \cos^{-1}\left[2\frac{\bar{i}_{a\theta}}{\bar{i}_{as}} - 1\right] \quad . \quad . \quad . \quad (2.23)$$

2.12 Efficiency

There are two possible definitions for the efficiency of a transductor. The thermal efficiency η is the ratio of the r.m.s. power dissipated in the load to the total r.m.s. power supplied. If the load resistance is R_L, and the internal resistance is r_a we have

$$\eta = \frac{R_L}{R_a} = \frac{R_L}{R_L + r_a} \text{ (see section 2.7)} \quad . \quad . \quad . \quad (2.24)$$

Alternatively we have the mean power efficiency in recognition of the fact that the output is normally considered in terms of mean volt-amperes ; this may be written as the ratio η_m of the mean power output to the total r.m.s. power supplied, i.e.

$$\eta_m = \frac{(\bar{i}_a)^2 R_L}{(q\bar{i}_a)^2 R_a} \text{ where } q = \text{the form factor,}$$

$$\text{giving } \eta_m = \frac{\eta}{q^2} \quad . \quad . \quad . \quad (2.25)$$

This is now considered in relation to the following example.

A transductor is required to operate into a large moving-coil milliameter of full-scale deflection 100 mA. and resistance 10 ohms.

If linear operation is achieved up to 70% of the maximum output, find

(*a*) The total power supplied.
(*b*) The thermal efficiency.
(*c*) The mean power efficiency.

Assume that the internal resistance of the transductor is 1 ohm, that saturation occurs sharply and that iron losses are negligible.

If the meter were operated from d.c. the power dissipated in the output would be

$$(0{\cdot}1)^2 \times 10 = 0{\cdot}1 \text{ watt.}$$

This is therefore the useful mean power output of the transductor. From equation 2.23 the firing angle at 70% of maximum output will be

$$\cos^{-1}(2 \times 0{\cdot}7 - 1) = 66{\cdot}5^\circ$$

giving $\theta = 1{\cdot}16$ radians

and hence from equation 2.22

$$q = 1{\cdot}37.$$

Thus the total power supplied will be

$$\begin{aligned} &(\bar{\imath}_a \times 1{\cdot}37)^2 \times R_a \\ &= (0{\cdot}1 \times 1{\cdot}37)^2 \times 11 \text{ watt} \\ &= 0{\cdot}207 \text{ watt.} \end{aligned}$$

The thermal efficiency η is $R_L/R_a = \dfrac{10}{11} = 0{\cdot}91$,

but the mean power efficiency will be

$$\frac{0{\cdot}1}{0{\cdot}207} = 0{\cdot}48.$$

This brings out the point that when supplying an electromagnetic load, the transductor supply is less efficient than a steady d.c. supply. This is due to the fact that the heating effect of a pulsating current is always greater than that of a constant current of the same mean value.

In practice the situation would not be quite so bad as depicted in the above example since the transductor pulses would be less sharp than in the ideal case ; the form factor q would probably be nearer 1·2 giving a mean power efficiency of about 60%.

If the load current is smoothed by means of capacitance or

inductance, then ideally the form factor approaches unity. The disadvantage of such an arrangement is that linearity no longer holds between mean input and mean output. There are, however, a number of advantages of smoothing. In particular it is found to cause a substantial increase of gain, particularly at low signal levels. This is fully discussed in Chapter 6.

The thermal efficiency of transductors varies considerably, depending upon the supply frequency and the size of the unit. Thus in order to be able to withstand a given supply voltage at low frequencies small transductors must have an a.c. winding of comparatively high resistance, and the ratio R_L/R_a may therefore be as low as 50% without overheating. At higher frequencies the number of a.c. turns, and hence the winding resistance, can be reduced for the same core size and applied voltage; the efficiency thus increases. It is in fact necessary to arrange for more efficient operation at high frequencies to avoid overheating. Large transductors at low frequencies also tend to be of higher efficiency than the smaller units.

2.13 Transductor Assemblies

In the preceding discussion the transductor has been represented by two transformers connected in series opposition. This is not normally regarded as a good method of transductor construction, however, and some of the methods used in practice are summarized below. There are three principal ways of arranging the cores and windings for single-phase transductors. Firstly, the method so far described of interconnecting two transformers. This is a fairly cheap method of construction and the arrangement is quite satisfactory in operation but suffers from the disadvantage that, although there is no resultant voltage of fundamental frequency across the control windings, the balance is obtained from two comparatively large anti-phased alternating voltages across each coil. This is liable to cause voltage breakdown of the individual windings.

One of the first methods of transductor construction was to employ cores of the shape shown in *Figure* 2.10 *a*, in which the control winding was on the centre limb and the a.c. windings on the outer limbs. The fundamental components of the alternating fluxes cancel in the centre limb, thereby giving cancellation of fundamental voltages in the control windings, while the polarizing fluxes pass through the centre limbs and thread the outer limbs.

This form of construction is also comparatively cheap and is still used in large power installations where economics are an important factor. It is also an advantage in some applications to saturate the centre limb of the core independently of the action due to the control windings thereby introducing special characteristics which are sometimes useful. For example, relays

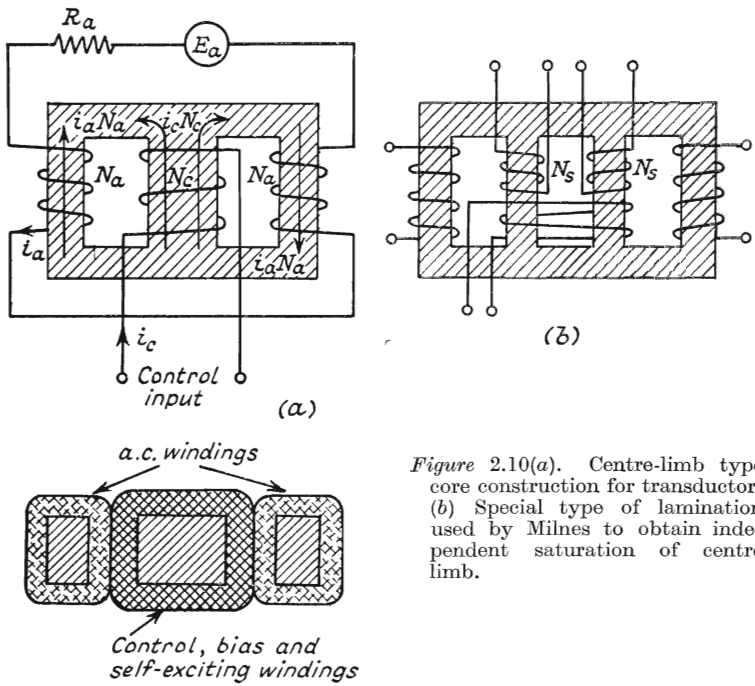


Figure 2.10(*a*). Centre-limb type core construction for transductor. (*b*) Special type of lamination used by Milnes to obtain independent saturation of centre limb.

required to have special characteristics may be driven from such an arrangement. Independent saturation of the centre limb can be achieved either by splitting this limb into two as shown in *Figure* 2.10*b* and placing saturating windings N_s on the extra limbs [2(b)], or by introducing an external flux from permanent magnets [2(c)] (Chapter 10).

In most cases, however, isolation of the centre limb is a disadvantage giving rise to poor zero stability and a tendency for the core to become permanently polarized when a large input signal is applied. This arises from the fact that the fundamental components of alternating flux in the centre limb are balanced out and the peak to peak flux swing is comparatively small. The

demagnetizing effect provided by the a.c. flux components is not therefore so pronounced as if these components linked the centre limb.

Probably the arrangements most commonly used for instrument amplifiers are those shown sectionally in *Figure* 2.11. In these the a.c. windings are wound separately on two closed cores, while the d.c. windings are wound jointly over both cores as shown.

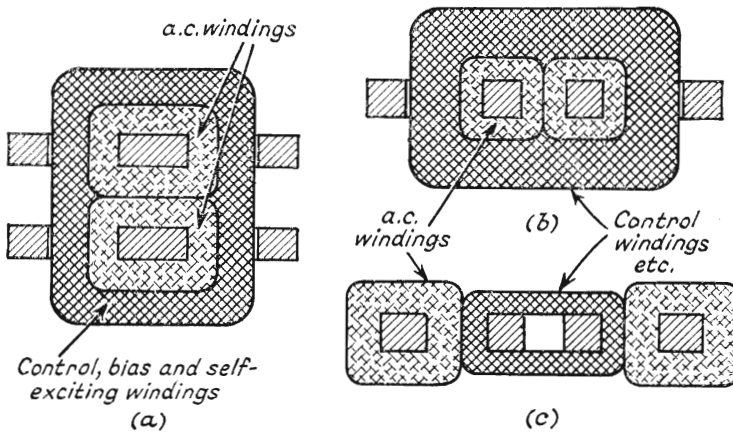


Figure 2.11. Methods of double core construction.
(*a*) *E* type cores.
(*b* and *c*) *U* type cores.

These methods have the advantage of a large flux swing occurring in each core every cycle, thereby helping to remove any permanent polarization. In this arrangement " turn by turn " cancellation of voltages occurs in the control windings. This type of construction is compact and has a good zero stability (Chapter 11), but is probably somewhat more expensive than the previous assemblies.

The two alternatives in (*a*) and (*b*) are, in effect, identical, the former applying to *E* type cores and the latter to the *U* types. The last arrangement (*c*) has the disadvantage that the coupling* between the a.c. and d.c. windings is slightly less effective than in (*a*) and (*b*) ; this leads to inferior linearity at large outputs. Its advantage lies in the smaller mean length of turn for the control windings, with consequent reduction of resistance and

* The fact that good coupling is required between the d.c. and a.c. windings should not be confused with previous statements that no coupling exists *in the unsaturated state* between these circuits. When one core is saturated the coupling between the a.c. and d.c. coils on the unsaturated core must be as tight as possible.

weight of copper for the same number of turns, or increased turns for the same resistance.

Further details of core construction, etc., are discussed in Chapter 13.

2.14 Control and Bias Windings

Variation of the transductor load current is effected by the total control ampere-turns linking the core. These may be supplied by one winding, or they may be the total effect of a number of windings. Thus the transductor may be used as a simple summing device. This property is an advantage in many applications where the output quantity is required to be dependent upon the sum of a number of variables. For example in position servo mechanisms it is often desirable to feed in rate terms to stabilize the system. This is done by applying the appropriate signal to an additional control winding on the transductor (section 13.23).

In particular, it is often necessary to bias the transductor to operate on a part of the characteristic which suits the particular application. This may be done by applying a constant input to additional windings. The effect of bias is simply to shift the characteristics (*Figures* 2.4 and 2.8) to the left or to the right, depending upon the direction in which the bias is applied in relation to the main control signal.

2.15 Summary

In this chapter the transductor amplifier has been introduced as a device which amplifies by virtue of its action as an electrically controllable gate. During the first part of every half-cycle of the supply frequency the supply voltage appears across the transductor, and for the remainder the voltage is applied to the load circuit. Increasing the control voltage reduces the gating period by causing the appropriate core to become saturated earlier in each half-cycle, thereby increasing the proportion of time for which the supply voltage is available to drive current through the load.

If the transductor has no self-excitation then a state of ampere-turn balance exists between the control windings and the a.c. windings; this tends to make the transductor a device with constant current output. It has a high output impedance and a low input impedance and the power gain is limited, although the amplifier is linear.

The power gain for a given power output and core size may be

increased considerably by the application of positive feedback or self-excitation. Under these circumstances the performance is dependent to a large extent on the permeability of the core.

A more detailed and general examination of the transductor is made in Chapters 3, 4 and 5. This is done in order to establish the flux relations in the core, to understand the transient operation and to examine the effect of altering some of the circuit parameters. An understanding of all these aspects is necessary in order to lead up to the more practical circuits in common use.

CHAPTER 3

TRANSIENT AND STEADY-STATE OPERATION OF IDEALIZED TRANSDUCTORS

Transient Operation of Transductors Without Self-Excitation

In Chapter 2 a brief account of transductor action was given by way of introduction to the subject, and it is now proposed to consider the operation in greater detail in order to understand the physical mechanisms involved, and hence to obtain data which will be of use to the engineer in design.

3.1 Simplification of the Magnetization Curve

Papers published on magnetic amplifiers have shown that considerable advance can be made towards a better understanding of the subject if the core magnetization characteristic is assumed to be represented by three straight lines of the general form shown in *Figure* 3.1*a*. These assumptions therefore not only neglect the hysteresis effect discussed in Chapter 1 but also ignore the knee of the magnetization curve and assume that the slope changes abruptly.

The characteristic is thus divided into distinct regions and, provided that the transductor continues to operate in one region, the equations are linear. When the values of m.m.f. or flux cause the core to operate in a different region, a new set of equations must be used.

Uno Lamm[3(a)] has given a full account of three-phase transductor operation primarily based on the assumption that the slope of the characteristic is finite before and after saturation, giving a magnetization curve of the form shown in *Figure* 3.1*a*.

A similar approach has been adopted by Krabbe[3(b)] for auto-

excited transductors. Both of the above treatments present a thorough picture of the mechanisms involved, but tend to be rather detailed for the unspecialized reader.

Gale and Atkinson [3(c)] and others [3(d,e,f)] analysed the transductor assuming the slope of the magnetization curve to be infinite before saturation and zero after saturation, giving a curve of the form shown in *Figure* 3.1*b*. These assumptions lead to a fairly clear physical picture of transductor operation, both in the transient and steady-state. The results of the analysis are not valid, however, when complete self-excitation is used. This is because infinite permeability implies zero magnetizing current which leads to infinite gain.

Milnes [3(g)] utilizes flux waveforms derived on the assumption of infinite slope to the magnetization curve with additional assumptions about the control circuit impedance. The current waveforms are then deduced from the flux waveforms on the basis of a magnetization curve such as that shown in *Figure* 3.1*c*. Thus the magnetizing current is taken into account and useful relations are obtained for the case of a self-excited transductor.

The above assumptions regarding the magnetic core are in general realistic for the high permeability nickel-irons and the agreement between theory and practice is good. Very often, however, the engineer may, on economic grounds, be forced to use silicon-iron cores having a comparatively low permeability and a less sharp knee to the magnetization curve. This occurs particularly in large transductors giving outputs ranging from 100 watts up to several kilowatts. In these cases a knowledge of the general principles usually enables the engineer to assess the parameters with sufficient accuracy for practical purposes.

3.2 Voltage-Time Integral

The analysis given in this chapter is based on the flux changes which occur in the cores during the cycle, these are derived from the voltage waveforms, and for this reason it is necessary to appreciate the physical significance of the voltage-flux relations as discussed in Chapter 1.

It was seen in section 1.5 that the total change of flux linking any coil is proportional to the area under the curve of induced voltage against time.

A magnetic core may be considered as a storage element in

which the quantity stored, the flux, is dependent upon the voltage-time integral applied to a coil linking the core.

In M.K.S. units the flux change is equal to the voltage-time integral per turn.

The conception of voltage-time integral when applied to an inductor is analogous to that of charge in a capacitor, where the total charge is given by the current-time integral. In the case of

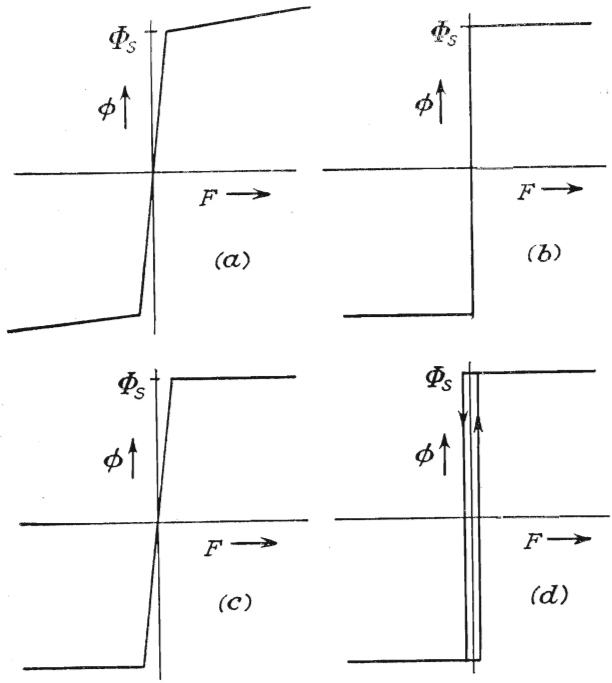


Figure 3.1. Various forms assumed for magnetization curve.

an infinite inductance, i.e. a coil linking a core of infinite permeability, the current in the coil is always zero although the flux may assume any value up to saturation.

In an iron-cored coil, the core of which saturates at a definite flux level, the total flux change is limited by saturation, and the voltage-time storage capacity of the core is therefore limited. Lamm [3(a)] and Krabbe [3(b)] refer to the voltage-time integral as degree of charge. They consider the core to be fully discharged when positive saturation is reached and fully charged at negative saturation.

If an iron-cored coil has a magnetization characteristic such as that shown in *Figure* 3.1*a*, the flux at any instant is fixed by the value of the m.m.f. In the case of the hypothetical characteristic shown in *Figure* 3.1*b*, however, the only flux datum available is that value which corresponds to saturation. Suppose that, at a certain time, the core flux is at the positive saturation value then, if the m.m.f. is zero, the flux will remain at that value (in the absence of noise) until a negative voltage is applied to the coil. When such a voltage is applied the flux will decrease by an amount proportional to the integral of voltage with respect to time. Thus, although the flux is indeterminate with respect to the m.m.f., a definite position on the curve may be established by considering the voltage-time integral.

It may be noted here that the assumption of infinite permeability is not necessary in order to postulate that the flux stays at the saturation value in the absence of any m.m.f. Such a state of affairs is in fact realized to a large extent in many practical core materials, in particular those having a rectangular hysteresis loop of the form shown in *Figures* 1.2*d* and 3.1*d*. In this case, although the m.m.f. is reduced to zero, the remanent flux in the core is very nearly equal to the saturation value. With certain types of transductor having a fast response it is sometimes preferable to use a core having this characteristic (see Chapter 9).

3.3 Basic Circuit Equations for a Simple Transductor

The circuit variables with which we are primarily concerned are the core fluxes, the transductor voltages and the currents in the load and control circuits. During the steady-state operation of a transductor the circuit variables are, in general, changing cyclically at the frequency of the supply and the values are precisely the same at corresponding times in successive cycles. The behaviour of the transductor over one cycle of the supply is governed by the initial conditions at the beginning of the cycle and when a transductor is operating in the steady-state, the initial conditions are the same at the beginning of every cycle.

If a control signal voltage is suddenly applied to the input of a transductor which had previously settled down to a steady-state of operation then a new set of initial conditions is introduced. This causes a transient which may persist for several cycles before the transductor finally settles down to a different steady-state

condition, the final conditions of flux, voltage, etc., at the end of any one cycle determining the initial conditions at the commencement of the following cycle. The main purpose of this chapter is to investigate these transients, and this will be done for a full-wave transductor without self-excitation.

A magnetization curve such as that shown in *Figure* 3.1*b* is assumed, and the equations employed at any instant depend upon the region of the magnetization curve in which the cores are operating. It is therefore necessary to know the times at which the transductor changes its zone of operation.

When the transductor is operating in the infinite permeability region the change-over occurs when the flux in a core reaches, say, the positive saturation value Φ_s. After this, the m.m.f. can increase, but no further flux change can occur in the positive direction. This state of affairs holds until the m.m.f. returns to zero, after which the operation is once again governed by the conditions applying to infinite permeability.

The circuit which we are about to examine is shown in *Figure* 3.2*a*. In addition to the assumption regarding the magnetization curve, it will also be assumed that iron losses, leakage inductance and self capacitance of windings are all negligible, the total control circuit resistance is lumped in R_c, and the total resistance in the a.c. circuit is R_a.

The control voltage E_c is considered to be positive, but the supply voltage E_a changes sign every half-cycle. The turns on each control and a.c. winding are N_c and N_a respectively, the connections to the a.c. windings on core B being reversed as shown. The arrows indicate positive values of the core fluxes ϕ_A and ϕ_B and when these fluxes change, e.m.f.s of magnitude e_{tA} and e_{tB} are induced in the a.c. coils, where

$$e_{tA} = N_a \frac{d\phi_A}{dt} \quad . \quad . \quad . \quad (3.1)$$

and

$$e_{tB} = -N_a \frac{d\phi_B}{dt} \quad . \quad . \quad . \quad (3.2)$$

Similar voltages equal to $\frac{N_c}{N_a} e_{tA}$ and $-\frac{N_c}{N_a} e_{tB}$ are induced in the control coils as shown. The flux changes occurring in cores A and B will be completely specified by the voltages e_{tA} and e_{tB} and in the following discussion our interest is primarily centred on these quantities, together with the control current i_c and the

rectified current $|i_a|$, which in this case corresponds to the load current.

Since there are two possible states for each core, either saturated or unsaturated, there are four possible conditions under which the circuit can operate. Equivalent circuits corresponding to each of these conditions are shown in *Figures* 3.2*b* to *e*.

(i) *Both cores unsaturated.* *Figure* 3.2*b* is the equivalent circuit when both cores are unsaturated. The a.c. and control circuit resistances are assumed to be negligibly small when compared with the corresponding reactances. The control voltage E_c and the supply voltage E_a are therefore opposed entirely by flux changes in the coils.

In the absence of any control voltage, e_{tA} and e_{tB} will each be equal in magnitude and sign to half the supply voltage. On positive half-cycles the flux in core A will be increasing while that in core B will be decreasing, and on negative half-cycles ϕ_B increases and ϕ_A decreases. When a small positive control voltage E_c is applied to the control circuit, e_{tA} becomes slightly more positive while e_{tB} becomes slightly less positive. Thus from equations 5 and 6, Appendix 1,

$$e_{tA} = N_a \frac{d\phi_A}{dt} = \frac{E_a}{2} + \frac{E_c N_a}{2N_c} \quad . \quad . \quad . \quad (3.3)$$

$$e_{tB} = -N_a \frac{d\phi_B}{dt} = \frac{E_a}{2} - \frac{E_c N_a}{2N_c} \quad . \quad . \quad . \quad (3.4)$$

The rate of increase of the flux ϕ_A is therefore greater than the rate of decrease of ϕ_B in the presence of a positive control voltage. There are therefore two rates of flux change applicable to either core; the algebraic value of these will be denoted by ψ_1 and ψ_2. The quantity ψ_1 is for increasing flux and applies to core A on positive half-cycles and core B on negative half-cycles of supply voltage ; while ψ_2 is for decreasing flux and applies to core B on positive half-cycles and core A on negative half-cycles. It is shown in Appendix 1 that in general when both cores are unsaturated

$$\psi_1 = \frac{E_c}{2N_c} + \frac{|E_a|}{2N_a} \quad . \quad . \quad . \quad (3.5)$$

$$\psi_2 = \frac{E_c}{2N_c} - \frac{|E_a|}{2N_a} \quad . \quad . \quad . \quad (3{\cdot}6)$$

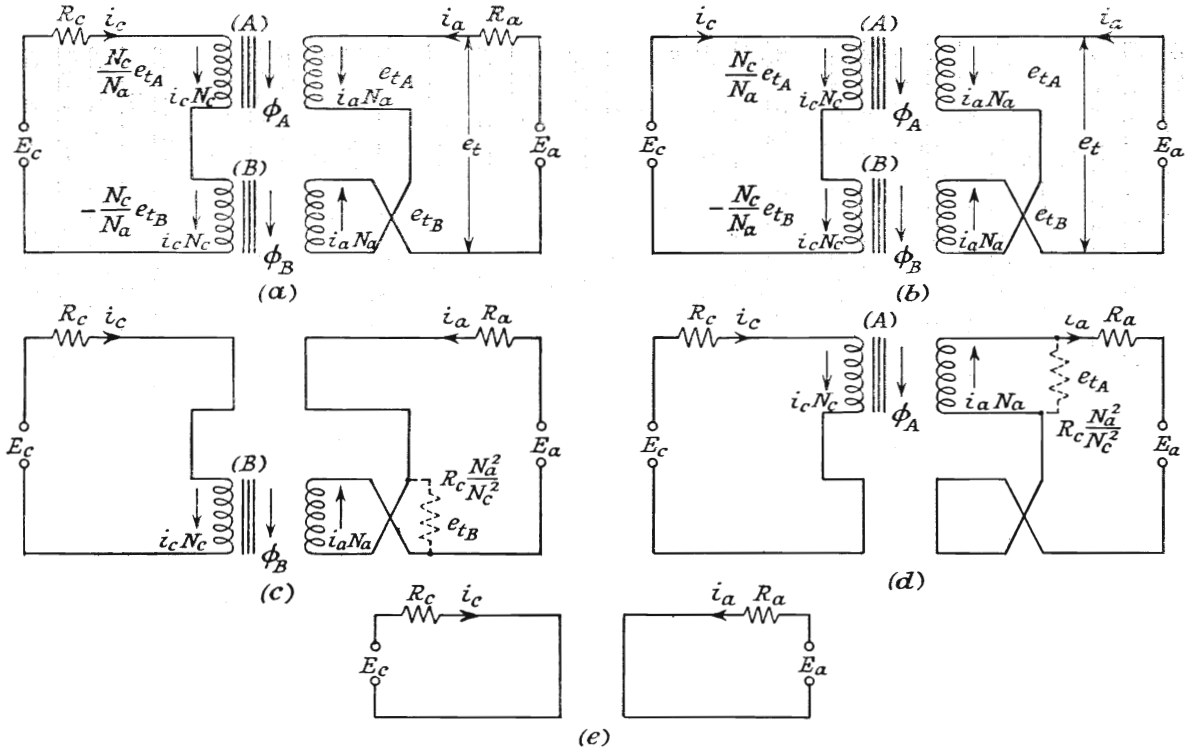


Figure 3.2. Equivalent transductor circuits.

(*a*) Simple circuit.
(*b*) Equivalent circuit when both cores are unsaturated.
(*c*) ,, ,, ,, core *A* is saturated.
(*d*) ,, ,, ,, core *B* is saturated.
(*e*) ,, ,, ,, both cores are saturated.

also $$|i_a| = 0^* \quad . \quad . \quad . \quad (3.7)$$

and $$i_c = 0 \quad . \quad . \quad . \quad (3.8)$$

The results show that when a positive control voltage E_c is applied and both cores are unsaturated the rate of increase of flux is greater than the rate of decrease by an amount $\frac{E_c}{N_c}$.

(ii) and (iii) *One core saturated.* Assuming that eventually the flux increase in one of the cores is sufficient to cause saturation in the positive direction then this will occur in core A during positive half-cycles and core B during negative half-cycles. Let us first consider that core A saturates. The coils on this core are assumed to have zero impedance when this occurs and the equivalent circuit is therefore as shown in *Figure* 3.2*c*.

The coils on core B are now mutually coupled so that although the m.m.f. in core B is zero, the control current and the load current suddenly assume finite values, since this inductor is acting like a transformer with finite primary and secondary impedances. This is therefore known as the conducting period. The control current and the load current will each consist of two components, one due to the control voltage and the other due to the supply voltage. The latter component will be positive.

The induced voltage e_{tB} will also be modified due to saturation of core A. The control circuit resistance R_c may be referred to the a.c. circuit as $R_c \frac{N_a^2}{N_c^2}$. (shown dotted in *Figure* 3.2*c*) and if R_c is small, only a small proportion of the supply voltage will appear across the coil. The flux changes in the unsaturated core will therefore be considerably reduced during the conducting period. This brings out the important point that when one core saturates, a discontinuity occurs in the flux relations for the *unsaturated* core, the discontinuity becoming more pronounced as R_c tends to zero.

It is worth noting, however, that in the special case where the referred resistance is equal to the a.c. circuit resistance no sudden change of induced voltage occurs in one coil when the core of the other coil saturates, since before and after a core saturates half the applied voltages will be supported across the coils on the unsaturated core.

* The current i_a is normally rectified and the circuit relations are therefore expressed in terms of $|i_a|$, the modulus sign indicating rectification.

The same reasoning applies to the circuit in *Figure* 3.2*d* in which core *B* has saturated during a negative half-cycle. The component of control current induced from the a.c. supply is positive as before. This is because although the supply voltage E_a has changed sign so also are the connections to the a.c. coil reversed.

It is shown in Appendix 1 (equations 16–19) that when either core is saturated (*Figures* 3.2*c* and *d*) the alternating current i_a and the control current i_c are given by the following relations where the upper signs refer to core *A* being saturated (positive half-cycles) and the lower signs to core *B* being saturated (negative half-cycles) thus:

$$i_a = \frac{\gamma}{1+\gamma}\left[\begin{matrix}+\\-\end{matrix}\frac{E_c N_a}{N_c} \begin{matrix}+\\+\end{matrix} E_a\right]\frac{1}{R_a} \quad . \quad . \quad . \quad (3.9)$$

and

$$i_c = \frac{1}{1+\gamma}\left[\begin{matrix}+\\+\end{matrix} E_c \begin{matrix}+\\-\end{matrix}\frac{E_a N_c}{N_a}\right]\frac{1}{R_c} \quad . \quad . \quad . \quad (3.10)$$

where

$$\gamma = \frac{N_c^2 R_a}{N_a^2 R_c} \quad . \quad . \quad . \quad (3.11)$$

It was also shown in Chapter 2 that the quantity γ is equal to the circuit power amplification G_p', for the special case of a transductor without self-excitation. Each current consists of a component due to the control voltage and one due to the supply voltage. Furthermore, since E_a changes sign every half-cycle the value of i_c is always positive while i_a changes sign every half-cycle with E_a.

If it can be assumed that a core desaturates when the supply voltage changes sign,* then by considering numerical values for i_a and i_c equations 3.9 and 3.10 may be expressed in the general form

$$|i_a| = \frac{\gamma}{1+\gamma}\left[\frac{E_c N_a}{N_c} + |E_a|\right]\frac{1}{R_a} \quad . \quad . \quad . \quad (3.12)$$

$$i_c = \frac{1}{1+\gamma}\left[E_c + \frac{|E_a| N_c}{N_a}\right]\frac{1}{R_c} \quad . \quad . \quad . \quad (3.13)$$

The m.m.f. in the unsaturated core is equal to the numerical difference between the control ampere-turns and the a.c. ampere-

* This is true for a square wave of supply voltage which is discussed in the following section. It is also very nearly true for a sine wave, but slight errors are introduced at the end of the half-cycle when $|E_a|$ becomes smaller than $E_c N_a / N_c$.

turns (see *Figures* 3.2c and *d*). The total m.m.f., however, must be zero since the core is unsaturated and therefore

$$i_c N_c = |i_a| N_a \quad . \quad . \quad . \quad (3.14)$$

The voltage induced in the a.c. coil on the unsaturated core is given by equations 13 and 14, Appendix 1, thus

$$e_{tB} \text{ or } e_{tA} = \left[\frac{E_a}{1+\gamma} \mp \frac{E_c N_a}{N_c} \cdot \frac{\gamma}{1+\gamma} \right] \quad . \quad . \quad (3.15)$$

where the upper signs refer to e_{tB} (core A being saturated), while the lower signs refer to e_{tA} (core B being saturated). When the circuit power gain $G_p' = \gamma$ is large the component due to the supply voltage E_a is considerably reduced.

The rate of flux change over this period (one core saturated) will be denoted by ψ_3 and therefore, on the same assumption as those applying to 3.12 and 3.13 during the conducting period, when one core is saturated the rate of flux change in the unsaturated core is

$$\psi_3 = \left[\frac{E_c}{N_c} \cdot \frac{\gamma}{1+\gamma} - \frac{|E_a|}{N_a} \frac{1}{1+\gamma} \right] \quad . \quad . \quad . \quad (3.16)$$

It will by now be evident that a given set of equations applies only to a specific state of operation, e.g. core A saturated, core B unsaturated. The conditions determining the times of change from one state of operation to another are governed by the fluxes and m.m.f.s. Thus assuming both cores to be unsaturated initially, equations 3.5, 3.6, 3.7 and 3.8 apply: as soon as the flux in either core reaches positive saturation then equations 3.12, 3.13 and 3.16 will be applicable until the m.m.f. in the saturated core returns to zero. This m.m.f. is equal to

$$i_c N_c + |i_a| N_a.$$

But since the control m.m.f. is equal to the a.c. m.m.f. due to ampere-turn balance in the unsaturated core (equation 3.14) the m.m.f. in the saturated core will be $2|i_a|N_a$; this becomes zero when the load current, i.e. the current in the a.c. windings, is zero. Therefore the time of desaturation is fixed by the condition $i_a = 0$.

(iv) *Both cores saturated.* This mode of operation is represented by the equivalent circuit in *Figure* 3.2e. It arises in practice when the control source has a high impedance (section 3.9). In this case the currents in the control and load circuits are independent, each being determined by the individual circuit parameters.

3.4 Response to Step Function Voltage Signal

The results of the previous section will now be used to investigate the transients arising when a control voltage is suddenly applied to the control circuit.

The following discussion will be simplified by making the core fluxes linear functions of time rather than sinusoidal functions. This is done by using a supply voltage which is a square wave of height $\hat{E}_a$, while the control signal is a constant direct voltage E_c applied to the control circuit at a definite time. By considering the circuit to be operating under these conditions, a graphical representation is considerably simplified while all the salient features are retained.

Referring to *Figures* 3. 3*a*, *b*, *c* and *d*, over the period 0 to T the control voltage (3.3*b*) is zero. The induced voltages are plotted as e_{tA} and $-e_{tB}$. These are symmetrical about the time axis and equal in magnitude to half the supply voltage. The shaded areas under the voltage time curves in *Figures* 3.3*c* and *d* are proportional to the changes in the fluxes ϕ_A and ϕ_B respectively. The flux waveforms are triangular and vary within the saturation limits, prior to the application of a control voltage. The rates of change of flux reverse at the end of every half-cycle when the supply voltage reverses (*Figure* 3.3*e*). The first two half-cycles define the steady-state operation for $E_c = 0$, in which case the rate at which the flux in either core increases is precisely the same as the rate at which it decreases.

At a certain instant of time, chosen for convenience as $t = T$. say, a positive control voltage E_c is applied to the control circuit, When this occurs the induced voltages e_{tA} and $-e_{tB}$ immediately become asymmetrical, the positive areas increasing and the negative areas decreasing ; these voltages are now given by equations 3.3 and 3.4. The flux waveforms are modified and the rates of increase ψ_1 become slightly greater, while the rates of decrease ψ_2 are correspondingly reduced with the result that the mean flux level, defined by $\phi_0 = \dfrac{\phi_A + \phi_B}{2}$ and shown chain-dotted, is progressively raised a small amount in each half-cycle.

This ultimately results in one core (core A, say) reaching the saturation level at a time t_{q1}, *Figure* 3.3*e*. At this point the mode of operation changes, the flux in core A now remains constant at the saturation value Φ_s while that in core B commences to

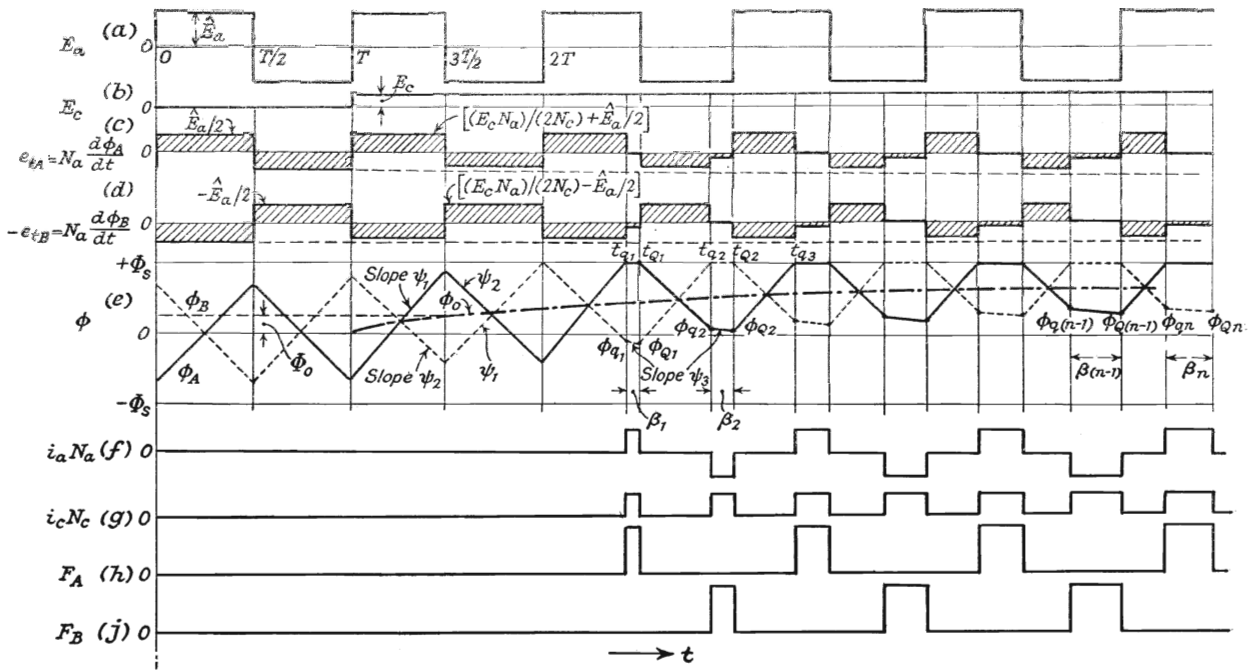


Figure 3.3. Transient operation of a square-wave excited transductor.

(*a*) Supply voltage, (*b*) control voltage, (*c* and *d*) coil voltages, (*e*) flux changes, (*f*) a.c. ampere-turns, (*g*) control ampere-turns, (*h*) m.m.f. in core *A*, (*j*) m.m.f. in core *B*.

change at the new rate ψ_3 in accordance with the modified value of e_{tB} given in equation 3.15 and shown in *Figure* 3.3*d*.

This state of affairs corresponds to the period of conduction during which the state of core A is determined by the m.m.f. F_A. The coils on core B provide mutual coupling between the a.c. circuit and the control circuit and the currents i_a and i_c in these circuits suddenly increase to values given by equations 3.9 and 3.10.

The magnitude of the current in the a.c. circuit is proportional to the alternating ampere-turns i_aN_a, this is plotted in *Figure* 3.3*f*. The control ampere-turns i_cN_c are plotted in *Figure* 3.3*g*. The core m.m.f.s F_A and F_B which are equal to the sum and difference of the a.c. and control ampere-turns respectively are plotted in *Figures* 3.3*h* and *j*, thus

$$F_A = i_cN_c + i_aN_a$$

and

$$F_B = i_cN_c - i_aN_a$$

The conducting period which commenced at t_{q1} lasts until the m.m.f. F_A in the saturated core becomes zero and it has been shown that this occurs when $i_a = 0$. In the case of square wave excitation this occurs at the end of the half-cycle of supply voltage, but with sine wave excitation desaturation may occur just after the end of the half-cycle when considering idealized cores (see section 3.5).

When the supply voltage E_a goes negative at time t_{Q1} the flux in core A starts decreasing while that in core B starts increasing from the value ϕ_{Q1}, and since both cores are unsaturated the rates of flux change are equal to ψ_2 and ψ_1 respectively. During the time interval from t_{Q1} to t_{q2} both cores are unsaturated so that the current in the a.c. windings i_a and the control current i_c are both zero.

This persists until core B saturates at a point t_{q2} slightly earlier than the corresponding point t_{q1} in the previous half-cycle.

In any half-cycle, the rate of increase of flux is constant, provided the control voltage is constant, and therefore the time at which a core saturates depends upon the initial flux in that core at the beginning of the half-cycle (e.g. ϕ_{Q1}, ϕ_{Q2}, $\phi_{Q(n-1)}$, ϕ_{Qn}, etc.). Similarly the rate of decrease of flux is constant and therefore the flux values in the unsaturated core at the beginning of conduction (e.g. ϕ_{q1}, ϕ_{q2}, etc.) depend only upon the time at which conduction starts.

The initial flux $\phi_{Q(n-1)}$ in the nth half-cycle is the final flux in

the unsaturated core at the end of the $(n - 1)$th half-cycle and examination of the diagram in *Figure* 3.3*e* shows that $\phi_{Q(n-1)}$ depends upon $\phi_{q(n-1)}$ which in turn depends upon the time at which conduction starts in the $(n - 1)$th half-cycle.

This leads to the important conclusion that the unsaturated core gives the transductor a " memory " effect so that the events occurring in any half-cycle are directly influenced by the events which occurred in the previous half-cycle. Thus, reading from the *Figure*, ϕ_{q1} determines ϕ_{Q1}, ϕ_{Q1} determines t_{q2} which determines ϕ_{q2} and so on. It follows, therefore, that the duration of the conducting period β_n in the nth half-cycle is determined by the magnitude of β_{n-1}.

It can be seen from *Figure* 3.3*e* that in the particular case shown there is a finite delay period consisting of a few half-cycles, before any output appears. This is because the initial flux swing was insufficient to saturate the core for small signal voltages and the first conducting period did not occur until the mean flux level had increased to some value Φ_0 such that saturation could occur in part of the cycle.

Normally transductors operate with a peak to peak flux swing which just fails to saturate the cores in the absence of a control voltage and the mean supply voltage $\bar{E}_{as}$ necessary to ensure this has been defined in Chapter 2. Otherwise the supply voltage $\bar{E}_a$ is some factor K of the saturation value where

$$\bar{E}_a = K\bar{E}_{as}$$

In the present case K can only be equal to or less than unity and is given by

$$K = \frac{\Phi_s - \Phi_0}{\Phi_s} \text{ (see also section 3.13).}$$

When dealing with transients the initial delay period is removed if $K = 1$, thereby ensuring that saturation occurs in the first half-cycle after application of a signal (*Figure* 3.4).

After the first half-cycle, the time of saturation t_q occurs at an increasingly early stage in each subsequent half-cycle until ultimately a steady-state is reached in which the total decrease of flux in one core over a half-cycle is equal to the total increase of flux in the other core. A study of *Figure* 3.3*e* shows that this mechanism is entirely dependent upon the rates of flux change ψ_1, ψ_2 and ψ_3. The last of these is the only one dependent upon

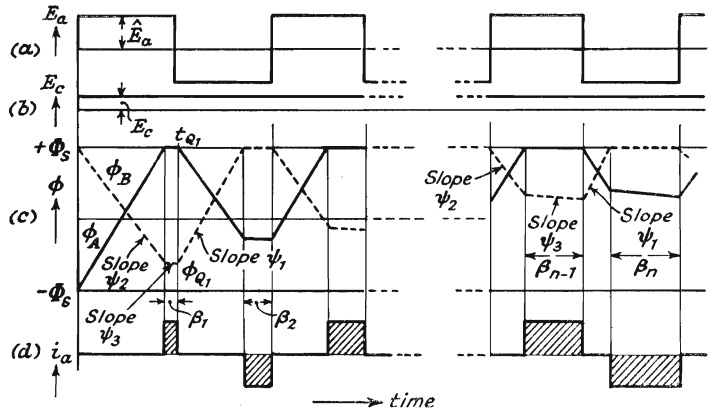


Figure 3.4. Build up of core fluxes when initial peak to peak flux = 2 Φs. (*a*) Supply voltage. (*b*) Control voltage. (*c*) Core fluxes. (*d*) Load current.

the circuit gain γ (equation 3.16) and hence ψ_3 can have any value independently of ψ_1 and ψ_2. Thus, if ψ_3 were zero, a steady-state would never be reached because, since the rate of increase ψ_1 is greater than the rate of decrease ψ_2, a net increase of mean flux must always occur over each half-cycle until the transductor is completely saturated the whole time. It follows that as the magnitude of the slope ψ_3 decreases the response becomes slower.

When the magnitude of ψ_3 is small it implies a large power gain γ and hence transductors having a large power gain also have a long time constant. The relationship between power gain and response time can be obtained analytically as follows.

The total change of flux over any time interval may be written : (final flux) = (initial flux) + (ψ × time interval), where ψ is the appropriate rate of flux-change over that particular interval. Referring to *Figure* 3.4*c* this relation may therefore be applied to core B in the first half-cycle ending at time $t_{Q1} = T/2$

$$\phi_{Q1} = \Phi_s + \psi_2\left(\frac{T}{2} - \beta_1\right) + \psi_3\beta_1 \quad . \quad . \quad . \quad (3.17)$$

and for the same core in the following half-cycle.

$$\Phi_s = \phi_{Q1} + \psi_1\left(\frac{T}{2} - \beta_2\right) \quad . \quad . \quad . \quad (3.18)$$

These expressions for β_1 and β_2 relate the duration of the conducting period in the second half-cycle to the duration of conduction

in the first half-cycle. If this is done for the nth half-cycle it is possible to build up a sequence of terms in which the nth term is found to be a function of the first term, the circuit constants and the value of n. From this one may obtain the steady-state mean load current and the time constant which is defined as the time elapse after switching on before the mean load current reaches a value $\left(1-\frac{1}{e}\right)$ of the steady value, where e is the base of natural logarithms.

This is done in Appendix 2, where it is shown that in the steady-state

$$\bar{\imath}_a = \frac{E_c N_c}{R_c N_a} = \bar{\imath}_c \frac{N_c}{N_a} \text{ (cf. equation 2.6)} \quad . \quad . \quad . \quad (3.19)$$

and the time constant

$$\tau = \frac{1}{2f}\left[1 + \frac{\log \frac{\gamma e}{\gamma + 1}}{\log \frac{\gamma + 1}{\gamma - 1}}\right] \quad . \quad . \quad . \quad (3.20)$$

Putting $y = \frac{1}{\gamma}$ the logarithmic terms may be expanded as a series which shows that as γ increases, the relation between the time-constant and power gain approaches the linear form given by

$$\tau = \frac{\gamma}{4f} \quad . \quad . \quad . \quad (3.21)$$

Thus for large values of power gain the simpler expression for the time constant is valid. Both equations show that at a given supply frequency the time constant is uniquely determined by the circuit power gain. If the response time is expressed as a number of cycles then the term in f disappears.

3.5 Sine Wave Supply Voltage

The same principles as those applying to a transductor excited with square waves are true for any waveform of voltage, the most usual combinations being a unidirectional step voltage input and a sine wave supply voltage, the analysis of the transient state using this combination is somewhat difficult, however, and a simple general expression for the time constant such as that obtained for the case of square waves is not readily derived.

In the case of sinusoidal excitation, the component of flux variation due to the control voltage varies linearly with time while the component due to the supply voltage varies as a cosine wave with time. This leads to fairly simple expressions for ψ_1, ψ_2 and ψ_3, but they do not readily lend themselves to the formation of a simple sequence. The case of sine wave excitation has been considered by Ramey [2(f)], but the signal E_c is assumed to be a rectified sine wave of the same frequency and phase as the supply voltage. The expression for the time constant is derived for one particular value of the input signal, and in this case the result is the same as that obtained for square waves in Appendix 2.

When the supply voltage is sinusoidal the flux relations are no longer linear but cosine and linear functions of time superimposed. The equations applying to these conditions are given in Appendix 3 and the corresponding waveforms showing the transient response are plotted in *Figure* 3.5. From these curves it is seen that, in common with the square wave case, the unsaturated core exhibits a discontinuity in the flux-time curve at the beginning of the conducting period. This discontinuity causes the unsaturated core to have a " memory " which influences its time of saturation in the following half-cycle, thereby causing the gradual build-up to a steady-state. It should also be noted that the cores do not come out of saturation until a short time after the end of the half-cycle. A core desaturates when the current i_a in the a.c. coil is zero. This current is due to two components of e.m.f. (equation 3.9) and it will not, therefore, be zero until the total e.m.f. which causes it is zero.

This occurs when the component due to the supply voltage just cancels the component due to the control voltage, and since these components are proportional to $\hat{E}_a \sin \omega t$ and $\frac{E_c N_a}{N_c}$, the desaturation time may be represented graphically as in *Figure* 3.5*a*. The intersection between the sine wave and the lower and upper horizontal lines $\frac{E_c N_a}{N_c}$ defines the times of desaturation for positive and negative conducting periods respectively.

In practice not much error is involved if it is assumed that the core desaturates at the end of the half-cycle, particularly in cases where the gain is large.

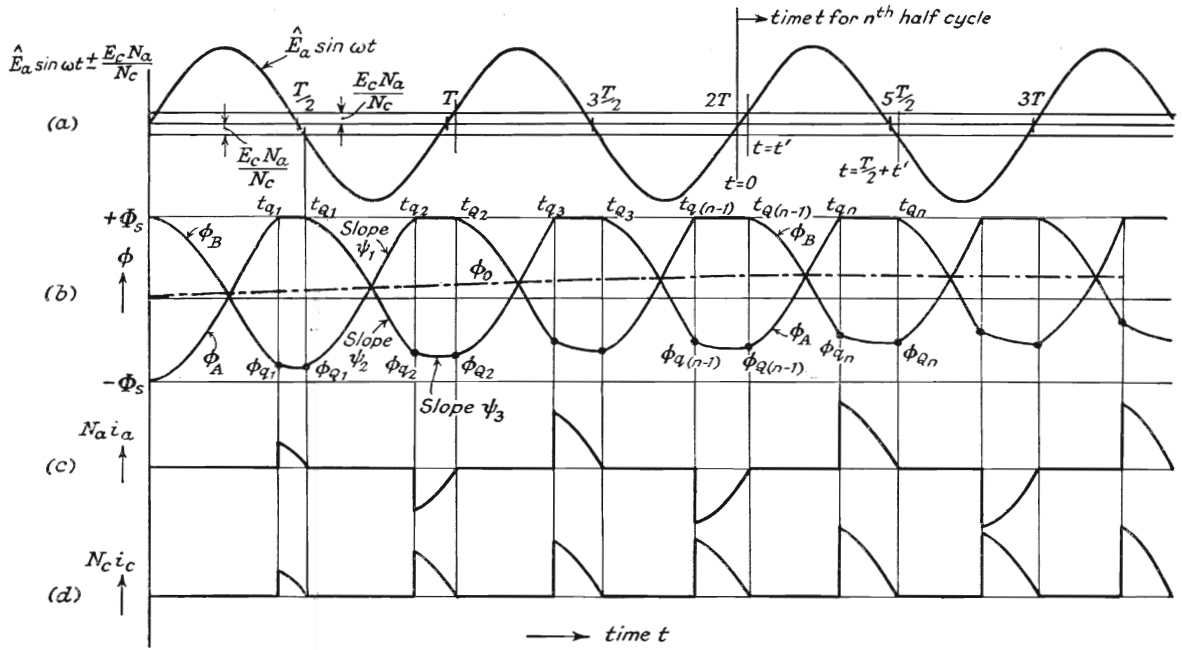


Figure 3.5. Transient operation of simple transductor with sinusoidal excitation.
(*a*) Waveform of supply voltage and control voltage. (*b*) Core fluxes. (*c*) Load ampere-turns. (*d*) Control ampere-turns.

Steady-State Waveforms

3.6 Derivation of Voltage Waveforms

The waveform of the steady-state current in the a.c. coils can be derived by determining the instantaneous induced voltage e_t across the a.c. windings and subtracting it from the supply voltage. This gives the voltage applied to the circuit resistance R_a and is proportional to the load current.

The total voltage e_t across the transductor (*Figure* 3.2*a*) is always equal to the sum of the induced voltages in the a.c. coils thus:

$$e_t = e_{tA} + e_{tB} \quad . \quad . \quad . \quad (3.22)$$

where e_{tA} and e_{tB} are given in equations 3.3, 3.4 and 3.15.

When both cores are unsaturated the transductor voltage is equal to the supply voltage, i.e.

$$e_t = \hat{E}_a \sin \omega t \quad . \quad . \quad . \quad (3.23)$$

When core A saturates no flux change can occur in this core and therefore $e_{tA} = 0$ so that

$$e_t = e_{tB} \quad . \quad . \quad . \quad (3.24)$$

similarly whenever core B is saturated $e_{tB} = 0$ and

$$e_t = e_{tA} \quad . \quad . \quad . \quad (3.25)$$

The waveforms of current and voltage are dependent upon the ratio of the control circuit resistance to the (control turns)^2, i.e. R_c/N_c^2. If this is very high, the fourth mode of operation is introduced in place of the mode corresponding to both cores being unsaturated. In most practical applications the impedance of the control circuit is not sufficiently high to cause this mode of operation ; there are, however, a few exceptions and we will therefore consider the effects of operating the transductor at three different values of control circuit impedance.

3.7 Waveforms when R_c/N_c^2 is Small

When the control circuit impedance is small for a given number of control turns, it is presumed that the control voltage is also small in order to maintain a finite control current. In these circumstances both coil voltages are zero whenever either core is saturated, because in one core the flux is constant at the saturation

value while in the other core flux changes are completely opposed due to the fact that the control circuit impedance is virtually a short circuit (see section 1.2). (This is also evident from equation 3.16, a small value of R_c/N_c^2 corresponds to a large circuit power gain γ and if γ is made very large and the control voltage

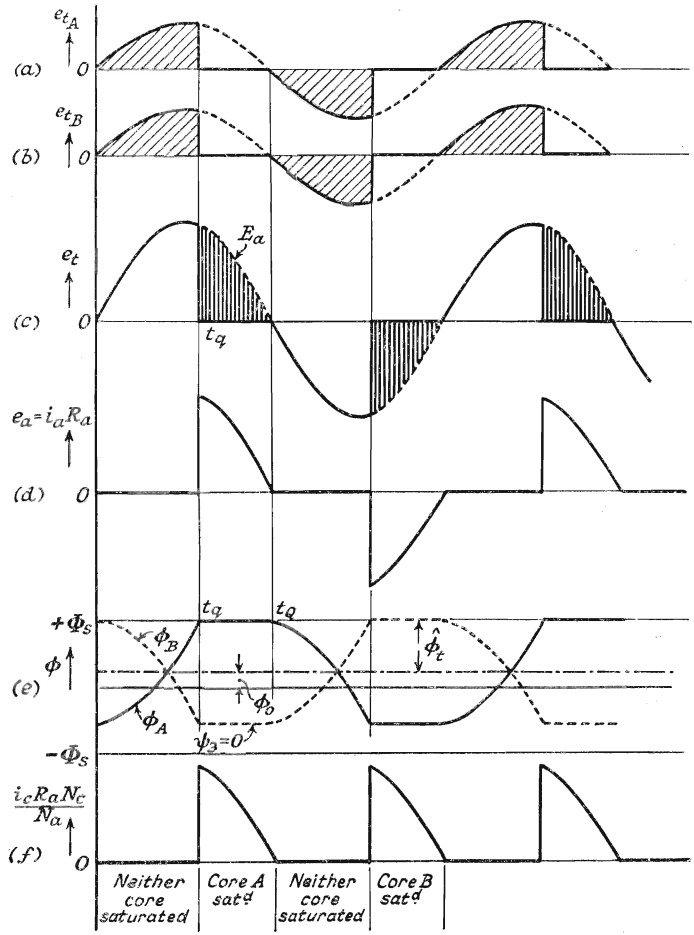


Figure 3.6. Steady-state voltage, current and flux waveforms for R_c and E_c very small.

(*a*) and (*b*) Individual coil voltages e_{tA} and e_{tB}.
(*c*) Transductor voltage e_t.
(*d*) Load voltage $i_a R_a$.
(*e*) Core fluxes.
(*f*) Control current $\times \frac{N_c}{N_a} R_a$.

very small, only a negligible voltage will be induced in the a.c. windings of the unsaturated core during the conducting period.)

The waveform of transductor voltage e_t is shown by the full-line curve in *Figure* 3.6*c*; it follows the curve of supply voltage until core A saturates at t_a, after which the transductor voltage drops to zero and the supply voltage is transferred to the total circuit resistance R_a until the end of the conducting period.

The voltages e_{tA} and e_{tB} across the a.c. coils are each equal to $\frac{1}{2}e_t$ and are shown in *Figures* 3.6*a* and *b*. The magnitude of the load voltage is represented by the shaded area under the dotted line in *Figure* 3.6*c*, being the difference between the supply voltage and the transductor voltage. This is in fact the resistive voltage drop $e_a = i_a R_a$ which is plotted in *Figure* 3.6*d*. The time of desaturation is defined by the condition $i_a = 0$ which therefore fixes the duration of the conducting period and enables the flux waveforms to be drawn (*Figure* 3.6*e*). The control current plotted in *Figure* 3.6*f* is scaled up by the factor $R_a \dfrac{N_c}{N_a}$ for direct comparison with the load current plotted as $i_a R_a$ in *Figure* 3.6*d*. In the present case, desaturation occurs at the end of the half-cycle of supply voltage.

These were the conditions considered in Chapter 2. It is of interest to note from the flux waveforms that the rates of flux change ψ_1 and ψ_2 are equal and that ψ_3 is nearly zero ; this implies a very long time constant corresponding as it does to a large circuit power gain.

3.8 Waveforms when Using an Intermediate Value of Control Circuit Impedance

Let us now consider a value of control circuit resistance such that $\dfrac{R_c}{N_c^2} = \dfrac{R_a}{N_a^2}$ which corresponds to a circuit power gain of unity. Once again so long as both cores are unsaturated the voltage across the transductor is equal to the supply voltage thus :

$$e_t = \hat{E}_a \sin \omega t$$

Core A saturates during positive half-cycles and since in this case the ratio γ is equal to unity it is seen from equations 3.15 and 3.24 that

$$e_t = e_{tB} = \frac{\hat{E}_a}{2} \sin \omega t - \frac{E_c N_a}{2N_c} \quad . \quad . \quad . \quad (3.26)$$

This is shown in *Figure* 3.7*b*.

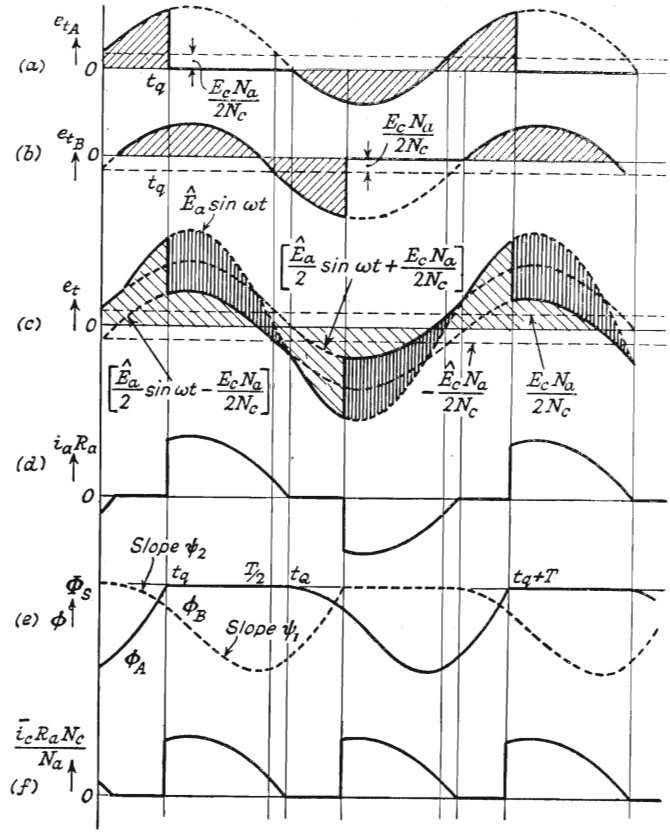


Figure 3.7. Waveforms showing steady-state operation of a transductor for which $R_c/N^2_c = R_a/N^2_a$.

(*a*) and (*b*) Individual coil voltages e_{tA} and e_{tB}.
(*c*) Transductor voltage e_t.
(*d*) Load voltage $i_a R_a$.
(*e*) Core fluxes.
(*f*) Control current $\times$ $R_a N_c/N_a$.

Similarly when core *B* saturates during negative half-cycles

$$e_t = e_{tA} = \frac{\hat{E}_a}{2} \sin \omega t + \frac{E_c N_a}{2N_c} \quad . \quad . \quad . \quad (3.27)$$

as shown in *Figure* 3.7*a*.

The waveforms shown in *Figure* 3.7*a* and *b* therefore represent the voltages across each coil, the zero line for each of these voltages is displaced from the true mean of the sine wave by amounts $\mp \frac{E_c N_a}{2N_c}$.

As before, the intersections between the displaced sine-wave and the zero line give the times of desaturation, and since the total flux change linking any one coil must be zero over a complete cycle the shaded areas must be equal about the zero lines.

The sum of the individual coil voltages gives the total transductor voltage at all times, this is shown as the full-line curve in *Figure* 3.7*c* where the curve of supply voltage is shown (partly dotted). Up to saturation, the curve of supply voltage defines the transductor voltage. When core A saturates at a time corresponding to t_q the transductor voltage switches to the new value given in equation 3.26. The output voltage is the difference between the supply voltage and the transductor voltage and is defined by the shaded area which is proportional to the load current shown in *Figure* 3.7*d*. Desaturation occurs at t_Q when the load current decreases to zero and once again the whole of the supply voltage is developed across the transductor until core B saturates and so on. The fluxes ϕ_A and ϕ_B are shown in *Figure* 3.7*e*; they are proportional to the areas under the voltage curves for e_{tA} and e_{tB} respectively. The condition corresponding to unity circuit power gain is that in which $\frac{R_c N_a^2}{N_c^2} = R_a$ and in this particular case, since $\gamma = 1$, it is seen from equations 3.6 and 3.16 that the flux rate ψ_3, applicable to conducting periods, is equal to the flux rate ψ_2, which applies to non-conducting periods. It therefore follows that no discontinuity occurs to the flux changes in the unsaturated core at the beginning of conduction. Therefore nothing happens to the unsaturated core to indicate saturation of the other core and since there is no " memory " effect the duration of a conducting period is independent of the duration of the previous conducting period. The device therefore, only requires one half-cycle to reach its steady-state. The power amplification, however, is unity. In this case also the control ampere-turns are equal to the load ampere-turns during the conducting period.

3.9 Waveforms when Using a High Value of R_c/N_c^2

A high impedance control circuit can be achieved either by means of series resistance in the control circuit or by series inductance. In the former case the circuit power gain is considerably less than unity due to the losses in the control circuit, while in the latter case, although the power gain may be appre-

ciable the presence of the inductance gives the control circuit a large time constant. Transductors may be used as d.c. current transformers for metering purposes (see Chapter 14), in which case the control coil consists of one turn formed by a bus-bar which passes through the space on the core normally occupied by a control winding. Although the resistance of the bus-bar is generally small, the ratio of resistance to inductance is very large since there is only one turn whose mean length is very many times greater than that normally encountered. This case, therefore, corresponds to a transductor having a high ratio of control circuit resistance to (control turns)^2, and as it is of some interest in practice the mode of operation will be described briefly.

When the control circuit resistance is large the control source may be looked upon as a constant current source, thus

$$i_c = \bar{i}_c = \frac{E_c}{R_c} \qquad . \quad . \quad . \quad (3.28)$$

substituting this in equation 3.15 gives the induced voltage

$$e_{tB} \text{ or } e_{tA} = \frac{E_a}{1+\gamma} \mp \frac{\bar{i}_c R_c \gamma}{1+\gamma}\left(\frac{N_a}{N_c}\right)$$

also when the control circuit resistance R_c is large the circuit power gain γ is small compared with unity therefore the induced voltage in the coil on the unsaturated core is

$$E_a \mp \frac{\bar{i}_c R_a N_c}{N_a}\left(\text{by putting } \gamma = \frac{R_a}{R_c} \cdot \frac{N_c^2}{N_a^2}\right)$$

so that when core A is saturated the transductor voltage is

$$e_t = e_{tB} = E_a - \frac{i_c R_a N_c}{N_a} \text{ (}\textit{Figure}\text{ 3.8}b\text{)} \qquad . \quad . \quad . \quad (3.29)$$

and when core B is saturated

$$e_t = e_{tA} = E_a + \frac{\bar{i}_c R_a N_c}{N_a} \text{ (}\textit{Figure}\text{ 3.8}a\text{)} \qquad . \quad . \quad . \quad (3.30)$$

Since the resistance in the control circuit is large, the control current is determined by the control voltage and is always constant at the value $\bar{i}_c$. The instantaneous e.m.f. supported by the a.c. circuit resistance R_a during the conducting period is $E_a - e_t$; and this is $\mp \frac{\bar{i}_c N_c}{N_a} \cdot R_a$, the sign depending upon which

core is saturated. It is now important to note two points :

(*a*) When one of the cores is saturated the a.c. winding on the unsaturated core has an effective impedance in the a.c. circuit which is due to its own inductance in parallel with the referred resistance from the control circuit (*Figures* 3.2 *c* and *d*). The former has been assumed infinite and the latter very large. Therefore, when one core is saturated, the current in the a.c. winding will be maintained at a constant value by the very large inductance of this winding.

(*b*) The m.m.f.s in the cores *A* and *B* are :

$$F_A = i_c N_c + i_a N_a$$

and $F_B = i_c N_c - i_a N_a$ respectively.

The control current is always finite and constant and since an infinitesimal m.m.f. will cause saturation, F_A and F_B can never both be zero simultaneously and hence both cores can never be unsaturated together.

Therefore in this particular case there must exist another mode of operation to enable the current in the a.c. windings to switch from $\frac{\bar{i}_c N_c}{N_a}$ to $-\frac{\bar{i}_c N_c}{N_a}$ once every half-cycle. This corresponds to the mode where both cores are saturated, in which case, of course, the two circuits are isolated from one another and the conditions are governed by the voltages and currents in the individual circuits (*Figure* 3.2*e*).

When the control current is very small the output voltage is rectangular and changes sign when the supply voltage is at a maximum or a minimum. As the control current increases the amplitude of the square wave of output voltage increases, always being equal to $\frac{\bar{i}_c N_c}{N_a} R_a$. Thus as in other cases the load ampere-turns $\bar{i}_a N_a$ balance the control ampere-turns $\bar{i}_c N_c$. Toward the upper end of the characteristic the square wave gradually becomes sinusoidal as shown in *Figures* 3.8*g* to *l*. Over the first part of the characteristic, saturation of one core coincides with desaturation of the other, both cores never being unsaturated together. *Figures* 3.8*a* and *b* show the voltage waveforms existing across the individual windings. The zero lines are displaced by amounts equal to $\pm \frac{i_c N_c}{N_a} R_a$ and the positive and

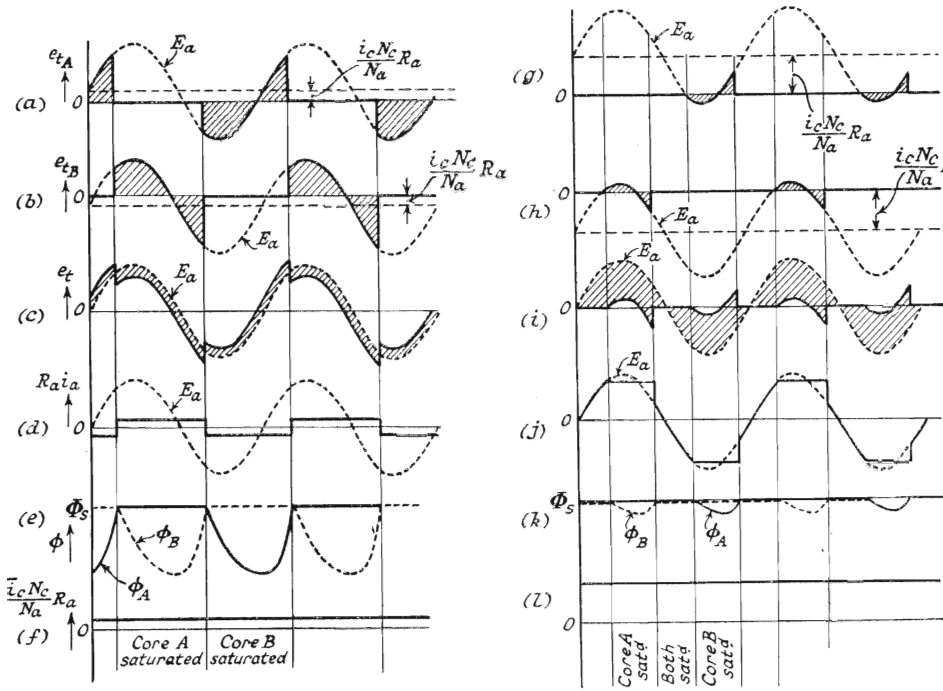


Figure 3.8. Waveforms showing steady-state operation when control circuit impedance is large. (*a*) to (*f*) small control current ; (*g*) to (*l*) large control current. (*a*) and (*g*) e_{tA}, (*b*) and (*h*) e_{tB}, (*c*) and (*i*) transductor voltage, (*d*) and (*j*) load voltage, (*e*) and (*k*) core fluxes, (*f*) and (*l*) control current $\times R_a \frac{N_c}{N_a}$.

negative shaded areas which define the flux changes are equal. The total transductor voltage e_t is equal to $e_{tA} + e_{tB}$ and is shown in Figure 3.8*c*.

When the control current is large the waveform of the voltages across each coil are shown in *Figures* 3.8*g* and *h*. It may be seen by comparison with (*a*) and (*b*) that as the period during which a core is unsaturated decreases there must come a time when both cores are simultaneously in the saturated state for an appreciable interval.

The shaded areas in *Figure* 3.8*c* and *i* denote the magnitude of the output voltage which is at any instant equal to the difference between the transductor voltage and the supply voltage. The output voltage is plotted to its own appropriate zero line in *Figures* 3.8*d* and *j*.

These two sets of diagrams show how the transition occurs between the square wave of load voltage and the final sine wave which must apply when complete saturation is reached.

The two extreme conditions of high and low control circuit impedance, discussed in this section and section 3.7, have been referred to as forced and natural magnetization respectively. In general the core m.m.f.s contain both odd and even harmonic components, the even harmonics are contributed by the current flowing in the control winding while the odd harmonics are due to the load current. If the control circuit has a low impedance the even harmonics of current can circulate freely; this condition has been termed natural magnetization. If the control circuit impedance is high the even harmonics, all of which flow in the control circuit, are blocked, and the coil voltages are distorted to contain even harmonic components; this is known as forced magnetization. Even harmonic distortion of the coil voltages e_{tA} and e_{tB} may be seen in *Figure* 3.8 where the conditions of forced magnetization apply. In the case for a low impedance control circuit (*Figure* 3.6) each coil voltage contains only odd harmonics.

It should be noted that the conditions of forced magnetization are applicable if a large choke is placed in the control circuit. In this case a power gain is achieved but the choke gives rise to delayed growth of control current.

3.10 Reasons for Limited Gain and Response Time

Returning to the low impedance control circuit and the waveforms in *Figures* 3.3 and 3.5, it has been shown that over the conducting period two significant conditions arise. Firstly a comparatively large pulse of control current is drawn from the signal source to maintain ampere-turn balance between the mean current in the a.c. windings and the control current. Secondly, the rate of flux change in the unsaturated core is modified during the conducting period, thus implying that the final flux at the end of the conduction is a measure of the duration of conduction. This final flux is the initial flux for the following cycle and therefore the performance in one cycle depends upon the conditions in the previous cycle, thereby giving rise to a cycle by cycle growth of the output to a steady-state condition.

The pulses of current in the control circuit are, in a sense, an unnecessary drain of energy from the control source which

inevitably limits the amplification. The mean control current in the steady-state is determined only by the control voltage and the circuit resistance. The magnetizing effect of a large proportion of this current, however, is opposed by the load current and only an infinitesimally small amount actually produces a change in the mean flux ϕ_0. The control voltage is therefore much larger than that which would be required to produce the same flux change if all the control current were available to magnetize the core.

Thus the limited power gain combined with a response time extending over several cycles are both disadvantages arising from the fact that a period of coupling occurs between the control circuit and the load circuit during each cycle. It will be shown in Chapter 4 that this coupling, via the cores, may be regarded as a negative feedback over the basic transductor circuit. The latter will be shown to have a high gain which is dependent upon the core properties. This inherent negative feedback, however, makes the overall gain small and the transductor linear and independent of the core properties as shown.

Principles of Self-Excitation

3.11 Self-Excitation

The energy drawn from the control source during a conducting period can be supplied partly or wholly from the main power source by using self-excitation. This is a form of positive feedback whereby the load current is rectified and made to flow in feedback windings similarly disposed to the control windings as shown in *Figure* 3.9*a*. The pulses of control current are demanded during the conducting period which is the time when the load current also flows as similar pulses. This process was in fact originally termed positive feedback, but self-excitation is now almost universally accepted as being preferable.

The circuit in *Figure* 3.9*a* is complicated by the presence of the self-excitation rectifier. In certain circumstances it is possible for the point M to become positive in relation to N in which case all four arms of the bridge become conducting and the current fed back i_f is not equal to the current in the a.c. coils i_a. This effect has important repercussions on the time constant, particularly in the practical case of a core with finite permeability ; it will be discussed in Chapter 8. Meanwhile the simple case will

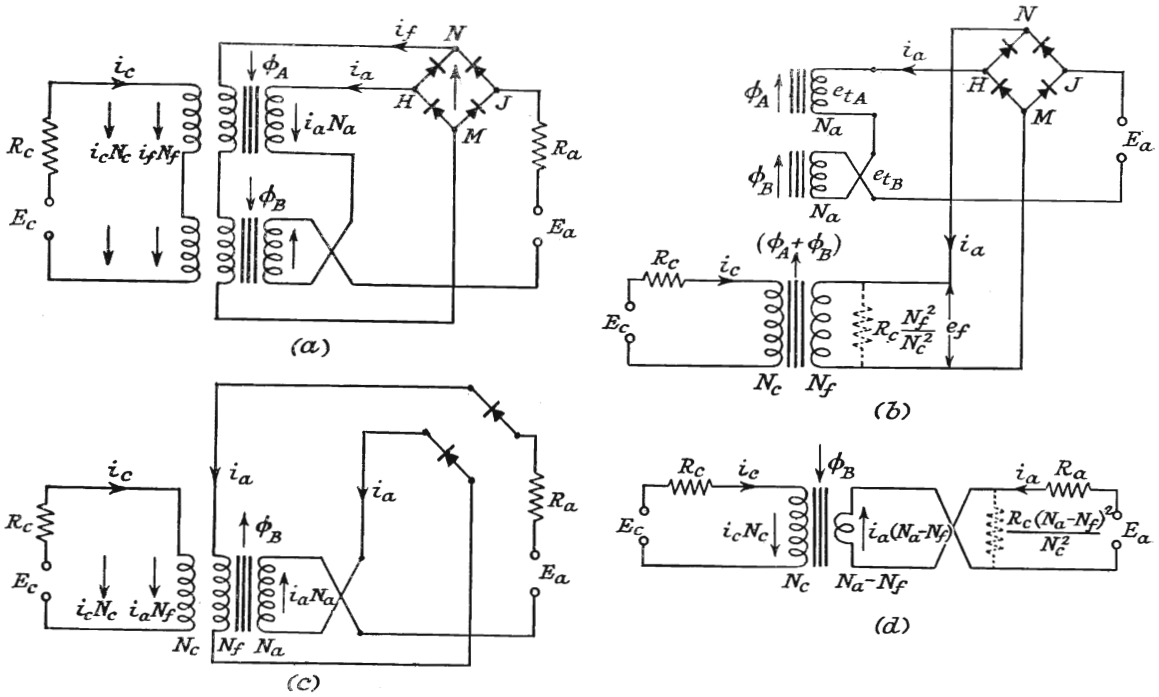


Figure 3.9. Equivalent circuits for self-excited transductors.

(*a*) Self-excited transductor. (*b*) Equivalent circuit when neither core is saturated. (*c*) and (*d*) Equivalent circuits when core *A* is saturated.

be considered on the basis of a paper by Gale and Atkinson.[3(c)] It will be assumed that the rectifiers act as a simple switch in which arms MH and JN are conducting when i_a is positive, and arms HN and MJ conduct when i_a is negative. It will also be assumed that the rectifier is ideal, that is to say, when it is conducting its resistance is zero and when non-conducting the resistance is infinite.

Under normal operating conditions, both the control current and the self-exciting current flow in such a direction as to assist one another. If the sign of the control current is reversed, however, then the self-exciting current i_f will oppose it and the gain will be reduced. Thus the amplifier will be sensitive to the signal polarity.

The relations between the circuit variables will now be determined for the case of a positive control voltage suddenly applied to the control windings. Considering the various possible modes of operation we have firstly the case when both cores are unsaturated.

The control windings and the self-exciting windings are both opposed to the a.c. windings, therefore the a.c. windings will not be coupled either to the control windings or the self-exciting windings. There will, however, be coupling between the latter pair of windings. This state of affairs can be represented by the equivalent circuit in *Figure* 3.9*b* in which the a.c. windings are not coupled to any other windings while the self-exciting windings are coupled to the control windings through a linear inductance in which the core flux is $\phi_A + \phi_B$ as shown.

There are now three components of induced e.m.f. supporting the supply voltage ; the induced e.m.f.s e_{tA} and e_{tB} across each a.c. winding, and the induced voltage e_f across the self-exciting windings (as before the load impedance is neglected during unsaturated periods). The control circuit resistance is small compared with the reactances, however, and therefore the component of voltage e_f will be small compared with e_{tA} and e_{tB} ; thus

$$E_a = e_{tA} + e_{tB} = N_a \frac{d(\phi_A - \phi_B)}{dt} \quad . \quad . \quad . \quad (3.31)$$

Considering the control circuit, if the control voltage is increased from zero to a positive value, a voltage will be induced across the self-exciting windings which tends to drive a current through the rectifier from N to M. This current cannot flow, however, since the rectifiers MJ and HN are cut off and therefore have infinite

resistance. Under these conditions the impedance of the self-exciting circuit as seen from the control circuit is very large and the equation for the control circuit during non-conducting periods will be the same as for the unself-excited transductor thus :

$$E_c = N_c \frac{d(\phi_A + \phi_B)}{dt} = \frac{N_c}{N_a}(e_{tA} - e_{tB}) \quad . \quad . \quad . \quad (3.32)$$

The load current and the control current are zero over this period, as in the case of the transductor without self-excitation. Therefore, provided the assumptions are valid, the operation of the self-excited transductor is the same as that of the simple transductor during the non-conducting period, and the values already calculated for the rates of increase and decrease, of flux, ψ_1 and ψ_2 in equations (3.5) and (3.6), are applicable to the present case.

The second mode of operation corresponds to that in which one of the cores is saturated (core A, say). Since core A is saturated all the coils on this core may be represented as short circuits. The equivalent circuit for this mode is shown in *Figure* 3.9*c*.

In this case the self-exciting turns N_f are opposed to the a.c. turns N_a on the unsaturated core, and since the same current flows through both windings, the m.m.f.s due to the two coils are in opposition. The circuit may therefore be simplified further to that shown in *Figure* 3.9*d*. This is identical to that shown for the transductor without self-excitation during the conducting period excepting that the a.c. turns N_a are now replaced by $(N_a - N_f)$. It must be remembered that although the rectifier is omitted in *Figure* 3.9*d*, this is only possible because we are considering one particular half-cycle. The purpose of the rectifier is to route the load current in such a direction through the self-exciting windings that it always aids the control current which implies that it always produces an m.m.f. opposing the current in the a.c. windings of the unsaturated core. This circuit completely defines the self-excited transductor during the conducting period and therefore the induced voltages, the currents and rates of flux change are obtainable from the previous expressions by substituting $N_a - N_f$ for N_a.

As previously the m.m.f. in the unsaturated core is zero so that

$$i_c N_c = |i_a|\,(N_a - N_f) \quad . \quad . \quad . \quad (3.33)$$

(cf. equation 3.14)

This shows that the ratio of load current to control current during conduction is increased by the factor $\frac{N_a}{N_a - N_f}$ when self-excitation is used. The control circuit resistance when referred to the a.c. circuit is equal to $R_c \frac{(N_a - N_f)^2}{N_c^2}$. This represents a considerable reduction and therefore the proportion of supply voltage appearing across the a.c. coil on the unsaturated core is less than it was for the unself-excited transductor. Hence a more pronounced discontinuity occurs in the flux changes for this core at the beginning of the conducting period. The rate of flux change in the unsaturated core during conduction is in fact

$$\psi_3 = \left[\frac{E_c}{N_c} \frac{\gamma_f}{1 + \gamma_f} - \frac{|E_a|}{(N_a - N_f)} \cdot \frac{1}{1 + \gamma_f}\right] \quad . \quad (3.34)$$

where γ_f is equal to

$$\frac{N_c^2}{(N_a - N_f)^2} \cdot \frac{R_a}{R_c} \quad . \quad . \quad . \quad (3.35)$$

(cf. equations 3.16 and 3.11)

The load current and control current may also be obtained by similar substitutions and it may easily be shown by direct integration of equation 3.33 that in the steady-state

$$\bar{\imath}_a = \frac{E_c}{R_c} \cdot \frac{N_c}{N_a - N_f} \quad . \quad . \quad . \quad (3.36)$$

Thus the current amplification, on being compared with that obtained without self-excitation in equation 3.19, is found to be increased in the ratio $\frac{N_a}{N_a - N_f}$. The circuit power amplification G'_p is increased as the square of this ratio and is in fact γ_f.

In the extreme case where the self-exciting turns are equal to the a.c. turns, no current is induced in the control circuit from the a.c. circuit and therefore the only current drawn from the control source is that required to magnetize the core. On the other hand, it is evident that the mean control current is fixed by the control voltage and the control circuit resistance, the control voltage required to produce a given change of load current is therefore infinitely small when the transductor is operating with complete self-excitation. Apart from the power gain, however, the operation of the idealized transductor in respect of the voltage

and current waveform is independent of the self-excitation. Since the power gain is usually high for self-excited transductors, the waveforms are similar to those shown in *Figure* 3.6 rather than *Figures* 3.7 or 3.8. The pulses of control current for a given output become smaller as the self-excitation is increased and the increased m.m.f. ratio leads to an increased gain or alternatively a reduced time-constant for a given gain. Before discussing these points, however, it is necessary to consider the relationships between the transductor voltage, the output voltage and the mean core flux.

3.12 Transductor Voltage and Output Voltage

In this and the following section it is assumed that during conducting periods the flux in the unsaturated core is maintained at a constant value as depicted by the diagrams in *Figure* 3.6*e*. This is implicit in the assumption that the power gain is large.

In Chapter 2 it was noted that the load line drawn on the voltage current characteristic was nearly linear; this may now be understood by referring to *Figure* 3.6*c* and studying the waveforms of transductor voltage and load voltage. Examination of these shows that there is never a time when a voltage exists across the load and transductor simultaneously, thus when one is finite the other is zero, the sum of the two giving the supply voltage. There is therefore never any time throughout the cycle when one of these quantities is positive and the other is negative, and thus for instantaneous values

$$|e_a| + |e_t| = |E_a|$$

This is the required condition for the addition of mean values as shown in section 1.4 and therefore

$$\bar{E}_a - \bar{e}_t = \bar{e}_a = \bar{\imath}_a R_a \quad . \quad . \quad . \quad (3.37)$$

The mean transductor voltage and the mean load current may be measured on a rectifier instrument and if their values are plotted on the no load voltage/current characteristics as a succession of points, one point for each value of control current, the locus of all such points will be a straight line XY as shown in *Figures* 2.3 and 2.7 extending over the range of the transductor. This gives the operating conditions of a transductor with resistive load for any control current.

The slope of the load line is $-R_a$ (assuming zero winding resistance), and if this is small the line becomes horizontal. A

load line having a small slope corresponds to a large maximum output current, and therefore the smallest permissible value of load resistance for a given transductor is usually limited by heating of the windings and low efficiency. While very large values of load resistance give rise to inadequate control ratios and low power output.

3.13 Mean Flux and Transductor Voltage

The mean flux ϕ_0 and the mean output voltage $\bar{e}_a$ have a considerable bearing on transductor properties both in the transient and the steady-states, and since the relationship between these two variables is one which will frequently arise, it will now be discussed in detail. When a transductor is operating in the steady-state the individual core fluxes ϕ_A and ϕ_B are varying symmetrically but anti-phased. The mean flux will be constant and defined as

$$\phi_0 = \tfrac{1}{2}(\phi_A + \phi_B)$$

During a transient, however, a slight asymmetry occurs in the core fluxes which gives rise to a change in the mean flux level ϕ_0. This latter change is only a small one when considered over one half-cycle but it builds up over several cycles, thereby causing a change of mean flux which occurs slowly by comparison with the flux variations in each core which occur at the frequency of the supply. Thus when a control voltage is applied to a transductor the mean flux level is progressively raised in each half-cycle until a steady-state is reached. During the transient, saturation occurs slightly earlier in successive half-cycles. This results in an increased output voltage and a similarly reduced voltage developed across the transductor.

The total flux change which can occur in either core is from negative up to positive saturation, i.e. $2\Phi_s$, and if the supply voltage is $\bar{E}_{as}$ then this flux change will be produced in the absence of a signal. If the supply voltage is appreciably less than this, however, then a finite signal must be applied before any output can appear.

The mean flux level ϕ_0 can therefore be defined in two parts thus : $\phi_0 = \Phi_0 + \phi_c$. Where Φ_0 is a constant and represents the amount by which the peak a.c. flux fails to reach saturation in the absence of a signal. While ϕ_c is the variable component in the mean flux and is applicable when the transductor is operating over the knee of the magnetization curve (see *Figure* 3.10).

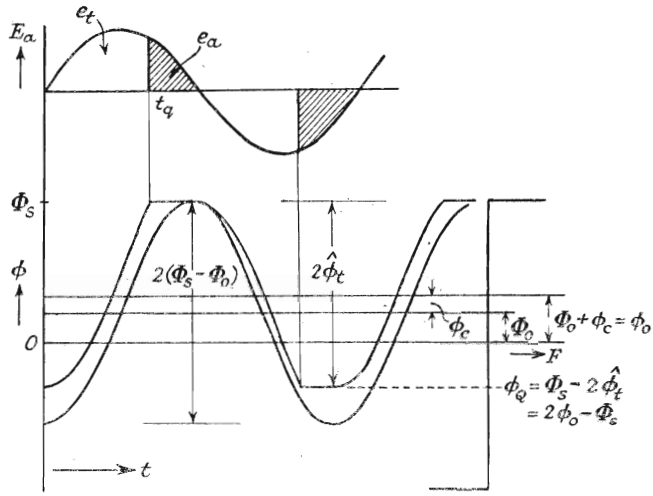


Figure 3.10. Variation of core fluxes in relation to idealized magnetization curve.

If the supply voltage is $\bar{E}_a$ the peak flux change in the absence of a signal is $\Phi_s - \Phi_0$ where

$$\frac{\bar{E}_a}{\bar{E}_{as}} = K = \frac{\Phi_s - \Phi_0}{\Phi_s} \qquad . \quad . \quad . \quad (3.38)$$

Assuming the transductor is just about to operate over the knee of the curve, if a control voltage E_c is applied, the mean flux level will be raised by an amount ϕ_c and the peak value of the alternating flux will be reduced from $\Phi_s - \Phi_0$ to $\hat{\phi}_t$ where

$$\phi_c = (\Phi_s - \Phi_0) - \hat{\phi}_t$$

An output voltage $\bar{e}_a$ will appear and the total voltage across the transductor will be reduced from $\bar{E}_a$ to $\bar{e}_t$, i.e. $\bar{e}_t/2$ across each coil.

From equation (1.23) we have,

$$\frac{\bar{e}_t}{2} \cdot \frac{1}{2fN_a} = \text{total flux change, } 2\hat{\phi}_t,$$

similarly

$$\frac{\bar{E}_a}{2} \cdot \frac{1}{2fN_a} = 2(\Phi_s - \Phi_0).$$

Therefore

$$\frac{1}{8fN_a}[\bar{E}_a - \bar{e}_t] = (\Phi_s - \Phi_0) - \hat{\phi}_t = \phi_c,$$

so that, assuming the waveforms are of the type shown in *Figure* 3.6, we have by combining with equation 3.37

$$\phi_c = \frac{\bar{e}_a}{8fN_a} = \frac{\bar{\imath}_a R_a}{8fN_a} \qquad . \quad . \quad . \quad (3.39)$$

This result does not utilize the ampere-turn balance equation and it is therefore applicable without serious error to all degrees of self-excitation and to practical core materials. This is an important point because equation 3.39 is used in Chapter 4 when the core is assumed to have a finite permeability.

From the preceding discussion we may now consider physically the sequence of events on applying a signal to the control circuit. If the control circuit resistance were actually zero, then the mean control current would increase without limit and eventually the conducting period would extend throughout the cycle. Therefore some resistance must be present however small, so that the ultimate mean value of the control current is limited to $\frac{E_c}{R_c}$.

After the control voltage E_c is switched on the mean core flux (control flux) increases by a small amount in each half-cycle thereby progressively increasing the duration of the conducting period and hence the fraction of the supply voltage applied to the external circuit. This causes a similar increase in the mean current in the a.c. windings which in any one cycle must be proportional to the mean control current due to the constraint imposed by ampere-turn balance.

Therefore the mean current $\bar{\imath}_a$ cannot increase beyond the limit imposed by the control current. The total change of control flux therefore adjusts itself to such a value that the a.c. ampere-turns are balanced against the control and the self-exciting ampere-turns ; that is when

$$(N_a - N_f)\bar{\imath}_a = \frac{E_c}{R_c} N_c,$$

and by combining equations 3.36 and 3.39 we get

$$\phi_c = \frac{R_a}{8fN_a} \cdot \frac{N_c}{N_a - N_f} \cdot \bar{\imath}_c \qquad . \quad . \quad . \quad (3.40)$$

This shows that when the idealized transductor is a.c. excited, the control flux is not determined by the control m.m.f. according to the magnetization curve, but rather tends to be governed by the requirement for ampere-turn balance.

Equation 3.40 applies with reasonable accuracy to transductors for which the self-exciting ampere-turns are not greater than about 80% of the a.c. ampere-turns. In practice, however, the proportion is very nearly 100% and in this case the elementary theory breaks down. Several factors have been neglected, however, the two most important being the finite permeability of the core prior to saturation and the effects arising due to imperfect rectifier characteristics both of which must be taken into account before the self-excited transductor can be examined on a practical basis. This will be done in the following chapter; meanwhile, however, these results will serve as a basis for comparing the properties of transductors with and without self-excitation.

Effect of Self-Excitation on the Properties of Ideal Transductors

3.14 Power Output of Self-Excited Transductor

For a given a.c. circuit resistance and efficiency the maximum power output is proportional to the square of the maximum supply voltage which can be supported by the transductor. Thus, if the a.c. fluxes are varying from positive to negative values just short of saturation the same equations will define the self-excited and the unself-excited transductor, since operation is within the unsaturated range (p. 74). Therefore the supply voltage $\bar{E}_{as}$ will be the same for both cases and the power output of the self-excited transductor will be the same as that for the transductor without self-excitation. Thus, by employing self-excitation, the power amplification has been increased for the same core size and power output.

In the simple transductor, the ampere-turns ratio was found to be unity, and in the self-excited case this ratio becomes equal to $\frac{N_a}{N_a - N_f}$. As the number of turns on the self-exciting winding approaches the number on the a.c. winding, this ratio tends to infinity and in the limit the device becomes unstable. In practice the maximum allowable value for the ampere-turns ratio varies considerably, depending primarily on the magnetic core and the rectifiers ; but 30 is a fairly average figure.

Taking the numerical case given in section 2.8, it is now seen that the useful power amplification has increased by a factor of 900, giving an amplification of 24,300 for a power output of one

watt. This assumes that the characteristic is linear; in practice this is not so and the improvement is not so great as that depicted here. (See Chapter 4.)

3.15 Time-Constant

A general expression will now be found for the time-constant of a transductor with any degree of self-excitation. The following theory assumes that the response is exponential, that core losses are negligible and that the power amplification is large compared with unity so that the time-constant will be large compared with the period of the a.c. excitation. It is also assumed that no other windings are coupled to the control windings.*

If a control voltage E_c is suddenly applied to the transductor at a time $t = 0$ from the initial conditions $\phi_0 = \Phi_0$ the initial growth of flux will be governed by the equation

$$E_c = N_c \frac{d\phi_A}{dt} + N_c \frac{d\phi_B}{dt} \dagger$$

and since the mean flux $\phi_0 = \frac{\phi_A + \phi_B}{2}$

we have $$\frac{E_c}{2N_c} = \frac{d\phi_0}{dt}.$$

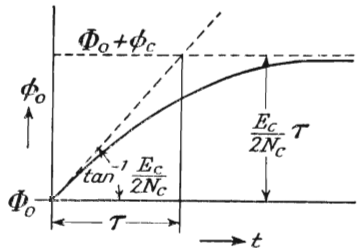


Figure 3.11. Curve showing growth of mean core flux with time.

This gives the slope of the tangent to the flux-time curve at the instant $t = 0$. It may easily be shown that the time-constant is defined by the point at which this tangent cuts the final value of flux $\phi_c + \Phi_0$ as shown in *Figure* 3.11.

* This implies that any voltage changes applied to the control circuit are in such a direction that the self-excitation rectifier blocks the flow of induced currents in the self-exciting windings. This is normally valid when the control voltage is increased, but not when it is decreased. In the latter circumstances the self-exciting windings are liable to be coupled to the control windings (Chapter 8).

† The initial condition specified here ensures that the transductor is operating over the linear range (Section 3.13). Provided the initial mean flux is equal to or greater than Φ_0 then the operation is linear. In practice this point should not arise as the a.c. supply voltage is adjusted to ensure that the operating range is a maximum and Φ_0 is therefore zero, in which case any initial flux may be chosen.

Thus
$$\int_0^{\tau} \frac{E_c}{2N_c} \cdot dt = \int_{\Phi_0}^{\phi_c + \Phi_0} d\phi_0$$

giving
$$\phi_c = \frac{E_c}{2N_c}\tau$$

therefore from equation 3.39
$$\frac{\bar{e}_a}{8fN_a} = \frac{E_c}{2N_c}\tau$$

where $\bar{e}_a$ is the mean output voltage appearing as a result of the control voltage E_c.

Thus
$$\tau = \frac{1}{4f} \cdot \frac{\bar{e}_a}{E_c} \cdot \frac{N_c}{N_a} \quad . \quad . \quad . \quad (3.41)$$

or multiplying and dividing by $\bar{\imath}_a/\bar{\imath}_c$
$$\tau = \frac{1}{4f} \frac{\bar{e}_a\bar{\imath}_a}{E_c\bar{\imath}_c} \cdot \frac{N_c\bar{\imath}_c}{N_a\bar{\imath}_a} \quad . \quad . \quad . \quad (3.42)$$

Equation (3.41) gives the time-constant in terms of the voltage amplification thus :
$$\tau = \frac{1}{4f} \cdot G_v \frac{N_c}{N_a} \quad . \quad . \quad . \quad (3.43)$$

Equation 3.42 expresses the time-constant in terms of the circuit power gain G_p' and the m.m.f. ratio G_m thus :
$$\tau = \frac{1}{4f} \cdot \frac{G_p'}{G_m} \quad . \quad . \quad . \quad (3.44)$$

Although these expressions for the time-constant do not depend, to a first approximation, on the cores having infinite permeability it must be remembered that they are only applicable when the power gain is large.

If there is no self-excitation the ampere-turns ratio is unity and equation 3.44 becomes
$$\tau = \frac{G_p'}{4f}$$

which is the same result as the limiting value obtained using the step-by-step treatment for the special case of zero self-excitation in section 3.4.

Thus it is seen that as the ampere-turns ratio G_m increases, the time-constant for a given power amplification is reduced. Self-excitation has the effect of increasing the ampere-turns ratio and

therefore, *for a given power gain* transductors have a faster response when using self-excitation.

In the numerical example in section 2.8, the useful power gain was found to be 27. In order to determine the time-constant we must find the circuit power gain from the relation

$$G'_p = G_p \frac{2r_{cw}}{R_c} \cdot \frac{1}{\eta}$$

where r_{cw} is the resistance of one control winding. Assuming that the transductor input is matched to the source $\frac{2r_{cw}}{R_c} = \frac{1}{2}$ and therefore $G'_p = 16{\cdot}9$. The time-constant given by equation 3.21 is ·085 seconds. If the ampere-turns ratio is increased by a factor of 30 then the time-constant of the self-excited transductor is now 2·5 seconds. The circuit power gain, however, has been increased by a factor of 900, and although self-excitation has increased the time-constant, the ratio of time-constant to power amplification has been reduced.

3.16 Effect of Over-Self-Excitation

If a transductor is over-self-excited then a triggering action results; this will be considered briefly.

The characteristic of a transductor without self-excitation is shown in *Figure* 3.12*a* in which $\bar{\imath}_a N_a$ is plotted against $\bar{\imath}_c N_c$.

If a self-exciting winding having N_f turns is also available, then, assuming perfect rectification, the mean self-exciting ampere-turns may be plotted against the mean alternating ampere-turns giving a staight line OQ of slope $\frac{N_a}{N_f}$.

The total mean direct m.m.f. required to produce a load current OL, for example, is LN of which LM is supplied by the self-excitation windings and MN is supplied by the control windings. If the magnitude of all lines such as MN is plotted against load current, the control characteristic for a self-excited transductor is obtained.

In the case of negative control currents the relevant value of control current is represented by lines such as MN_1. This is greater than LM and therefore a larger negative control current is required than in the unself-excited case. This leads to a characteristic such as that shown in *Figure* 3.12*b*.

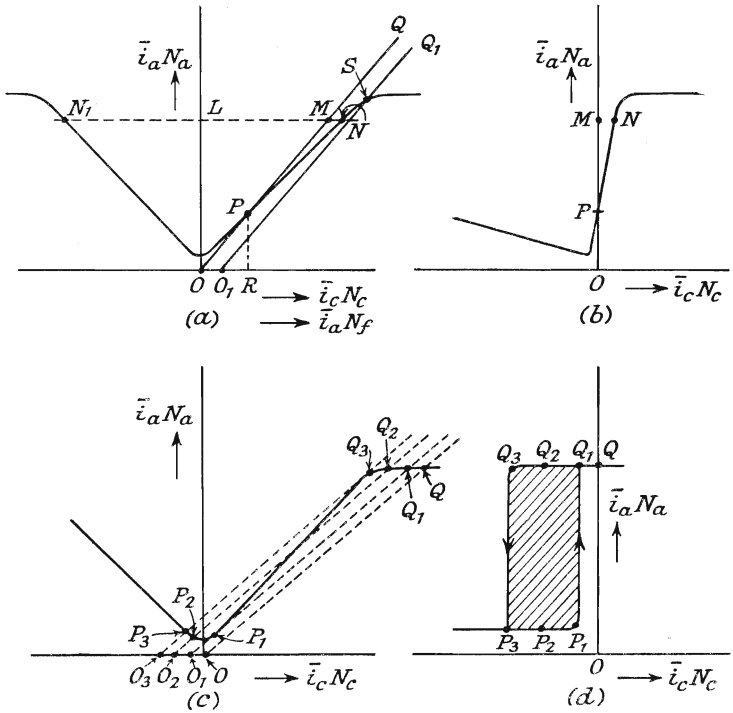


Figure 3.12. Graphical constructions for ideal transductor with varying degrees of self-excitation.

(*a*) Feedback locus with $N_f < N_a$.
(*b*) Control characteristic with $N_f < N_a$.
(*c*) Feedback locus with $N_f > N_a$.
(*d*) Control characteristic with $N_f > N_a$.

In the absence of a signal, the operating point must be on both the straight line and the curve, and therefore the load current is given by PR. In order to determine the working point for any control current OO_1 say, it may be found by drawing O_1Q_1 parallel to OQ, and the new intersection S gives the required working point (*Figure* 3.12*a*).

If $N_f > N_a$ then the device becomes unstable and the load current will trigger for certain critical values of control current. This may be seen from *Figure* 3.12*c*. The feedback line for zero control current is OQ and the working point is Q ; as the control current increases in the negative direction Q moves to the left, e.g. along Q, Q_1, Q_2, etc. If the control current is increased by a small amount beyond OO_3 then the device triggers and the load current suddenly decreases to a value corresponding to the point

P_3. On decreasing the control current, the load current assumes values corresponding to P_3P_2, etc., until at P_1 the current triggers back to Q_1. The characteristic of a transductor which is over self-excited is shown in *Figure* 3.12*d*, in which the shaded area is unstable.

3.17 Summary

In this chapter the transductor has been considered on the basis of idealized core characteristics. It has been shown that following the application of a control voltage to the input, the output voltage builds up to a steady-state value over a number of cycles which, if large, is approximately proportional to the circuit power gain G_p'. When the gain is large it is found that the relationship between gain and time-constant is given by

$$\tau = \frac{G_p'}{4f}$$

and this result is the same for square wave as for sine wave excitation.

The steady-state mode of operation of the transductor also depends upon the circuit power amplification. It is shown that with a low impedance control circuit, i.e. a high power gain, the waveforms of voltage and current are pulses having the form of parts of a sine wave, while with a high impedance control circuit the current waveform tends to be square. In either case ampere-turns equality very nearly exists between the control circuit and the load circuit over the operating part of the transductor characteristic. It is also shown that the mean output voltage, corresponding to the ungated portion of the supply voltage, is decided only by the mean flux level in the cores thus :

$$\bar{e}_a = 8fN_a\phi_c.$$

The greater part of the control current flows during conducting periods over which time the load current flows also. These two currents provide m.m.f.s which act in opposition and hence the actual mean flux change occurring is very small for a given control voltage.

The power gain of this circuit is therefore limited by the opposing effect of the a.c. ampere-turns.

This effect may be partly or completely eliminated by applying self-exciting ampere-turns to the core, these can be arranged to balance the a.c. ampere-turns and thereby enable the control

current to produce a much greater change in the mean flux level for a given control voltage. In addition, however, self-excitation increases the ampere-turns ratio G_m from unity up to some comparatively large value, with the result that two valuable improvements are obtained by the use of self-excitation, firstly an increased power amplification for the same maximum power output and secondly a greater ratio of power amplification to time-constant. The general form of the time-constant equation is in fact given by

$$\tau = \frac{1}{4f} \frac{G'_p}{G_m}.$$

CHAPTER 4

THE SELF-EXCITED TRANSDUCTOR

4.1 Limitations of the Simplified Treatment

THE analysis of the transductor given in the previous chapter has explained the transient and steady-state operation of the device assuming ideal characteristics. When, in the self-excited transductor, the self-exciting turns were made equal to the a.c. turns, the current gain was found to be infinite. When the permeability is assumed finite this is not true, however, and in practice the gain is largely determined by the properties of the core and self-excitation rectifier. The time-constant also tends to be greater than that predicted due to the fact that, in certain circumstances, currents can circulate in the self-exciting windings through the bridge rectifier from M to N (*Figure* 3.9*a*). When this occurs the only resistance in series with these windings is their own resistance and that of the rectifier. The e.m.f.s induced in this closed circuit therefore cause currents which tend to delay changes of the mean flux in the cores during each half-cycle. Thus a smaller increment of flux occurs per cycle and therefore a larger number of cycles are required before the transductor reaches its steady-state of operation. In the case of a d.c. input the steady-state is defined by the maximum value of the control current and this is only limited by the resistance of the control windings.

By considering the rectifier characteristic to be divided into linear zones and utilizing the equations appropriate to each zone it would doubtless be possible to approximate as closely as desired to the practical case. Such a treatment, however, would be very laborious due to the frequent discontinuities arising either in the core or the rectifier and it is doubtful whether a result obtained by these methods would justify the work involved even if such a

result were obtainable in a general form. The reason for this is that core materials and rectifiers are only reproducible to within certain limits, these limits will therefore dictate the degree of accuracy to which it is worth while analysing the problem.

In this chapter transductor performance will be considered on the basis of a more realistic magnetization curve combined with circuit conditions usually encountered in practice. This leads to equations which are directly applicable to design problems.

Hitherto the transductor load has been considered as a part of the a.c. circuit resistance. Usually the load is some d.c.-operated device such as a meter or relay. In these circumstances the necessary rectifier is, of course, considered as a part of the a.c. load. Sometimes the d.c. load is placed in series with the self-exciting windings; this often gives rise to inferior operating characteristics, particularly when the load is inductive, but makes the provision of a load rectifier unnecessary.

Before discussing the transductor, however, the operation of the self-excitation rectifier must be examined, since its behaviour during the period when both cores are unsaturated has an important bearing on the circuit performance, particularly where the response time is concerned.

4.2 Action of the Self-Excitation Rectifier

Additional time lags and modified steady-state operation of a transductor arise due to the closed circuit formed by the rectifier through the self-exciting windings. Such a closed circuit only exists, however, when all the elements in the rectifier are conducting. The circumstances under which these conditions arise depend upon the effective impedance connected to the d.c. terminals of the bridge rectifier. The self-exciting windings are partly resistive and partly inductive, furthermore they are coupled to the control windings when both cores are unsaturated and therefore the impedance of the control circuit will affect the impedance of the feedback circuit as seen by the rectifier. These factors must be examined before it is possible to consider the actual operation of the transductor itself. In the first instance the problem of inductively loaded rectifiers will be considered without particular reference to the transductor.

Referring to the circuit in *Figure* 4.1*a* the bridge rectifier is connected to an a.c. supply through a high impedance Z. The d.c. terminals of the rectifier are connected to an inductive circuit

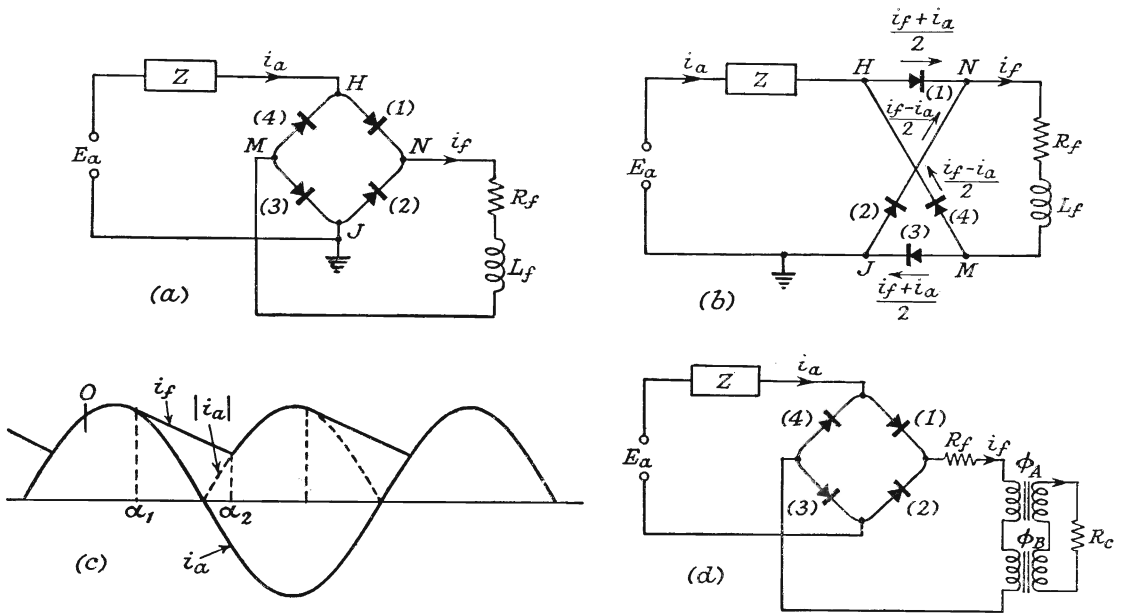


Figure 4.1. Diagrams showing rectifier-fed inductive circuits and the resulting relaxation periods during rectification.

(*a*) and (*b*) Inductive circuit connected to bridge rectifier. (*c*) Waveforms of alternating and direct current. (*d*) Rectifier-fed coupled circuit for self-exciting winding.

and the instantaneous currents flowing in the a.c. and d.c. circuits are i_a and i_f respectively. This may be compared with the equivalent circuit for the self-excited transductor with both cores unsaturated (*Figure* 3.9*b*) in which the impedance Z corresponds to the a.c. windings in series with R_a while the d.c. inductive load corresponds to the self-exciting winding N_f. In the previous case it was assumed that when the a.c. was positive the rectifier elements (1) and (3) were conducting, and when it was negative, elements (2) and (4) were conducting, the rectifier unit was therefore considered to act like a switch in which the switching was governed by the sign of the current in the a.c. circuit. In these circumstances the d.c. i_f must always be equal to the rectified a.c. $|i_a|$ since at any instant there is only one closed circuit and therefore i_a and i_f are the same current; the rectifier merely routes the a.c. always in the same direction through the coil.

A metal rectifier, however, does not behave like a switch in this sense and the conduction or non-conduction of a rectifier element is determined by the voltage and current relations applying to that particular element.

Typical voltage/current curves for a number of different types of rectifier element are shown in *Figure* 4.2 where it is seen that the voltage required to cause a current flow through the rectifier in the positive (forward) direction is much less than that required in the negative (reverse) direction. As the positive current decreases to zero, the resistance of the element rapidly increases and becomes very high when the voltage across the element becomes negative. A bridge rectifier has four such elements and the resistance of each one will be dependent upon the voltage across it. In the first instance it will be assumed that each element is ideal, that is to say, the reverse resistance is infinite, but the slope changes discontinuously at the origin and the forward resistance becomes very low.

A rectifier element will therefore conduct until the current flowing in it decreases to zero, after which its resistance becomes very large and remains so until the voltage across it just goes positive.

When the element is conducting it has a low impedance and the current can either increase or decrease. A decrease in the current can be considered as a negative current superimposed on the positive current, and therefore when conducting the element is capable of allowing components of current to flow in the reverse

direction provided the total element current is greater than zero. On the other hand, when a reverse voltage exists across an element, current cannot flow *in either direction* until the reverse voltage is zero. The circuit in *Figure* 4.1*a* can be redrawn in the less familiar form shown in *Figure* 4.1*b*. If the a.c. i_a is assumed to be positive as shown, then considering i_a to be increasing near to its peak value at O say (*Figure* 4.1*c*), the voltage across the d.c.

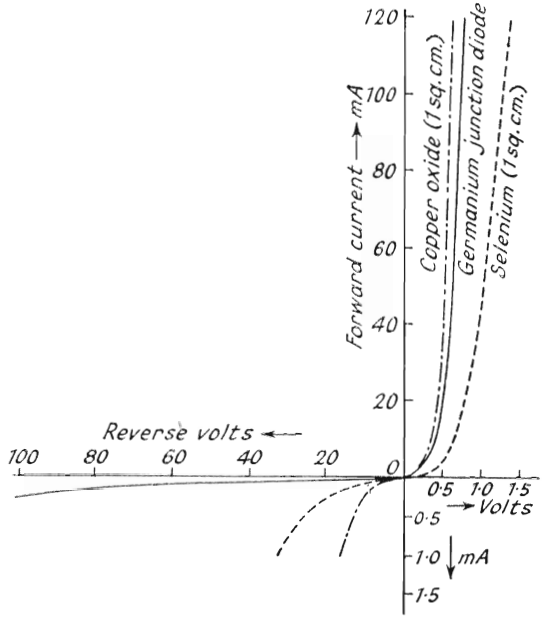


Figure 4.2. Characteristics of copper oxide, selenium and germanium rectifiers.

load will be due to the e.m.f.s in the coil and in the resistance. The e.m.f. in the coil opposes the increase of current and that in the resistance opposes the current. Both e.m.f.s therefore tend to make N and H positive with respect to J and M; elements (2) and (4) are therefore cut off and the current in the coil is the same as that in the a.c. circuit. When the a.c. passes its peak the resistance still opposes the current, but the e.m.f. $L_f \frac{di_f}{dt}$ in the coil now reverses and tends to *maintain* the current. As the current decreases there comes a time when the induced e.m.f. in the coil is greater than that in the resistance, and when this occurs (at a time corresponding to α_1) M and J become positive with

respect to H and N. As soon as this happens, elements (2) and (4) conduct and d.c. circulates in the parallel paths MJN and MHN; this current dies away with a time-constant equal to $\frac{L_f}{R_f}$, while the a.c. decreases to zero.

During the time when all four elements are conducting, the current in elements (1) and (3) is equal to $\frac{i_f + i_a}{2}$ and the current in (2) and (4) is $\frac{i_f - i_a}{2}$. When the a.c. changes sign and starts increasing negatively, the current in (1) and (3) will become zero at α_2, that is, when $i_f = |i_a|$.

The previous events are now repeated except that the a.c. flows through the inductance via elements (2) and (4) while (1) and (3) are cut off.

The rectifying cycle is therefore divided into two periods, the current forcing period and the relaxing period. In the former case the d.c. is equal to the numerical value of the a.c., while in the latter case the d.c. is partly smoothed and is not numerically equal to the a.c. since the rectifier provides an additional path through which the current tends to be maintained in the inductance.

If the impedance across the rectifier is purely resistive, no relaxation occurs since there is no induced e.m.f. tending to maintain the current. If, on the other hand, it is inductive and the time-constant is large, the rate of decay during relaxation periods will be small so that the d.c. will be substantially constant and equal to the peak a.c. In this case all four rectifiers are conducting throughout the greater part of the cycle.

Considering the transductor over the period when both cores are unsaturated, the inductance across the terminals of the rectifier corresponds to that of the self-exciting windings which, as we have seen, are magnetically coupled to the control windings (*Figure* 4.1*d*) ; the impedance in the control circuit will therefore influence the current in the self-exciting circuit. If the control circuit impedance is very small, then variations of the self-exciting current induce equal and opposite variations of current in the control circuit by normal transformer action. The resultant current variations are such as to produce no change in the total flux ($\phi_A + \phi_B$) linking the windings. Furthermore, the control circuit resistance makes the self-exciting windings appear substantially resistive and it follows that no relaxation of the

rectifier occurs so that the self-exciting current is always equal to the current in the a.c. windings.

If the control circuit resistance is large, the effective impedance, seen by looking into the self-exciting windings, will be substantially the inductance of these windings, and since, in general, their resistance is small the time-constant of the self-exciting circuit is large. The current therefore tends to remain constant during relaxation periods although the alternating current is varying.

Thus whether the control circuit impedance is high or low, the currents are such as to maintain a constant value of total flux $(\phi_A + \phi_B)$ during unsaturated periods. It should be noted that this does not impose a restriction on the individual fluxes ϕ_A and ϕ_B, which are in fact varying over their full range; it does, however, restrain them in such a way that their mean value remains constant. Equal and opposite changes in the fluxes ϕ_A and ϕ_B cannot be detected in the self-exciting circuit or in the control circuit since they produce e.m.f.s which cancel.

4.3 Amplification of Self-Excited Transductor in the Steady-State

In the previous chaper the current-amplifying properties of the transductor were considered, and although the current amplification is relevant in many cases, occasions frequently arise when it is more convenient to consider the output voltage in terms of the control voltage. There are three reasons for this preference. Firstly, the input signal is usually a voltage applied to the control circuit; the resulting control current tends to lag behind this voltage, thereby introducing a time delay. By considering the control current as the input, this time delay is ignored. Secondly, since the transductor is a gating device in an a.c. circuit, when saturation takes place during the cycle the supply voltage is ungated and applied to the load circuit ; the output of the transductor is therefore a voltage corresponding to the ungated portion of a sine wave. Thirdly, in certain cases the load current is not equal in magnitude to the current in the a.c. windings. This applies when the load is partly reactive (see Chapter 6) and the actual current flowing in the load is obtained by considering the voltage applied to the load, namely, the output voltage from the transductor. The general expression for the voltage amplification will therefore first be derived, from which the current amplification will be readily determined for the special conditions of a resistive load and steady-state operation.

The last of these points makes it necessary to introduce some new symbols for the load circuit impedance. The resistance R_a has been defined as the total resistance in the a.c. circuit which will include the load R_L and the total internal impedance r_x. If rectifiers are included in this path their impedance is taken into account by adding up the total number of conducting elements in the current path and assuming each one to have a constant resistance.

If the load is purely resistive and fed through a bridge rectifier, the load current i_L is equal in magnitude to the transductor current i_a at all times. In addition, the output voltage divided by this current gives the total resistance R_a whether we consider mean or instantaneous values, thus

$$\frac{e_a}{i_a} = \frac{\bar{e}_a}{\bar{i}_a} = \frac{\bar{e}_a}{\bar{i}_L} = R_a \text{ (for d.c. resistive load)} \quad . \quad . \quad (4.1)$$

If, however, the load is partly inductive or capacitive some of these relations no longer apply due, for example, to the combination of a bridge rectifier with an inductive load. The following symbols will now be introduced and defined in order to treat the general case :

(1) R_a = total a.c. circuit resistance in forward direction.

(2) $\bar{R}_a = \frac{\bar{e}_a}{\bar{i}_a}$.

(3) $\bar{R}_{aL} = \frac{\bar{e}_a}{\bar{i}_L}$.

Particular values for $\bar{R}_a$ and $\bar{R}_{aL}$ for the various types of rectifier load will be found in Chapter 6 in terms of the circuit constants, etc. Meanwhile for a resistive load

$$\bar{R}_a = \bar{R}_{aL} = R_a = R_L + r_x.$$

4.4 Assumptions

In the following discussion it will be assumed that the core magnetization characteristic is of the form shown in *Figure* 3.1c, in which the slope is large but finite in the unsaturated region. Saturation is assumed to occur suddenly when the flux reaches the value Φ_s after which the slope is zero.

It is also assumed that the total resistance R_a in the a.c. circuit is small compared with the unsaturated reactance $2X_{aw}$ of the

a.c. windings. This implies a large value for the control ratio given by $m = 2X_{aw}/R_a$ (section 2.5).

When using self-excitation the unsaturated reactance of the a.c. circuit is still determined by that of the a.c. windings alone. The self-exciting windings do not limit the current reactively since they are either coupled to the control windings having a low impedance, or, alternatively, if the control circuit impedance is high, full relaxation occurs and all four elements of the self-exciting bridge rectifier are conducting, thereby providing a circuit for the a.c. which does not include the self-exciting windings. With most core materials the transductor control ratio is usually over 15 and is sometimes as high as 50 and therefore the assumption of a high reactance/resistance ratio is justified.

Iron losses will be neglected at first but their effects will be considered in Chapter 5.

It was shown in Chapter 3 that the flux changes in the unsaturated core during the conducting period are partly due to the control voltage and partly due to the component of supply voltage which appears across the coil linking the unsaturated core. In the case of the transductor with self-excitation or a low control circuit resistance, the flux change over this period is very small, and the following discussion will be based on the fact that in the steady-state the flux in the unsaturated core remains constant throughout the conducting period. The flux changes are therefore assumed to be of the form shown in *Figure* 3.6*e*. It will be remembered that these conditions, corresponding to a very small value of ψ_3, are given by a high circuit power gain. On this basis the m.m.f.s magnetizing the core can be calculated knowing the core reluctance, and hence a relationship can be obtained between the control current and the mean transductor current. The transductor circuit with self-excitation is shown in *Figure* 4.3 where the total a.c. circuit resistance comprises the load R_L plus the a.c. and self-exciting winding resistances $2r_{aw}$ and $2r_{fw}$ respectively; in addition the forward resistance of two rectifier elements $2r_{ar}$ should be included together with the a.c. source impedance r_{ag}, thus $R_a = R_L + r_x$

where r_x, the total internal resistance, is in this case given by

$$r_x = 2(r_{aw} + r_{fw} + r_{ar}) + r_{ag}.$$

The load can be placed in series with the self-exciting windings, in which case the same rectifier will serve for both the load and

self-excitation; both types of connection will be discussed. Firstly, the circuit shown in *Figure* 4.3 will be considered.

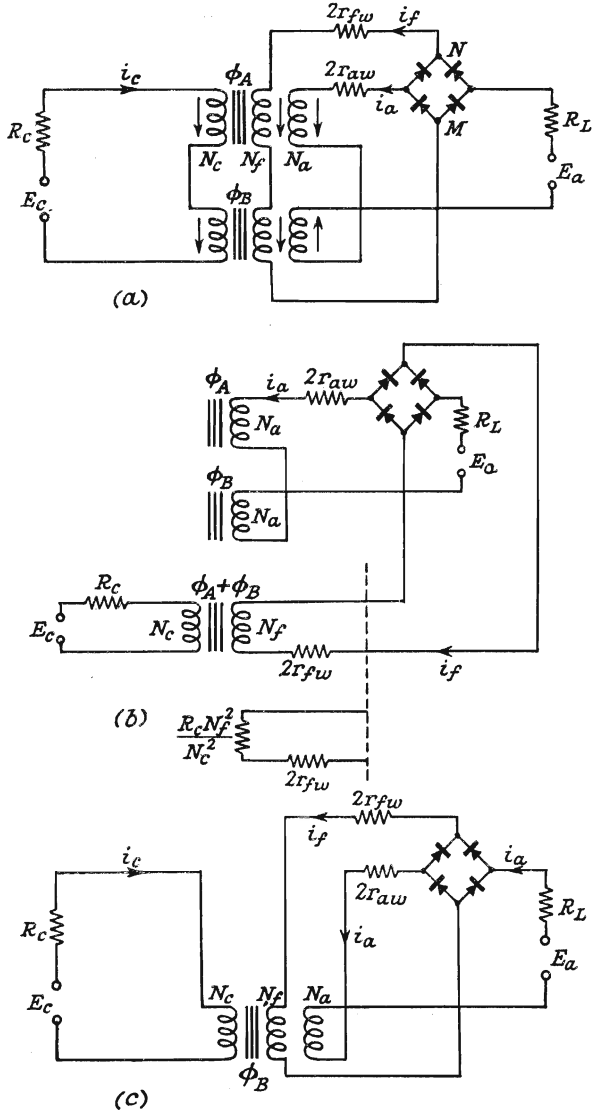


Figure 4.3. Circuits of self-excited transductor.

(*a*) General circuit.
(*b*) Circuit applicable to the time when both cores are unsaturated.
(*c*) Circuit applicable to the time when one of the cores is saturated.

4.5 Analysis of Voltage Amplification

Referring to *Figure* 4.4*a*, the flux changes in cores (*A*) and (*B*) are shown plotted to a vertical time scale. The mean flux ϕ_0 is shown and the a.c. fluxes vary symmetrically about ϕ_0 with a peak value $\hat{\phi}_t = \Phi_s - \phi_0$. These fluxes are drawn in relation to the magnetization curve (*Figure* 4.4*b*).

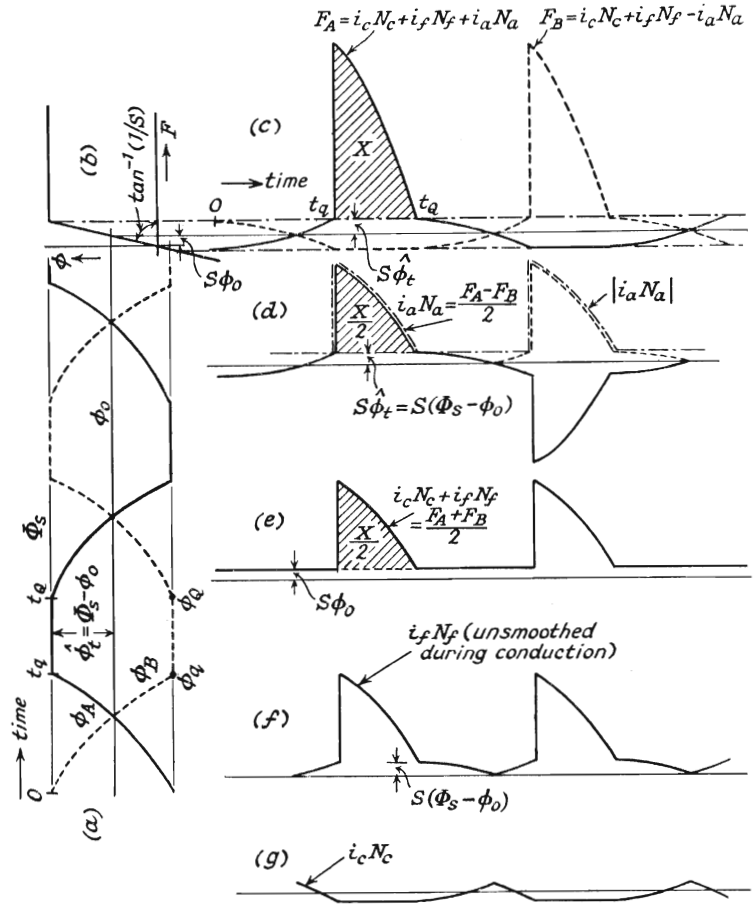


Figure 4.4. Flux, m.m.f., and current waveforms for a self-excited transductor.

(*a*) Flux variations.
(*b*) Magnetization curve.
(*c*) m.m.f. variations.
(*d*) m.m.f. due to alternating ampere-turns.
(*e*) m.m.f. due to sum of control and self-exciting ampere-turns.
(*f*) Ampere-turns due to self-excitation for $R_c = 0$.
(*g*) Ampere-turns due to control current for $R_c = 0$.

If the mean supply voltage is some fraction $K = \dfrac{\Phi_s - \Phi_0}{\Phi_s}$ of the optimum value $\bar{E}_{as}$, then the mean flux can increase up to Φ_0 without causing saturation during any part of the cycle. The total flux swing corresponding to the mean supply voltage $K\bar{E}_{as}$ will be $2(\Phi_s - \Phi_0)$ while the flux swing corresponding to the supply voltage $\bar{E}_{as}$ will be $2\Phi_s$ (section 3.13). The mean flux ϕ_0 therefore consists of a flux Φ_0 which is just sufficient to cause saturation and a further flux ϕ_c, the magnitude of which will determine the length of the conducting period and hence the mean output voltage where

$$\phi_0 = \Phi_0 + \phi_c \qquad . \quad . \quad . \quad (4.2)$$

If $K = 1$, then Φ_0 is zero and any finite mean flux will cause saturation.

If the mean flux increases due to an increase of control voltage, then assuming the core is already being driven into saturation over part of the cycle, the peak alternating flux $\hat{\phi}_t$ decreases and the time of saturation t_q occurs earlier in the half-cycle, thereby lengthening the conducting period and increasing the mean output voltage as described in Chapter 3.

By projecting points on the flux curves on to the magnetization curve, the m.m.f. $S\phi$ in each core may be drawn for the period when both cores are unsaturated, the m.m.f.s are shown plotted to a horizontal time scale in *Figure* 4.4*c*. Over the conducting period the m.m.f. in the unsaturated core is constant, and, furthermore, if the transductor is fully self-excited, then, since $i_f = i_a$ over this period, it is seen from *Figure* 4.3*c* that no resultant ampere-turns are applied to the unsaturated core from the a.c. supply. Hence the m.m.f. in the unsaturated core during the conducting period must be that due to the control current only. The flux ϕ_q corresponding to this m.m.f. is shown in *Figure* 4.4*a*. Over the period from $t = 0$ to $t = t_q$ the flux in core (B) is decreasing. Before the conducting period it decreases from the saturation value to the value ϕ_q by the combined action of the supply and control voltages. Although the flux waveforms are assumed to be as shown in *Figure* 4.4*a*, in practice this is not quite true and during conduction a further slight change of flux occurs to the final minimum value ϕ_Q; this latter change of flux is due to the voltage-time applied to the control circuit only, it is of course accompanied by a corresponding small change of control current ; both these changes will be neglected, so that $\phi_Q = \phi_q$ and the control current is assumed constant during conducting periods.

The total m.m.f. in the saturated core during conduction is equal to twice that due to the alternating current i_a since the same current flows in the a.c. and self-exciting windings (see section 3.3). For the present discussion, however, it is not necessary to evaluate the m.m.f. over this period and it is sufficient to denote the shaded area (*Figure* 4.4*c*) over the interval t_a to t_Q by the symbol X.

Since the m.m.f. due to the self-excitation always aids that due to the control then from *Figure* 4.3*a* the instantaneous m.m.f.s in the cores, as we have already seen, are given by

$$F_A = (i_c N_c + i_f N_f) + i_a N_a \quad . \quad . \quad . \quad (4.3)$$

and

$$F_B = (i_c N_c + i_f N_f) - i_a N_a \quad . \quad . \quad . \quad (4.4)$$

therefore subtracting

$$i_a N_a = \frac{F_A - F_B}{2} \quad . \quad . \quad . \quad (4.5)$$

and adding

$$i_c N_c + i_f N_f = \frac{F_A + F_B}{2} \quad . \quad . \quad . \quad (4.6)$$

Referring to *Figure* 4.4*c* the m.m.f.s F_A and F_B each consist of three components.

(1) An m.m.f. of peak value $S\hat{\phi}_t$ which varies within the saturation limits and corresponds to the alternating flux variations.
(2) A constant m.m.f. $S\phi_0$ which corresponds to the mean flux level.
(3) The m.m.f. corresponding to the shaded area X.

If the total m.m.f.s F_A and F_B in each core are subtracted then the constant component $S\phi_0$ disappears; the resulting m.m.f. is proportional to the alternating current i_a as given by equation 4.5. This m.m.f. due to i_a is shown in *Figure* 4.4*d*. It consists of the shaded portion of area $X/2$ and a small variation whose peak value is $S\hat{\phi}_t$. The curve corresponding to the transductor current when rectified is shown dotted as $|i_a N_a|$.

If the core m.m.f.s are added then the m.m.f. due to the control and self-exciting currents is obtained according to equation 4.6. In this case the small variations of peak value $S\hat{\phi}_t$ cancel, but the mean m.m.f. $S\phi_0$ remains. In addition we have as before the shaded component equal in area to $X/2$.

Since both of these latter components shown in *Figure* 4.4*e* are always positive we may add the mean values giving :

$$\bar{i}_c N_c + \bar{i}_f N_f = S\phi_0 + \frac{2}{T}\frac{X}{2} \quad . \quad . \quad . \quad (4.7)$$

If 100% self-excitation is used nearly the whole of this m.m.f. required during the conducting periods is supplied by the self-excitation winding, and the only control current required is the small component which determines the minimum flux ϕ_Q and hence the mean magnetization level in the core. For smaller proportions of self-excitation, the control current must be larger in order to supply a part of the m.m.f. corresponding to the shaded area X in addition to the magnetizing component, and in the case where there is no self-excitation then *Figure* 4.4*e* is the waveform of control current, *d* and *e* are then directly comparable with *Figures* 3.6*d* and *f*.

It is important to note that the particular curve bounding the area X has been chosen arbitrarily and in this case represents the waveform of m.m.f. which would be obtained with a resistive load and a sinusoidal supply, any other waveform gives the same result, however, since X remains unspecified throughout this treatment.

During conduction the self-exciting current is equal to the rectified alternating current $|i_a|$, but during non-conducting periods its value depends upon the rectifying action as described in section 4.2.

Case 1 : *Control circuit resistance large.* In this case the current i_f is assumed to be constant during the non-conducting period; this means that all four arms of the bridge rectifier are undergoing relaxation and the waveform of the self-exciting current i_f will then correspond to the chain-dotted line in *Figure* 4.4*d* thus

$$\bar{i}_f N_a = S(\Phi_s - \phi_0) + \frac{2}{T}\frac{X}{2} \quad . \quad . \quad . \quad (4.8)$$

Case 2 : *Control circuit resistance small.* If the control circuit resistance is very small, then from the results in section 4.2 the self-exciting current i_f will be equal to the rectified alternating current $|i_a|$ not only during the conducting periods, but throughout the cycle. Small variations will therefore occur in the magnitude of the self-exciting

current during the non-conducting period which will cause similar induced currents to flow in the control circuit as shown in *Figures* 4.4*f* and *g*.

If it is assumed that these small variations within the limits of saturations are sinusoidal then their mean value will be $\frac{2}{\pi} \times$ peak value and therefore in the case of a low control circuit impedance we have

$$\bar{i}_f N_a = \frac{2}{\pi} S(\Phi_s - \phi_0) + \frac{2}{T}\frac{X}{2} \quad . \quad . \quad . \quad (4.9)$$

Both equations 4.8 and 4.9 may be written in the common form

$$\bar{i}_f N_a = \lambda S(\Phi_s - \phi_0) + \frac{2}{T}\frac{X}{2} \quad . \quad . \quad . \quad (4.10)$$

where λ varies from $\frac{2}{\pi}$ up to unity as the control circuit resistance increases.

Thus, combining (4.7) and (4.10)

$$\bar{i}_c N_c + \lambda S\Phi_s = \bar{i}_f(N_a - N_f) + (1 + \lambda)S\phi_0.$$

The current flowing in the a.c. circuit during non-conducting periods is very small compared with that flowing during conducting periods, and having established the self-exciting current as accurately as possible, equation 4.7 may be combined with 4.10 and the mean current $\bar{i}_a$ in the load then assumed equal to $\bar{i}_f$.

Thus replacing $\bar{i}_f$ by $\bar{i}_a$ gives

$$\bar{i}_c N_c + \lambda S\Phi_s = \bar{i}_a(N_a - N_f) + (1 + \lambda)S\phi_0 \quad . \quad . \quad (4.11)$$

This relationship is the ampere-turns balance equation taking into account the core reluctance. The simplified form of this given in equation 3.36 is obtained by putting $S = 0$.

If there is more than one control winding, or if other closed windings are coupled to the control winding, the ampere-turn balance equation may be written in the more general form

$$\bar{F}_c + \lambda S\Phi_s = \bar{i}_a(N_a - N_f) + (1 + \lambda)S\phi_0 \quad . \quad . \quad . \quad (4.12)$$

where $\bar{F}_c$ is the total mean control m.m.f. due to all control windings.

Although in this case the steady-state condition has been stressed the same fundamental relation applies to any cycle of

the supply frequency during a transient provided the response time of the transductor is large compared with the period of the supply frequency. The transient and frequency response is considered in Chapter 8, however, and for the present we are primarily interested in the behaviour of the transductor after transients have disappeared.

The following equations are also applicable to the circuit variables

$$\phi_c = \frac{\bar{e}_a}{8fN_a} \quad \text{(equation 3.39)}$$

in the steady-state $\bar{\imath}_a = \dfrac{\bar{e}_a}{\bar{R}_a}$ (by definition)

and $$\bar{\imath}_c = \frac{E_c}{R_c}$$

also $$\phi_0 = \Phi_0 + \phi_c.$$

By substituting these expressions into the ampere-turn balance equation (4.11), an expression for the voltage amplification is obtained (see Appendix 4). Thus

$$\bar{e}_a = \frac{g_v E_c}{1 + g_v h_v} + \frac{\bar{E}_{as}}{1 + g_v h_v}\left[\frac{K(1+\lambda) - 1}{(1+\lambda)}\right] \quad . \quad . \quad . \quad (4.13)$$

where $$g_v = \frac{4}{\pi(1+\lambda)} \frac{\omega M_{ca}}{R_c} \quad . \quad . \quad . \quad (4.14)$$

$$h_v = \frac{R_c}{\bar{R}_a} \cdot \frac{N_a - N_f}{N_c} \quad . \quad . \quad . \quad (4.15)$$

$$K = \text{supply voltage factor } \bar{E}_a/\bar{E}_{as} = \frac{\Phi_s - \Phi_0}{\Phi_s}$$

and as before $\bar{E}_{as} = 8\Phi_s afN_a \quad . \quad . \quad . \quad (4.15a)$

The constant λ lies between $\dfrac{2}{\pi}$ and unity depending upon the control circuit resistance, and M_{ca} is the mutual inductance between an a.c. coil and a control coil when coupled by an unsaturated core. It is shown in section 1.2 that this is equal to $\dfrac{N_a N_c}{S}$ where S is the core reluctance and is in turn equal to $l/\mu a$ (section 1.1). In this context μ, the permeability, is the

slope of the magnetization curve in the unsaturated region; it is termed the sensitivity or gain permeability, thus :

$$M_{ca} = \frac{\mu a N_a N_c}{l}.$$

Equation 4.13 gives the steady-state relation between the control voltage and the mean output voltage for a transductor with any degree of self-excitation and a mean supply voltage $\bar{E}_a$ equal to $K\bar{E}_{as}$. The relationship is given by a straight line of slope $G_v = \dfrac{g_v}{1 + g_v h_v}$ having an intercept on the ordinate (*Figure* 4.5*a*(i)). This intercept corresponds to the output voltage for zero control voltage and is dependent upon the magnitude of the supply voltage through the factor K.

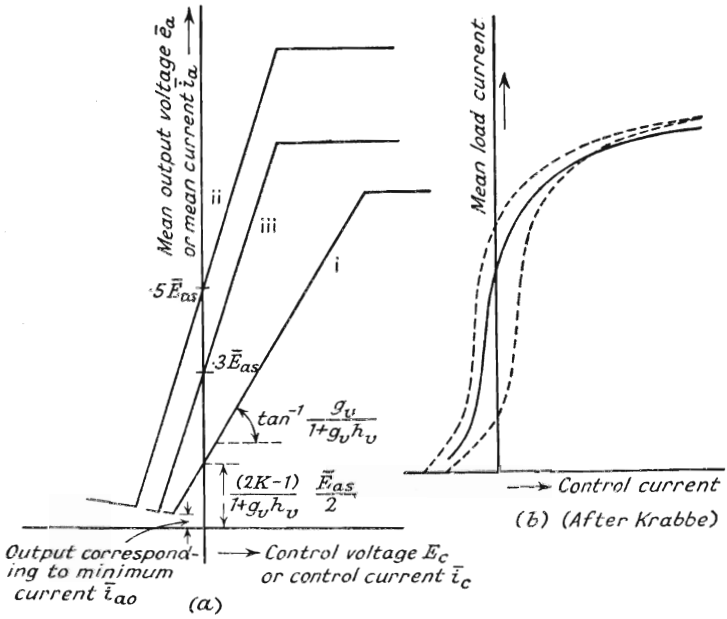


Figure 4.5. Curves showing amplification characteristics of transductor with resistive load.

(*a*) Theoretical curves based on idealized core :

(i) Incomplete self-excitation
(ii) Complete self-excitation $K = 1.0$
(iii) Complete self-excitation $K = 0.8$.

(*b*) Practical control characteristic showing similarity to magnetization curve. Nickel-iron core, magnetization curve shown dotted.

At this stage it is important to realize that nowhere has it been necessary to invoke the condition that the supply voltage is sinusoidal, and hence the above results apply equally well to a square wave of supply voltage. This is primarily because, whatever waveforms apply during conduction, the transductor current i_a is opposed either by a similar current flowing in the self-exciting windings or by a similar current induced in the control windings.

The constant λ affects to a small extent both the slope and the intercept on the characteristics; theoretically it varies from unity down to $\frac{2}{\pi}$ (see p. 101) as the control circuit resistance R_c decreases from a large value to a small value. In future it will be assumed that $\lambda = 1$ unless otherwise stated and this will imply a smooth control current for the condition of 100% self-excitation.

At this stage it is felt necessary to explain an apparent contradiction in the foregoing reasoning. In order to justify the flux waveforms in *Figure* 4.4*a*, it was necessary to assume a low control circuit resistance. Now one is asked to assume that $\lambda = 1$ which implies a high control circuit resistance.

The characteristic feature of the flux waveforms is that, during conduction, the flux in the unsaturated core does not change. This actually depends upon the *referred* value of R_c being small, and during conduction the value of R_c referred to the a.c. circuit is $R_c\left[\frac{N_a - N_f}{N_c}\right]^2$ (*Figure* 3.9*d*). If the self-excitation is appreciable $N_f \rightarrow N_a$ and the control circuit resistance referred to the a.c. circuit is very small.

To meet the second condition ($\lambda = 1$) we are concerned with the referred resistance of the control circuit when both cores are unsaturated and in this case the comparable quantity is the value of R_c referred to the self-exciting circuit, i.e. $R_c\left[\frac{N_f}{N_c}\right]^2$ (*Figure* 3.9*b*) which in practice will be considerably greater than $R_c\left[\frac{N_a - N_f}{N_c}\right]^2$.

Let us now consider the voltage amplification G_v for the transductor under certain conditions of self-excitation. In the special case where the self-excitation is complete, the self-

exciting turns N_f are equal to the a.c. turns N_a and therefore $h_v = 0$. Also putting $\lambda = 1$ gives the result

$$G_v = g_v = \frac{2}{\pi} \frac{\omega M_{ca}}{R_c} \quad . \quad . \quad . \quad (4.16)$$

The mutual inductance M_{ca} can be very large and will be dependent upon the core reluctance and dimensions. Thus if the self-excitation is complete the voltage amplification of the transductor is primarily dependent upon the supply frequency and the properties of the core as would be expected. The above results may now be compared with those obtained in Chapter 3 for the case of zero self-excitation.

If there is no self-excitation in the present case, then h_v is a maximum and equal to $\frac{R_c}{\bar{R}_a} \cdot \frac{N_a}{N_c}$.

In this case $g_v h_v = \frac{2}{\pi} \frac{\omega M_{ca}}{R_c} \cdot \frac{R_c}{\bar{R}_a} \cdot \frac{N_a}{N_c}$

$$= \frac{2}{\pi} \frac{\omega L_{aw}}{\bar{R}_a} = \frac{2}{\pi} \frac{X_{aw}}{\bar{R}_a} \quad \text{(section 1.2)}$$

which is usually large compared with unity since X_{aw} refers to the unsaturated reactance of an a.c. coil. Therefore when there is no self-excitation ($N_f = 0$) the expression for the gain, $\frac{g_v}{1 + g_v h_v}$, may be approximated to $\frac{1}{h_v}$.

Thus for the condition of no self-excitation

$$G_v = \frac{\delta \bar{e}_a}{\delta E_c} = \frac{\bar{R}_a}{R_c} \cdot \frac{N_c}{N_a}.$$

This is independent of the core characteristic and if the load is resistive, we have $\bar{R}_a = R_a$ and the current amplification may be written down as

$$\frac{\delta \bar{i}_a}{\delta \bar{i}_c} = \frac{\delta \bar{e}_a}{\delta E_c} \cdot \frac{R_c}{R_a} = \frac{N_c}{N_a}$$

which is the same result as that obtained in Chapter 3, where it was shown that for the condition of no self-excitation the current amplification is dependent only upon the turns ratio of the control and a.c. windings.

4.6 Transductor Current Amplification

The current amplification will now be considered for the case of a resistively loaded transductor under steady-state conditions. A general expression may be obtained directly from equation 4.13 by putting $\bar{e}_a = R_a\bar{i}_a$ and $E_c = R_c\bar{i}_c$ then dividing both sides of equation 4.13 by R_a gives

$$\bar{i}_a = \frac{g_i}{1+g_ih_i}\,\bar{i}_c + \frac{2K-1}{2}\cdot\frac{\bar{E}_{as}}{(1+g_ih_i)} \quad . \quad . \quad . \quad (4.17)$$

(for smooth control current $\lambda = 1$)

$$\text{where } g_i = \frac{2}{\pi}\frac{\omega M_{ca}}{R_a} \quad . \quad . \quad . \quad (4.18)$$

$$h_i = \frac{N_a - N_f}{N_c} \qquad \text{(See also Appendix 4)} \quad . \quad . \quad . \quad (4.19)$$

It should be noted that the expression in the denominator g_ih_i is equal to the corresponding expression g_vh_v applicable to the voltage gain. Thus the equation only differs from the voltage gain expression by the appropriate impedance factors. Thus, if the self-excitation is complete, then the self-exciting turns N_f are equal to the a.c. turns N_a, in which case $h_i = 0$ and the mean load current is

$$\bar{i}_a = \frac{2}{\pi}\frac{\omega M_{ca}}{R_a}\,\bar{i}_c + (2K-1)\frac{\bar{E}_{as}}{2R_a} \quad . \quad . \quad . \quad (4.20)$$

where the gain has now become equal to g_i. If the mean supply voltage is reduced, the gain is unaffected but the factor K decreases and the standing current for zero signal gets smaller. This is shown in *Figure* 4.5*a* where theoretical current transfer characteristics are shown for the condition of 100% self-excitation. Curve (ii) is for the case where the supply voltage is at its maximum value $\bar{E}_{as}$, while curve (iii) is for a supply voltage of 0·8 $\bar{E}_{as}$.

The minimum load current $\bar{i}_{a0}$ must occur when both cores are unsaturated throughout the cycle of supply voltage; this will be when the total direct m.m.f. is zero and will generally correspond to a negative control current which just balances the positive ampere-turns supplied by the self-exciting winding.

The effect of large negative control currents may be considered as follows. Referring to *Figures* 4.4*a* and *d* it is seen that on *positive half-cycles* of the alternating current core (B) is

unsaturated. If the mean flux were made negative by applying a sufficiently large negative signal, however, then the transductor would operate over the negative half of the magnetization curve and core (B) would be unsaturated on *negative half-cycles* of i_a. Now the ampere-turn balance equations relate to the unsaturated core and furthermore the sign of the self-exciting m.m.f. acts always in the same direction due to the rectifier. Therefore the alternating m.m.f. in the unsaturated core has undergone a phase reversal of 180° in relation to the self-exciting m.m.f. Referring to the equivalent circuit in *Figure* 3.9*d* the effective turns providing a.c. excitation are now given by $N_a + N_f$ and therefore the control current is called upon to supply even greater balancing ampere-turns than would be the case if no self-excitation were applied. The gain is therefore greatly reduced and is in fact given by changing the sign of N_f in equation 4.19.

4.7 Analogy with Negative Feedback Amplifier

Readers familiar with closed-loop control systems will probably have recognized the expressions for the transductor gain of the form $\frac{g}{1+gh}$ as being those of a negative feedback amplifier in which the internal gain is g and the gain in the feedback loop is h (see Chapter 7). This is shown by the block diagrams in *Figures* 4.6*a* and *b*, which represent such an amplifier. *Figure* 4.6*a* refers to the transductor as a voltage amplifier. The input voltage E_c is opposed by a proportion of the output, and the difference is then amplified to give the mean output voltage $\bar{e}_a$.

Thus from the diagram referring to the amplifier of gain g_v

$$\bar{e}_a = g_v(E_c - \bar{e}_a h_v)$$

giving

$$\bar{e}_a = \frac{g_v}{1 + g_v h_v} E_c \text{ as before.}$$

It is important to distinguish between the negative feedback referred to above, and the so-called positive feedback due to self-excitation. In the latter case, the simple transductor having a current gain $\frac{N_c}{N_a}$ was considered to be the fundamental transductor element, the application of positive feedback due to self-excitation was then shown to increase the gain. The errors which arise due to neglecting the core reluctance have already been

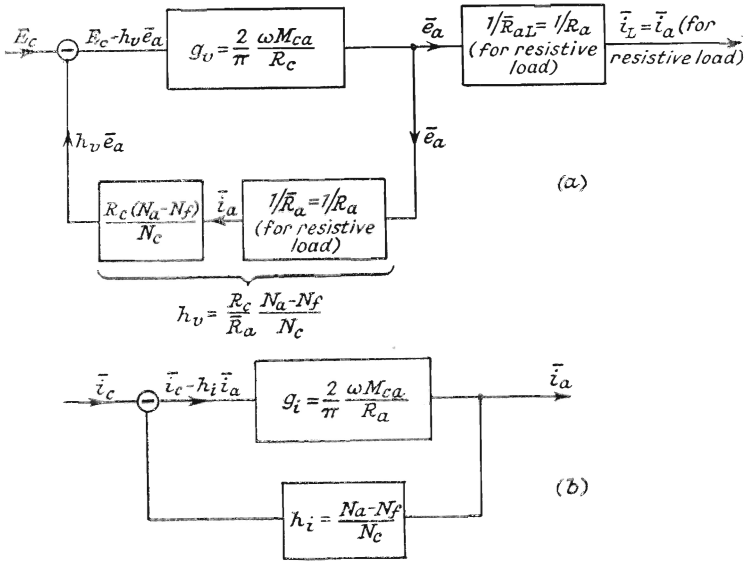


Figure 4.6. Block diagrams of feedback systems representing a transductor.
(*a*) Voltage amplifier.
(*b*) Current amplifier (for resistive load).

stressed. In the present case, the transductor with 100% self-excitation is considered to be the basic element, while the absence of self-excitation gives rise to the negative feedback properties inherently due to the fact that the ampere-turns must balance during the conducting period. Thus the effect of self-excitation is virtually to " open " the closed loop. This conception shows the complete dependence of the basic unit upon the core reluctance, but when considered as in Chapter 3 the part played by the core is not apparent.

Although at first sight the block diagram representation may appear somewhat artificial, there is a reasonable justification for it which may be seen by considering the physical mechanisms as follows.

The output voltage e_a appears during the conducting period and over this period it is very nearly equal to the supply voltage. The block diagram indicates that a proportional h_v of the output voltage opposes the effect of the control voltage and therefore it follows that according to the diagram a proportion h_v of the supply voltage should oppose the control voltage during conducting periods.

This has already been found to apply to the transductor ; for example, when core (A) saturates a fraction of the supply voltage opposes flux changes in core (B) due to the control voltage so that:

$$N_c \frac{d\phi_B}{dt} = E_c \frac{\gamma_f}{1+\gamma_f} - \frac{E_a N_c}{N_a - N_f} \cdot \frac{1}{1+\gamma_f} \quad \text{(equation 3.34)}$$

and putting $\gamma_f = \left[\frac{N_c}{N_a - N_f}\right]^2 \frac{R_a}{R_c}$

gives $$N_c \frac{d\phi_B}{dt} = \frac{\gamma_f}{1+\gamma_f} \quad [E_c - h_v E_a].$$

Thus during conduction the control voltage is opposed by the component $h_v E_a$ and since the supply voltage considered over conducting periods is equivalent to the output voltage it may be said that the control voltage is opposed by a fraction of the output voltage $h_v e_a$.

In the general case the mean output voltage $\bar{e}_a$ gives rise to a mean current $\bar{i}_a$ flowing in the transductor windings and a current $\bar{i}_L$ flowing in the load. When the load is resistive these two currents are equal, but otherwise their ratio to the voltage is given by the terms $\bar{R}_a$ and $\bar{R}_{aL}$ respectively (see Chapter 6).

The former term appears in the feedback loop and therefore, although the output voltage is applied to the load, the feedback is essentially that of the transductor current $\bar{i}_a$.

Thus, considering the operation in terms of currents and using the expressions for g_i and h_i, the block diagram may be redrawn to depict a current amplifier, in which case the input $\bar{i}_c$ is balanced against a proportion of the output $\bar{i}_a$ as shown in *Figure* 4.6*b*.

It is not suggested that this particular way of considering transductor operation is anything more than one of convenience to those more acquainted with servo systems and negative feedback amplifiers than with transductors, and from our point of view it is unnecessary to go further than the basic current gain equations to see that when no self-excitation is applied to the transductor the output current will be almost independent of everything excepting the input. This brings out the high impedance output of the transductor without self-excitation and the high degree of linearity normally associated with transductors operating under these conditions. If the self-excitation turns are made larger than the turns on the a.c. windings then the inherent

feedback term h_i becomes negative, and $1 + g_i h_i$ tends to zero; when this occurs the device becomes unstable as described in section 3.16.

4.8 Ampere-Turns Ratio

The gain of the transductor may be expressed in terms of the ampere-turns ratio rather than of currents (Appendix 4), thus, for the case of a smooth control current ($\lambda = 1$)

$$\frac{d(\bar{\imath}_a N_a)}{d(\bar{\imath}_c N_c)} = G_m = \frac{g_m}{1 + g_m h_m} \quad . \quad . \quad . \quad (4.21)$$

3(g)

where

$$g_m = \frac{2}{\pi} \frac{\omega L_{aw}}{\bar{R}_a} = \frac{2}{\pi} \frac{X_{aw}}{\bar{R}_a} \quad . \quad . \quad . \quad (4.22)$$

and

$$h_m = \frac{N_a - N_f}{N_a} \quad . \quad . \quad . \quad (4.23)$$

it is seen that the mutual inductance M_{ca} is now replaced by the self-inductance L_{aw} of an a.c. coil.

Although the current or voltage amplification are convenient quantities to consider for design purposes, the ampere-turns ratio is more useful for the comparison of transductor characteristics, since the former may be given any value simply by variation of the control turns while the latter is limited by the characteristics of the core or the rectifier. The current amplification is, of course, equal to the ampere-turns ratio when the a.c. turns are equal to the control turns.

The quantity g_m is the ampere-turns ratio for the case of 100% self-excitation. In practice it depends primarily upon the core materials, rectifier and load and is usually limited to a definite number which is seldom higher than 80 and with lower-grade core materials and rectifiers is often as low as 15 (Chapter 13).

Thus it has been shown that the gain of a transductor may, in all cases, be expressed in the form

$$G = \frac{g}{1 + gh}$$

where the suffixes v, i and m refer to voltage, current and m.m.f. ratios respectively. The table at the end of this chapter summarizes the various ways of specifying transductor amplification.

4.9 The Control Characteristic

The steady-state control characteristic is usually plotted in terms of the load current and the control current, sometimes the characteristic is generalized and it then applies to a given core material rather than a given transductor. It will be shown (section 5.13) that in the case of a transductor with 100% self-excitation the control characteristic is approximately given by the core magnetization curve. In practice it is found that the characteristic corresponds quite closely to the descending portion of the hysteresis loop as has been demonstrated by Krabbe [3(b)] for auto-excited transductors (*Figure* 4.5*b*). This may be explained to some extent on the basis of the facts so far considered.

It has been shown that when the flux in a core is at its minimum value ϕ_Q the only m.m.f. acting on the core is the control m.m.f. If the control circuit impedance is fairly large ($\lambda \rightarrow 1$) the control current will be substantially constant throughout the cycle and will therefore define the flux ϕ_Q according to the magnetization curve. When both cores are unsaturated this minimum flux will be just at negative saturation so that the corresponding control ampere-turns will be those necessary to produce this flux, i.e. $-S\Phi_s$. As the control ampere-turns are increased the minimum flux level ϕ_Q rises until eventually it coincides with the positive saturation flux. When this occurs the transductor delivers full power to the load.

As the minimum flux ϕ_Q increases so also does the mean flux ϕ_0 increase at half the rate with respect to control current, and the relation between the control ampere-turns and the mean core flux is therefore defined by the magnetization curve. Since the mean flux is proportional to the mean output voltage and hence the mean load current, it follows that the curve of load current plotted against control current will be of the same form as the magnetization curve as shown in *Figure* 4.5*b*.

It is apparent that the amplification of a fully self-excited transductor can alternatively be derived from these considerations. Thus if the total change of output current is divided by the total change of control current, the mean current amplification is obtained, i.e.

$$g_i = \frac{\bar{E}_{as}/R_a}{2S\Phi_s/N_c}$$

substituting for $\bar{E}_{as}$ and S gives

$$g_i = \frac{2}{\pi}\frac{\omega M_{ca}}{R_a}$$

which is the same result as that obtained previously for the special case of complete self-excitation.

When considering the output change it was assumed that the minimum load current is zero, and therefore that the total range is given by the maximum output. This, of course, is not the case, for when both cores are unsaturated a current equal to $\frac{\bar{E}_{as}}{2X_{aw}}$ flows in the load. This can be taken into account and the amplification is then found to be

$$g_i = \frac{2}{\pi}\left[\frac{\omega M_{ca}}{R_a} - \frac{N_c}{2N_a}\right].$$

In general the simpler formula is sufficiently accurate for all purposes and the correction is usually not worth applying due to the limited accuracy with which the properties of the core can be predicted.

Factors Influencing the Characteristics of Self-Excited Transductors

The characteristics of self-excited transductors are almost identical to those of auto-excited transductors. The latter are described in the following chapter and the influence of circuit parameters and components will be discussed at that stage. There are, however, a few points worth mentioning here which arise in practice and follow directly from the results of the previous sections.

4.10 Proportion of Self-Excitation

The effect of various proportions of self-excitation are illustrated by the curve in *Figure* 4.7. These curves apply to a mumetal-cored transductor with resistive load, details of which are given below :

$a = \cdot 675 \times 10^{-4}$ m^2. ($\cdot 675$ sq. cm.)

$l = \cdot 086$ m. ($8 \cdot 6$ cm.)

$N_a = 500$ turns

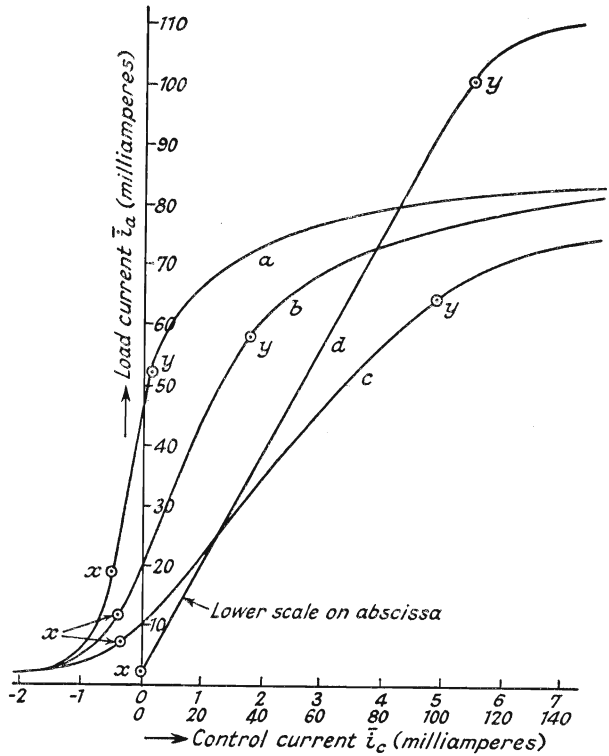


Figure 4.7. Control characteristics for self-excited transductor using mumetal cores and various degrees of self-excitation ($gm = 60$).

(a) $h_m = 0$.
(b) $h_m = \cdot 03$.
(c) $h_m = \cdot 06$.
(d) $h_m = 1 \cdot 0$ (lower abscissa scale).

$N_c = 500$ turns
$N_f = 470, 485, 500$ turns
$R_a = 800$ ohms (with self-exciting windings)
$R_a = 650$ ohms (without self-exciting windings)
$R_L = 500$ ohms
$f = 400$ c.p.s.
$\bar{E}_{as} = 75$ volts.

When the transductor is fully self-excited (curve (a)), the form of the characteristic closely resembles that of the magnetization

curve (section 4.9 and 5.13). The linear range is limited, the points x and y defining where the slope has decreased to two-thirds of its maximum value. Two factors limit the useful range of the transductor with self-excitation, firstly, the total circuit resistance; this includes that of the feedback rectifier and windings and therefore reduces the maximum current which can flow in the load for a given supply voltage. Moreover, as the self-excitation is increased, the form of the curve more nearly resembles that of the magnetization curve which itself shows marked non-linearity even at flux densities considerably below the saturation value. In the present example the current amplification is the same as the ampere-turns ratio and the maximum slope corresponds to a value of $g_m = 60$. Curves (*b*) and (*c*) are for the same condition, but with the feedback turns reduced to 485 and 470 respectively, for which h_m is ·03 and ·06. The maximum slope of these curves is correspondingly reduced to 21·5 and 13, the linear range of curve (*c*), on the other hand, is nearly double that of curve (*a*). Curve (*d*) (lower abscissa scale) shows the control characteristic for the same transductor without self-excitation ($h_m = 1$). In this case the ampere-turns ratio is not quite unity; the linear range, however, is three times that of the transductor with complete self-excitation.

The minimum useful load current occurs at the points marked (x), and when operating under practical conditions some means of balancing is usually incorporated in such a way that this value corresponds to zero output. The maximum useful load current which occurs on the characteristic at the points (y) is then given by the difference between the load current at (y) and the load current at (x). The useful power output P is proportional to the square of this difference.

In section 2.8 it was shown that the product of the power amplification and power output were a constant for a given transductor without self-excitation, and it was further shown in section 3.14 that this product is considerably increased by applying self-excitation. In the latter example, however, no account was taken of the reduced range due to poor linearity. In the present example the useful power output is reduced by a factor of about 9, while the power amplification is increased as the square of the ampere-turns ratio, i.e. $g_m^2 = 3{,}600$, thus the product of power amplification and power output over the linear range has increased by a factor of 400 for a given core size.

4.11 Load Resistance and Power Amplification

When considering the transductor with complete self-excitation it has been shown that the current amplification is inversely proportional to the total resistance in the a.c. circuit including that of the feedback rectifier and windings, unlike the transductor without self-excitation in which the current gain is found to be almost independent of the a.c. circuit resistance. The effect of the resistance on the gain is shown in *Figure* 4.8, in which the resistance R_a is plotted against $\frac{1}{\text{gain}}$ for three different conditions of operation. Curve (*c*) shows the effect of variations of resistance when no self-excitation is applied. The amplification only varies by about 5% for changes of resistance from 0 to 2,000 ohms, while in case (*a*), which corresponds to 100% self-excitation, the gain is proportional to $1/R_a$. The fully self-excited transductor may in fact be considered as a voltage generator (section 5.14), the e.m.f. of which is proportional to the control current or the control voltage and the internal resistance equal to the total resistance of the output circuit excluding the load.

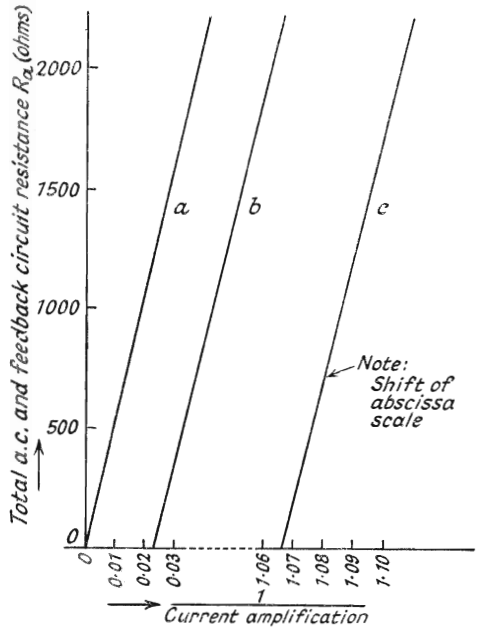


Figure 4.8. Curves showing effect of circuit resistance R_a on the current amplification of a transductor with various degrees of self-excitation.
(*a*) $h_m = 0$.
(*b*) $h_m = \cdot 03$.
(*c*) $h_m = 1 \cdot 0$ (abscissa scale shifted to the left).

Thus :
$$R_a = R_L + r_x \qquad . \quad . \quad . \quad (4.24)$$
and for zero load resistance the mean alternating current is given by the mean output voltage divided by the internal resistance
$$\bar{\imath}_a = \frac{2}{\pi} \frac{\omega M_{ca}}{r_x} \bar{\imath}_c \qquad . \quad . \quad . \quad (4.25)$$

If the output is connected in series with a load R_L then the current flowing in the load will be reduced as the resistance of the load increases. For a given transductor the maximum power output at a fixed control current occurs when the load resistance is matched to the internal resistance, i.e. when

$$R_L = r_x.$$

Thus in the case of the fully self-excited transductor maximum power amplification and maximum power output both occur when the load is matched to the transductor.

As the proportion of self-excitation is reduced, however, the inherent negative feedback increases due to the increased demands from the control circuit for ampere-turn balance. As we have seen the negative feedback tends to stabilize the mean output current and the output impedance of the transductor progressively increases. The mean output current on no load (neglecting the constant term) is given by equation 4.17 which may be written

$$\bar{\imath}_a = \frac{2}{\pi} \frac{\omega M_{ca} \bar{\imath}_c}{(1 + g_i h_i) r_x}, \text{ since } R_a = r_x \quad . \quad . \quad . \quad (4.26)$$

The denominator of this equation is the internal impedance which has increased by the factor $(1 + g_i h_i)$. In the case of an unself-excited transductor this is equal to $\left(1 + \frac{2}{\pi} \frac{\omega L_{aw}}{r_x}\right)$ and therefore the internal impedance is

$$r_x + \frac{2}{\pi} X_{aw}.$$

4.12 Limitations due to the Self-Excitation Rectifier

The characteristics of the self-excitation rectifier often place severe limitations on transductor performance and the conditions under which these rectifiers operate are therefore relevant factors in design. This will be considered more fully in the following chapter, but a few points will be mentioned here. So far, the rectifier has been considered ideal, and when the transductor is conducting it has been assumed that the ratio of the alternating ampere-turns to the self-exciting ampere-turns is given by the corresponding turns ratio $\frac{N_a}{N_f}$.

To be exact, however, this should be expressed fully as the ratio $\frac{\bar{\imath}_a N_a}{\bar{\imath}_f N_f}$, and it immediately becomes clear that if the self-

exciting current $\bar{i}_f$ does not bear a constant relationship to the mean current in the a.c. windings $\bar{i}_a$ then this ratio will vary, and will, in fact, be equivalent to an apparent change in the number of turns on the self-exciting winding. Referring to *Figure* 4.3*c*, if the rectifier is considered to have a reverse resistance, then during the conducting period part of the alternating current will not be routed through the self-exciting windings, but will be by-passed through this reverse resistance, so that for a given control current, the self-exciting current will be numerically less than the current in the a.c. windings, and complete self-excitation would not be obtained even with $N_f = N_a$. If the reverse resistance of the rectifier were reliably constant, however, this would not matter unduly, since the turns on the self-exciting windings could be increased by an appropriate amount to compensate for the loss of current fed back and thereby maintain the required m.m.f. for self-excitation.

Unfortunately, however, the reverse resistance is a function of the reverse voltage on the rectifier (*Figure* 4.2) which will in turn depend upon the input signal. Therefore, if the transductor is arranged to be stable at all parts of the characteristic it may, unless suitable precautions are taken, suffer from severe non-linearity due to changes in the operating conditions of the rectifier.

The quantity h_m in the expression for the ampere-turns ratio of a self-excited transductor may be written in full as

$$h_m = \frac{\bar{i}_a N_a - \bar{i}_f N_f}{\bar{i}_a N_a}$$

and in the case of complete self-excitation this is not zero as previously assumed, but equal to

$$h_m = \frac{\bar{i}_a - \bar{i}_f}{\bar{i}_a}, \text{ since } N_f = N_a.$$

Also if i_f changes by $+ \delta i_f$, then assuming i_f is approximately equal to i_a we have

$$\delta h_m = - \frac{\delta i_f}{i_f}.$$

The curves (*a*) and (*b*) in *Figure* 4·7 show that if the self-exciting current were to change by 3% the gain would change by a factor of nearly 3, and it is therefore clear that unless special precautions are taken, the characteristic will become seriously non-linear. If the m.m.f. ratio is excessive, then variations in the

rectifier become more serious. Provided the reverse voltage on the rectifier is small, the reverse resistance does not vary unduly (*Figure* 4.2), and it is found that if the rectifier reverse voltage is kept to within a certain value (section 5.16), depending upon the type of rectifier used, non-linear effects due to the rectifier are usually no more significant than effects arising due to the non-linear magnetization curve of the core. In addition to non-linearity, however, similar parasitic effects arise due to ambient temperature variations which also influence the reverse characteristics of metal rectifiers; this gives rise to zero drift and gain drift (see Chapter 11).

The above considerations all show that it is most important to know the maximum reverse voltage which appears across the elements of the self-exciting rectifier.

The reverse voltage across the rectifier is considered in relation to the waveforms in *Figure* 4.9*a* and *b*. Curve (*a*) shows dotted the current in the self-exciting coils for various conditions

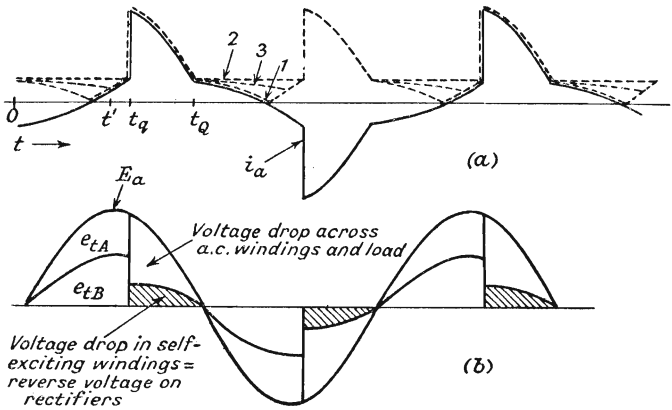


Figure 4.9. Curves showing reverse voltages on self-excitation rectifier.
(*a*) Self-exciting current (dotted).
(*b*) Curve showing division of supply voltage throughout cycle.

of control circuit impedance, and curve (*b*) shows the distribution of voltage across the circuit elements during different parts of the cycle.

Case (1). *Low Control Circuit Impedance.* When the control circuit impedance is small (section 4.2) the current in the self-exciting windings follows curve (1) in *Figure* 4.9*a* and is always numerically equal to the load current (full line). Over the period $t = 0$ to $t = t_q$ (*Figure* 4.9*b*) both cores are unsaturated. The

impedance across the d.c. terminals of the rectifier is very small, however, since the control circuit has a low impedance which can be referred to the self-exciting circuit and considered to appear across the windings (*Figure* 4.3*b*). Due to the large impedance in the a.c. circuit, the alternating current is small, and although two of the rectifier elements are non-conducting, the reverse voltage across their terminals is a negligible proportion of the total applied voltage which is equally divided between the two a.c. windings (*Figure* 4.9*b*). When one of the cores saturates at t_q, however, the whole supply voltage is available to drive current through the circuit resistance. Part of this resistance, that of both the self-exciting windings, is in the d.c. circuit of the rectifier, however, and therefore a proportion $2r_{fw}/R_a$ of the total supply voltage will appear across the non-conducting elements as a reverse voltage during the conducting period; this is shown by the shaded area in *Figure* 4.9*b*.

Case (2). *High Control Circuit Impedance.* If the control impedance is infinite and that of the self-exciting circuit small, then the current i_f follows curve (2) ; throughout the unsaturated period all four valves are therefore conducting and the reverse voltage is zero. During the saturation period the conditions are the same for case (1).

If the self-exciting circuit has a high resistance the rectifiers will not be relaxing throughout the non-conducting period, and during the period when forcing occurs (curve 3) it can be shown that a reverse voltage of half the supply voltage appears across non-conducting rectifier elements. It must be remembered, however, that these conditions are only applicable if the control circuit impedance is very large.[4(a)] This would be true if the control source were a pentode valve, for example, and when operating under these conditions it is necessary to take steps to ensure that the self-exciting rectifiers are not being overrun in terms of reverse voltage.

Although the resistance of the control circuit has been stressed as being the important consideration, in actual fact this is only true for a given inductance and the important consideration is the ratio of resistance to inductance which may be expressed as the ratio $\frac{R_c}{N_c^2}$ for a given core. Thus if the number of turns is doubled, the resistance will increase by a factor of four, but the same mode of operation will apply. In practice it is found that

the reverse voltage on the rectifier should be calculated on the basis of the resistive voltage drop in the self-exciting windings; further considerations are discussed in relation to the auto-excited transductor.

4.13 Connection of Load in Self-Excitation Circuit

Up to the present the load has been considered to be connected in series with the a.c. windings, a separate rectifier being used to rectify the load current when necessary. The load may, however, be placed in series with the self-exciting windings, in which case the load current is already rectified by the self-excitation rectifier. At first sight this appears to be a desirable arrangement since only one rectifier is required. It will be remembered, however, from case (1) in the previous section that the reverse voltage appearing across the rectifier was a fraction $\frac{2r_{fw}}{R_a}$ of the total supply voltage where $2r_{fw}$ was the resistance in the self-exciting circuit. In the present case the total resistance includes the load and is therefore equal to $2r_{fw} + R_L$ so that the reverse voltage $\frac{(2r_{fw} + R_L)}{R_a} \bar{E}_a$ is very nearly equal to the supply voltage.

In order to underrun this rectifier adequately for good linearity and zero stability, a comparatively large number of discs would be required in each rectifying element, which would be quite unnecessary for the load rectifier if it did not also have to perform the functions of the self-excitation rectifier. Therefore, for the same performance, the rectifier unit would be more costly and larger than if separate rectifiers were used. In addition, if a large number of rectifier discs are employed, the forward resistance increases and the circuit efficiency becomes smaller. There is, however, one advantage of this circuit which is often important, this relates to the dynamic performance of the transductor, which will be considered in Chapter 8. It has already been mentioned that when a control voltage is applied to the control circuit, the growth of flux in each cycle can be delayed due to the slugging action of the self-exciting windings. When the control circuit time-constant is very small, the time-constant of the transductor is primarily governed by that of the self-excitation circuit. Therefore, if the load is included in this circuit the time-constant will decrease and more rapid response of the transductor will be obtained.

4.14 Table

Ratio expressed by G.	Appropriate value of g.		Appropriate value of h.
	$\lambda = 1$	$\lambda = 2/\pi$	
$\frac{\text{Output voltage}}{\text{Control voltage}} = \frac{\delta \bar{e}_a}{\delta E_c} = G_v$ . . .	$g_v = \frac{2}{\pi} \frac{\omega N_a N}{l} a\mu \cdot \frac{1}{R_c}$	$\frac{4}{(2+\pi)} \frac{\omega N_a N_c a\mu}{l} \frac{1}{R_c}$	$h_v = \frac{N_a - N_f}{N_c} \cdot \frac{R_c}{R_a}$
$\frac{\text{Output current}}{\text{Control current}} = \frac{\delta \bar{i}_a}{\delta \bar{i}_c} = G_i$ . . .	$g_i = \frac{2}{\pi} \frac{\omega N_a N_c a\mu}{l} \cdot \frac{1}{R_a}$	$\frac{4}{(2+\pi)} \frac{\omega N_a N_c a\mu}{l} \cdot \frac{1}{R_a}$	$h_i = \frac{N_a - N}{N_c}$
$\frac{\text{Output voltage}}{\text{Control current}} = \frac{\delta \bar{e}_a}{\delta \bar{i}_c} = G_{vi}$ . . .	$g_{vi} = \frac{2}{\pi} \frac{\omega N_a N_c a\mu}{l}$	$\frac{4}{(2+\pi)} \frac{\omega N_a N_c a\mu}{l}$	$h_{vi} = \frac{N_a - N_f}{N_c} \cdot \frac{1}{R_a}$
$\frac{\text{a.c. m.m.f.}}{\text{Control m.m.f.}} = \frac{\delta(\bar{i}_a N_a)}{\delta(\bar{i}_c N_c)} = G_m$ . . .	$g_m = \frac{2}{\pi} \frac{\omega N_a^2 a\mu}{l} \cdot \frac{1}{R_a}$	$\frac{4}{(2+\pi)} \frac{\omega N_a^2 a\mu}{l} \cdot \frac{1}{R_a}$	$h_m = \frac{N_a - N_f}{N_a}$

In these expressions it should be noted that the term R_a is equal to R_a when dealing with resistively loaded transductors. Also that μ is defined as the gain permeability or sensitivity permeability and refers to the slope of the magnetization curve rather than the true permeability. In the present context, however, these two are synonymous since the curve has been assumed linear up to the knee of the characteristic.

CHAPTER 5

TRANSDUCTORS WITH AUTO-EXCITATION

AUTO-EXCITED transductors [5(a)] are now frequently employed in place of the circuits using self-excitation. Although the circuit principles are almost the same as those applying to the latter, the performance, size and cost of transductors using auto-excitation are, in general, more favourable. A description of the auto-excited transductor will now be given followed by a discussion of the practical limitations which in most cases apply equally to transductors using self-excitation.

5.1 Circuit Operation of Auto-Excited Transductor [3(b), 5(b)]

When considering the transductor with complete self-excitation, it was shown that the main function of the self-exciting windings is to provide ampere-turns to the unsaturated core which just balance those due to the alternating current during the conducting period. The result of this is that during conducting periods the total alternating m.m.f. in the unsaturated core is zero. In the complete absence of self-excitation it has been pointed out that the balancing ampere-turns are provided by a current which flows in the control windings and is drawn from the control source, thereby limiting the ampere-turns ratio to unity.

In the auto-excited transductor, the circuit is arranged in such a way that alternating current is prevented from flowing in the coil on the unsaturated core. This removes the possibility of large induced currents in the control circuit and makes the transductor behave like one with complete self-excitation.

The circuit of a conventional auto-excited transductor is shown in *Figure* 5.1*b*. The control circuit is the same as for the self-excited transductor. The a.c. windings are connected in parallel and the supply is fed in at the points *HJ*. On positive half-cycles

the current is routed through the coil EF, and on negative half-cycles through GC due to the rectifier elements (1) and (2).

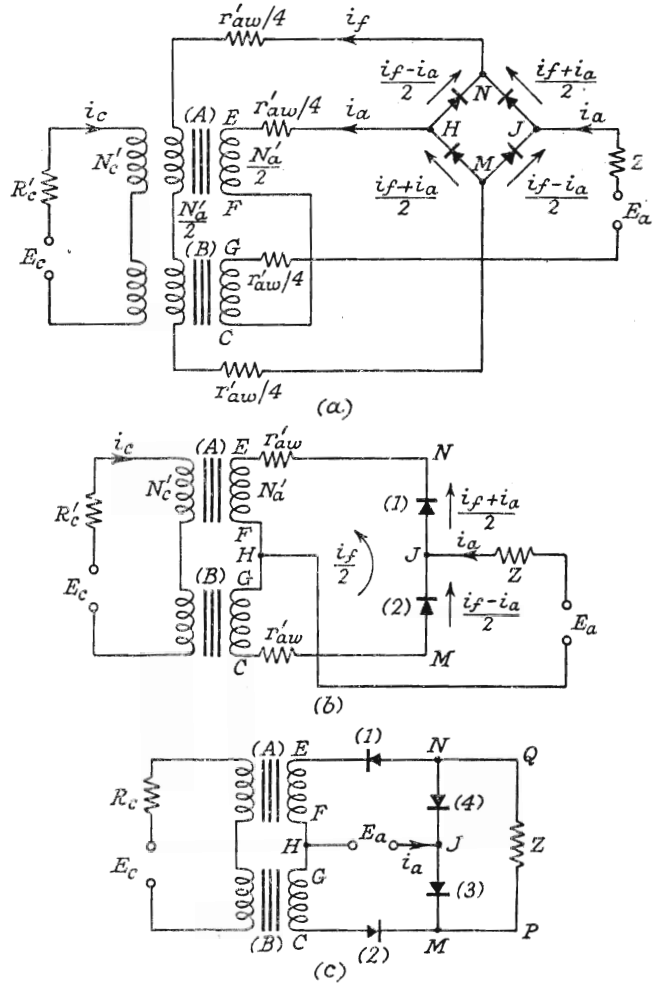


Figure 5.1. Self- and auto-excited transductors.

(*a*) and (*b*) Circuits showing conditions for equivalence.
(*c*) Auto-excited bridge (Group 2) connections.

Thus during conducting periods only the coil on the saturated core is subjected to the supply voltage and hence no induced current can flow in the control circuit since one core is saturated and the supply to the other is cut off. The load is placed in the

main circuit, it may be either an a.c. load as shown, or fed through a separate bridge rectifier.

An alternative connection, available when a d.c. load is required, is shown in *Figure* 5.1c. In this case two more rectifiers are placed between the common point J and the auto-excitation rectifiers (1) and (2). The load is placed across the terminals PQ and the other supply line is connected to the common point of the two coils at H. During a positive conducting period core A saturates and current flows along the path $JMPQNEH$. Elements (2) and (4) are cut off and therefore no current flows in the coil linking core B ; the direction of the load current is from P to Q. During negative conducting periods core B saturates and current flows through $HCMPQNJ$ while the rectifier elements (1) and (3) are cut off ; the current again flows through the load from P to Q.

Returning to *Figure* 5.1*b*, it is seen that the closed circuit $MNEC$ contains both a.c. windings and two rectifiers. During unsaturated periods this circuit could maintain a current in one direction if the total inductance in the circuit were large and both rectifiers able to conduct. These conditions will apply if the control circuit resistance is sufficiently high (p. 101), otherwise a current is induced in the control circuit during unsaturated periods, and, as in the case of the self-excited transductor, no relaxation can take place.

Let us consider the waveforms of voltage and current for the auto-excited transductor in which the conditions of complete relaxation apply to the rectifiers (1) and (2) during non-conducting periods, and the load is assumed to be resistive.

A control voltage E_c is applied and after several cycles the transductor reaches a steady-state of operation where cores A and B saturate on alternate half-cycles due to the mean flux level produced by a signal. On positive half-cycles rectifier (1) conducts and the supply voltage is applied to the a.c. coil on core A; similarly on negative half-cycles the supply is applied to B while rectifier (1) is cut off.

Thus during the latter part of the first (positive) half-cycle of supply voltage shown dotted in *Figure* 5.2*a*, core A is saturated, element (2) is cut off and the whole supply voltage corresponding to the shaded area is available to drive current through the circuit resistance; the load current is therefore large and equal to the current in the a.c. coil A, it is also in phase with the supply voltage. As the supply voltage decreases towards zero, the current

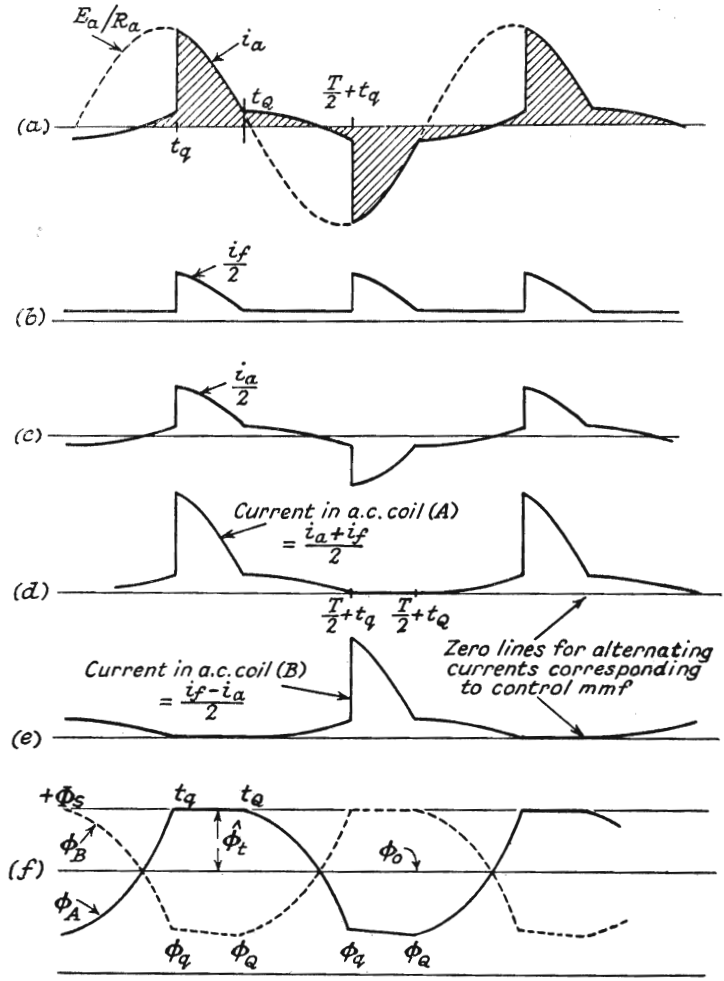


Figure 5.2. Waveforms of currents and fluxes for auto-excited transductor.
(*a*) Load current.
(*b* and *c*) Components of a.c. coil currents.
(*d* and *e*) Coil currents.
(*f*) Core fluxes.

eventually reaches such a value that the total m.m.f. in core A is less than that required to saturate the core. When this occurs at a time t_Q, core A desaturates and a component of current is maintained round the closed circuit $ECMN$ by the self-induced e.m.f. due to the inductance of the coils in series. It is assumed that this component does not decay appreciably during the period

when both cores are unsaturated due to the large effective inductance in the closed circuit and it follows that both rectifier elements (1) and (2) conduct over this interval.

The voltage E_a, which has changed sign, is therefore applied to both coils in parallel and the load current which is now small divides equally between the two a.c. coils ; the current in each element may therefore conveniently be considered to consist of two components. A direct circulating component $i_f/2$, which is numerically equal to half the load current during conducting periods, but constant when both cores are unsaturated, and an additional component which is equal to half the load current. The latter changes sign every half-cycle and flows in opposite directions in the two coils while the former always flows in the same direction, thus at all times we have

$$\text{current in coil } A = \frac{i_f + i_a}{2} \quad . \quad . \quad . \quad (5.1)$$

$$\text{current in coil } B = \frac{i_f - i_a}{2} \quad . \quad . \quad . \quad (5.2)$$

where i_a changes sign every half-cycle.

It should be noted that the circulating current $i_f/2$ is introduced as a convenient conception. During conducting periods the actual current in the unsaturated coil is equal to zero. It is supposed, however, that the coil current is equal to the difference between the circulating current and half the load current. During conduction these currents are equal in magnitude and therefore the resultant is zero.

The components of current are shown in *Figure* 5.2*b* and *c*, while the resultant coil currents are shown in *d* and *e*. Over the interval following t_Q, core *A* departs from saturation while core *B* proceeds towards saturation from the minimum flux level ϕ_Q, these flux changes therefore cancel and produce no resultant e.m.f. in the control circuit.

Core *B* saturates when the negative area under the voltage-time curve corresponds to the necessary flux change; this occurs at a time $T/2 + t_q$ at which stage the current in the a.c. coil A becomes zero and the rectifier (1) cuts off so that current flows through the load and coil *B* and is limited only by the circuit resistance.

The flux changes occurring in the cores are shown in *Figure* 5.2*f*. Over the interval from t_q to t_Q the current in the coil *B*, *Figure* 5.2*e*, is zero and therefore any flux existing

in the core over this interval is due entirely to the control m.m.f. The curves shown in *Figures* 5.2*d* and *e* are proportional to the alternating m.m.f.s in cores *A* and *B*.

As in the self-excited transductor, the mean load current depends upon the saturation time t_a, which is determined by the minimum level ϕ_Q reached by the flux in the previous half-cycle and, as shown above, this is given by the control current only.

The flux in core *B* starts increasing from ϕ_Q at time t_Q. Simultaneously the flux in core *A* starts decreasing from the saturation value. This latter decrease of flux occurs in two parts. The first part, from t_Q to $(t_a + T/2)$, is due to the combined action of the control voltage and supply voltage (since rectifier (1) is conducting). The second part is during the conducting period when the rectifier element (1) is cut off and therefore the further slight decrease of flux to ϕ_Q is due only to the voltage-time applied from the control circuit. This flux change, which is linear with respect to time, is comparatively small if the power amplification is large, as for the self-excited transductor.

5.2 Similarity Theorem for Auto-Excited Transductors

From the preceding discussion it is evident that the auto-excited transductor is, in many respects, identical to the series transductor operating with complete self-excitation ($N_f = N_a$).

Furthermore, just as the self-excited transductor consists of three circuits, namely the control circuit, the a.c. circuit and the self-excitation circuit, so also does the auto-excited transductor consist of a control circuit, an a.c. circuit and a closed circuit *ECMN* respectively. The latter corresponds to the self-excitation circuit closed through the terminals *MN* of the bridge rectifier (*Figure* 5.1*a*).

If an auto-excited and self-excited transductor are connected to equal supply voltages E_a, equal control voltages E_c, and deliver power into equal load impedances Z, then it may be shown that the external electrical characteristics are identical in all respects provided appropriate circuit constants are introduced. Thus, considering the circuit in *Figure* 5.1*b*, the following *numerical values* of the circuit constants are applicable :—

(1) Number of turns on each a.c. coil $= N'_a$
(2) Number of turns on each control coil $= N'_c$
(3) Resistance of each a.c. winding $= r'_{aw}$
(4) Number of rectifier discs in each element $= S'_1$
(5) Resistance of control circuit $= R'_c$

It is shown in Appendix 5 that this circuit has the same external electrical characteristics as a fully self-excited transductor to which the following circuit constants apply numerically :

(1) Number of turns on each a.c. and self-exciting winding $= \frac{1}{2}N'_a$

(2) Number of turns on each control winding $= N'_c$

(3) (*a*) Resistance of each a.c. winding $= \frac{1}{4}r'_{aw}$

(*b*) Resistance of each self-exciting winding $= \frac{1}{4}r'_{aw}$

(4) Number of rectifier discs in each element of the bridge $= \frac{1}{2}S'_1$

(5) Resistance of control circuit $= R'_c$

In common with these conditions is of course the added requirement that the cores are identical in both cases.

The circuit in *Figure* 5.1*b* has been termed the group (1) connection for auto-excited transductors, and the circuit (*c*) the group (2) connection. [3(b)] The latter is similar but not identical to a self-excited transductor in which the load is connected in series with the self-exciting windings.

5.3 Current Amplification of Auto-Excited Transductor

From the preceding theorem an expression for the current amplification can be written down.

If M'_a is the numerical value of the mutual inductance between an a.c. coil and control coil in the auto-excited transductor, the mutual inductance in the *equivalent* self-excited transductor is $\frac{M'_a}{2}$.

The current amplification for the auto-excited transductor is given by analogy as that of the equivalent self-excited transductor, and from the results in the previous chapter this is

$$G_i = \frac{2}{\pi}\frac{\omega(\frac{1}{2}M'_{ca})}{(R_L + r'_x)} = \frac{1}{\pi}\frac{\omega M'_{ca}}{(R_L + r'_x)}.$$

Now the numbers N'_a, N'_c, r'_{aw}, etc., apply to the auto-excited transductor. They have exactly the same meaning in relation to the windings and cores, however, as the symbols N_a, N_c, r_{aw}, etc., in Chapter 4 for the self-excited transductor.

There is no reason, therefore, why the latter set of symbols

should not be used since, for example, both N_a and N'_a denote the number of a.c. turns on a core. Using these symbols, the general expression for the current gain G_i of the auto-excited transductor is given by :

$$G_i = \frac{1}{\pi} \frac{\omega M_{ca}}{(R_L + r_x)} = \frac{g_i}{2} \quad . \quad . \quad . \quad (5.3)$$

Thus for an auto-excited transductor the current amplification is seen to be one-half that obtained for a fully self-excited transductor having the same turns on the coils. In the former case, however, if both transductors are wound with the same wire, the internal resistance is only one-quarter that of the latter ; in addition, the full winding space is not utilized, since there are no self-exciting windings.

The operation was considered in section 5.1 on the assumption that the control circuit impedance was sufficiently high for a constant circulating current to flow round the path *ECMN* when both cores were unsaturated, thereby maintaining the rectifiers in the relaxed condition and making $\lambda = 1$. If, however, the control circuit resistance of the auto-excited transductor is low, then, once again, by analogy with the equivalent self-excited transductor :

$$G_i = \frac{2}{2 + \pi} \frac{\omega M_{ca}}{R_L + r_x}.$$

5.4 Ampere-Turns Ratio of Auto-Excited Transductor

The ampere-turns ratio (section 4.8) is obtained from the current amplification and the turns ratio, thus for the auto-excited transductor :

$$G_m = \frac{N_a}{N'_c} \frac{1}{\pi} \frac{\omega M'_{ca}}{(R_L + r'_x)} = \frac{1}{\pi} \frac{\omega L'_{aw}}{(R_L + r'_x)} \quad . \quad . \quad . \quad (5.4)$$

The ampere-turns ratio of the *equivalent* self-excited circuit is only half this value since the self-inductance is $\frac{1}{4} L'_{aw}$, i.e.

$$G_m = \frac{2}{\pi} \frac{\omega(\frac{1}{4} L'_{aw})}{R_L + r'_x} = \frac{1}{2} \frac{1}{\pi} \frac{\omega L'_{aw}}{R_L + r'_x}.$$

This apparent anomaly does not invalidate the similarity theorem, however, since the ampere-turns ratio is purely an internal characteristic which is introduced as a convenient ratio by which to assess transductor performance, and has no more significance than the fact that the turns on the a.c. windings are not equal in the two equivalent circuits.

5.5 Output Impedance of Auto-Excited Transductor

If the load is resistive the total circuit resistance defined by R_a will be the internal resistance r_x plus the load R_L. In the equivalent self-excited transductor r_x is equal to the total resistance in the a.c. and self-exciting windings, plus the forward resistance of two arms of the bridge rectifier, i.e.

$$\begin{aligned} r_x &= 4(\tfrac{1}{4}r'_{aw}) + 2(\tfrac{1}{2}S_1)\,(r'_{ar}) \\ &= r'_{aw} + S_1\, r'_{ar}. \end{aligned}$$

By analogy, therefore, this is the internal resistance of the auto-excited transductor, namely the resistance of one a.c. winding and the corresponding auto-excitation rectifier containing S_1 discs.

Using the general symbols, this is

$$r_x = r_{aw} + S_1 r_{ar}.$$

If the load R_L is placed across the d.c. terminals of a bridge rectifier, the impedance of the load rectifier should be added to the internal impedance in the above equation.

5.6 Transductors with Parallel Connected A.C. Windings

A special case of the similarity theorem arises when the self-excitation and auto-excitation rectifiers are all short circuited (this is a permissible condition since a general rectifier characteristic was assumed in proving the theorem). From this it follows by inspection of *Figures* 5.1*a* and *b* that a transductor with parallel connected a.c. windings and no self or auto-excitation is equivalent to one with series connected windings having half the turns and an additional pair of windings which are inductively coupled to the control windings and short circuited.[5(c)] The effect of these shorted coils leads to slugging of the response (section 8.4) and therefore the transductor with parallel windings has a much longer response time than one with series connected windings.

Since the current amplification of the equivalent series transductor in the steady-state is equal to the turns ratio $\dfrac{N'_c}{\frac{1}{2}N'_a}$, this must also be the current amplification of the parallel transductor.

Using the general symbols it therefore follows that the current gain of the parallel connected transductor is given by

$$G_i = 2\frac{N_c}{N_a}$$

which is twice that of a series transductor with the same number of turns.

The supply voltage for the parallel transductor may also be obtained by analogy with the *equivalent* series transductor thus :

$$\bar{E}_{as} = 8\Phi_s f\,(N'_a/2) \text{ (see equation 4.15}a\text{)}$$

i.e. $$\bar{E}_{as} = 4\Phi_s f\,N'_a$$

or using the general symbols

$$\bar{E}_{as} = 4\Phi_s f\,N_a$$

which is half that applicable to a series transductor *with the same number of turns*. Therefore, if the a.c. windings of a given transductor are connected in parallel rather than series, the current gain is doubled while the voltage gain is halved. The power gain is the same but in the former case an additional time delay is introduced. This is due to the closed circuit formed by the a.c. windings connected in parallel which introduce an additional slugging effect.

5.7 Gain Adjustment by Feedback

The voltage or current gain of the auto-excited transductor is dependent on the mutual inductance M_{ca}, while the ampere-turns ratio is dependent upon the self-inductance L_{aw}. As in other types of amplifier the gain can be varied by introducing either a positive or a negative feedback. In the transductor with self-excitation it was shown that such a feedback was automatically introduced if the turns on the self-exciting winding were not equal to the turns on the a.c. winding, and only in the special case of 100% self-excitation where the turns were equal was the gain of the circuit equal to g, otherwise the gain was

$$G = \frac{g}{1 + gh}.$$

The feedback is positive if $N_f > N_a$ making h negative, and the feedback is negative if $N_f < N_a$ so that h is positive.

In the auto-excited circuit, ignoring rectifier leakage, the value of the feedback factor h for the ampere-turns ratio is zero; it may, however, be introduced with either a positive or a negative sign by the addition of a small number of extra turns on each core connected in series and inductively coupled to the control winding.

When dealing with the auto-excited circuit the symbols g' and h' will be used with suitable suffixes thus :

$$g'_i = \frac{1}{\pi} \frac{\omega M_{ca}}{(R_L + r_x)} = \frac{1}{2} g_i.$$

The effect of negative feedback will now be considered with reference to the m.m.f. ratio; from this one may easily deduce the current or voltage amplification for a resistive load if required thus :

$$\text{m.m.f. ratio } G_m = \frac{\bar{i}_a N_a}{\bar{i}_c N_c} = \frac{\frac{1}{\pi} \frac{\omega L_{aw}}{(R_L + r_x)}}{1 + \frac{1}{\pi} \frac{\omega L_{aw}}{(R_L + r_x)} \cdot h'_m} = \frac{g'_m}{1 + g'_m h'_m}$$

$$\text{current gain } G_i = \frac{G_m N_c}{N_a}$$

$$\text{voltage gain } G_v = \frac{G_m N_c}{N_a} \frac{R_L + r_x}{R_c}.$$

Feedback may be applied to an auto-excited transductor from the a.c. circuit through a bridge rectifier as shown in *Figure* 5.3*a*. The arrows marked $i_c N_c$ show the direction of the m.m.f.s due to the control windings (not shown), and it is seen that in this case the feedback is positive since it introduces ampere-turns $i_a N_b$ which aid the control. Negative feedback can equally be introduced by reversal of the windings. The turns N_b correspond to the difference between the a.c. and self-exciting turns in the self-excited transductor.

Thus :

$$h'_m = \mp \frac{N_b}{N_a} \text{ cf. equation 4.23} \quad . \quad . \quad . \quad (5.5)$$

where the negative sign applies to positive feedback and the positive sign to negative feedback. The circuit in *Figure* 5.3*a* may alternatively have the load connected in series with the feedback windings, which will in general have only 1% or 2% of the a.c. turns.

An alternative feedback circuit is shown in *Figure* 5.3*b*. In this case the feedback windings are connected as a group (2) load. The main load can be either in the a.c. circuit or in series with the feedback.

The circuit shown in *Figure* 5.3*c* has the advantage of not

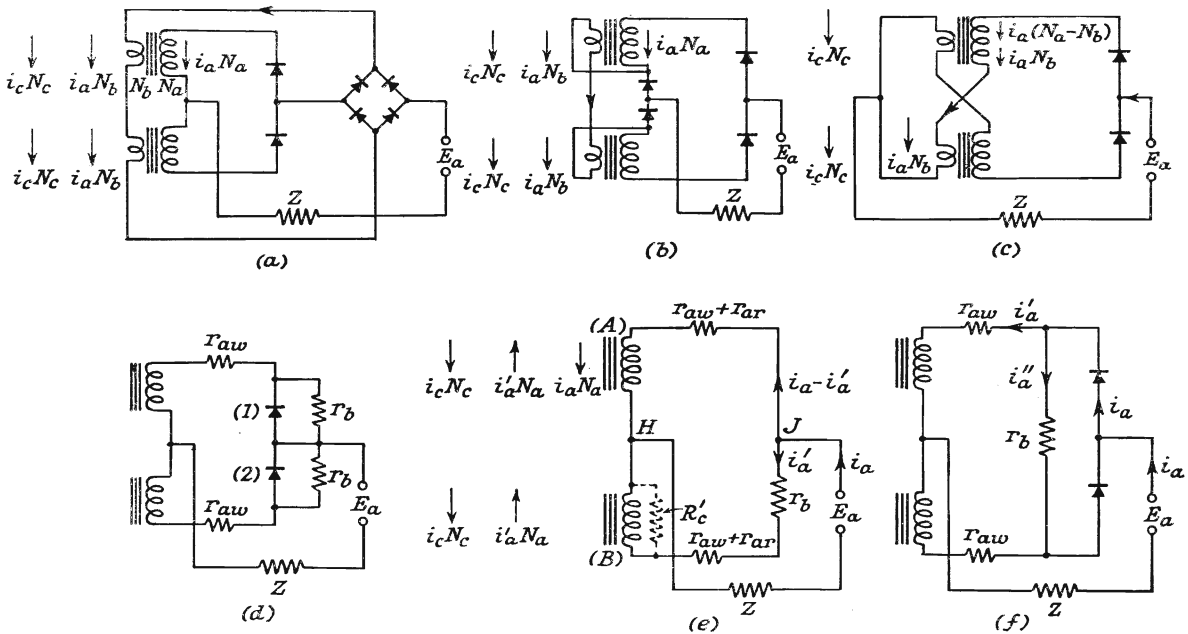


Figure 5.3. Feedback circuits for auto-excited transductors.

requiring a feedback rectifier. In order to understand this circuit each a.c. coil may be imagined to be divided into two windings : a coil having $N_a - N_b$ turns immediately in series with another coil having N_b turns making the total up to N_a. The latter portion of the a.c. coil together with the feedback coil N_b on core B provides positive feedback ampere-turns $i_a N_b$ on each core which aids the control on positive half-cycles. On negative half-cycles the a.c. coil on core B and the feedback coil on core A are used. Using this circuit the effective a.c. turns are $N_a - N_b$ and the feedback turns are N_b, therefore

$$h'_m = -\frac{N_b}{N_a - N_b}.$$

If the feedback windings N_b are reversed then by considering the a.c. winding as divided into $N_a + N_b$ turns and $-N_b$ turns the conditions then apply to negative feedback and

$$h'_m = \frac{N_b}{N_a + N_b}.$$

These and other similar feedback circuits were published originally by Krabbe.[3(b)]

A very simple method of applying negative feedback to an auto-excited transductor is given in *Figure* 5.3*d*. This consists of a shunt resistor r_b placed across each of the auto-excitation rectifiers. The circuit is redrawn for a positive conducting period in *Figure* 5.3*e* in which the rectifier (1) is a short circuit and rectifier (2) an open circuit. The current divides at J, most going through the saturated coil and rectifier of resistance $(r_{aw} + r_{ar})$ but a small fraction through the shunt resistance r_b, the winding resistance and the resistance referred from the control circuit. For simplicity the latter component has been neglected and the forward resistance of the conducting rectifier considered as a part of the a.c. winding, whence the current in the a.c. coil B will be

$$i'_a = \frac{r_{aw} + r_{ar}}{2(r_{aw} + r_{ar}) + r_b} \cdot i_a.$$

The resultant m.m.f. due to the a.c. coil A will therefore consist of a component $i_a N_a$ and a further component opposing this equal to $i'_a N_a$ as shown. The m.m.f. due to the a.c. coil B will be $i'_a N_a$, the directions of these m.m.f.s being shown by the

arrows in *Figure* 5.3*e*. Thus negative feedback is provided by the current i'_a and the feedback factor is given by

$$h'_m = \frac{N_a i'_a}{N_a i_a} = \frac{r_{aw} + r_{ar}}{2(r_{aw} + r_{ar}) + r_b} \quad . \quad . \quad . \quad (5.6)$$

If r_b is zero, then the feedback factor is a half and the circuit is that of a simple parallel connected transductor with an ampere-turns ratio equal to 2. It may be noted that almost the same result is obtained if the resistor r_b is placed across the a.c. windings as shown in *Figure* 5.3*f*.

This method is limited to cases where a negative feedback is required; it can never be used to increase the gain of an auto-excited transductor. The chief advantage of this form of feedback is its simplicity added to the fact that it can be applied to an existing transductor without internal modification; it is particularly useful as a method of trimming when circuits are being finally set up.

5.8 Other Methods of Reducing Transductor Gain

Methods of reducing the transductor gain described in the last section all depend upon reducing the m.m.f. ratio.

The gain may also be reduced, however, by reducing the control turns for a given control circuit resistance or alternatively increasing the control circuit resistance for a fixed number of control turns. The ampere-turns ratio is independent of both these parameters and thus the current or voltage gain may be reduced without altering the ampere-turns ratio. This inevitably results in a mismatch between the signal source and the transductor input and it will be shown in section 11.23 that such a state of affairs causes a deterioration of the zero stability. On the other hand this arrangement has a better dynamic performance. It will be remembered that, for a given supply frequency, the ratio of power gain to response time is proportional to the ampere-turns ratio and therefore if the gain is reduced by reducing this ratio the response time is not so short as it would be if the gain were reduced by adjustment of the control circuit parameters. Thus in this respect the design of a transductor is largely a matter of compromise, depending upon the application.

The following example illustrates this :

A transductor designed for a given set of conditions has a circuit power gain of 10^4, an ampere-turns ratio of 40 and a

time-constant of 0·2 seconds. Thus the ratio of circuit gain to time-constant is $5 + 10^4$. Now the power gain actually required is 1,000, and this may be achieved either by reducing the m.m.f. ratio or by reducing the control turns but maintaining a constant control circuit resistance.

By using feedback the m.m.f. ratio may be reduced by a factor of $\sqrt{10}$ to $\frac{40}{\sqrt{10}}$. The feedback h'_m will be given by

$$\frac{40}{\sqrt{10}} = \frac{40}{1 + 40h'_m}$$

giving $h'_m = \cdot 054$.

The ratio of power gain to time-constant will be reduced to

$$\frac{5 \times 10^4}{\sqrt{10}} \text{ (equation 3·44)}$$

and since the gain is 1,000 the time-constant will be

$$\frac{1{,}000\sqrt{10}}{5 \times 10^4} = \cdot 063 \text{ seconds.}$$

Alternatively the gain may be reduced by reducing the control turns by a factor $\sqrt{10}$, this will not alter the m.m.f. ratio, however, and the ratio of power gain to time-constant will therefore remain the same, thus for a power gain of 1,000, the time-constant will be ·02 seconds.

5.9 Effect of Bias

A constant biasing m.m.f. is often applied to transductors in order to operate them over a certain part of the characteristic. One of the effects of auto- or self-excitation is to produce a horizontal shift in the control characteristics due to the d.c. ampere-turns provided by the minimum load current through the auto- or self-exciting circuit. This shift is seldom that predicted by the theory (equation 4.20) and in practice the standing load current is usually higher than the theoretical value. For this reason it is not possible to predict exactly what the standing current will be and it is therefore desirable to supply a constant biasing m.m.f. to the core. This is readily achieved by supplying a constant current in an additional control winding having a small number of turns and fed through a high impedance in order to

avoid introducing extra time delays. If there are N_k turns on the bias winding and the current is constant and equal to $\bar{i}_k$ then the bias current is equivalent to a constant control current equal to

$$\frac{\bar{i}_k N_k}{N_c} \quad . \quad . \quad . \quad (5.7)$$

5.10 Power Amplification, Output and Efficiency

The power amplification of an auto-excited transductor with resistive load may be expressed in terms of the current amplification, thus the circuit power amplification is equal to

$$G'_p = \left[\frac{1}{\pi}\frac{\omega M_{ca}}{R_a}\right]^2 \cdot \frac{R_a}{R_c} \quad . \quad . \quad . \quad (5.8)$$

while the useful power amplification is

$$G_p = \left[\frac{1}{\pi}\frac{\omega M_{ca}}{R_a}\right]^2 \cdot \frac{R_L}{2r_{cw}} \quad . \quad . \quad . \quad (5.9)$$

The power amplification may also be considered more directly as the total power output divided by the total power input. As the a.c. fluxes vary cyclically at the supply frequency it has been seen that when the flux is set to the minimum value ϕ_Q this flux is due only to the control m.m.f. (p. 127), and if the control current is smoothed, it must define the mean value of the control current.

Now the minimum possible value for the flux ϕ_Q corresponds to negative saturation and if, during a given half-cycle, the flux in a core reaches this value then in the following half-cycle there should be no output. The maximum possible value of ϕ_Q corresponds to positive saturation and in this case the following half-cycle would be conducting throughout.

Thus the full range of outputs from zero to maximum is given by the corresponding range of values for the minimum flux ϕ_Q from negative to positive saturation. This is equivalent to a corresponding variation of control m.m.f. over the whole of the unsaturated range of the core. Now for a given core and winding area, a given m.m.f. is associated with a specific value of applied volt-amperes since $i_c^2 N_c^2 \propto i_c^2 \cdot 2r_{cw} \propto E_c i_c$ (equation 2.16). Therefore the change of input volt-amperes required to produce the total change of output is fixed. The maximum power output is usually limited by heating of the windings, this is also a function of the core and coil geometry and therefore

a useful criterion for a transductor core can be expressed by the ratio :

$$\frac{\text{maximum volt-amperes controlled in the output}}{\text{input volt-amperes required to saturate the core}}.$$

The maximum current which can be controlled in a given load depends upon the temperature rise which can be tolerated in the transductor and this in turn depends upon the current density and the actual power dissipated in the windings.

Both the current density and the power generated are assessed from the r.m.s. current flowing in a winding. Thus if at maximum output the mean load current is $\bar{\imath}_{a\theta}$, the corresponding r.m.s. load current will be $q\bar{\imath}_{a\theta}$ where q is the form factor (section 2.11). The power generated in a resistance r_{aw} will therefore be

$$q^2\bar{\imath}_{a\theta}^2 r_{aw}.$$

When applied to an a.c. winding, however, this current only flows for half the time in any one winding so the actual power dissipated in a single winding will be

$$\tfrac{1}{2}\, q^2\bar{\imath}_{a\theta}^2\, r_{aw} = i'^2 r_{aw}.$$

Thus the r.m.s. current flowing in one a.c. winding is given by i' where

$$i' = \frac{q}{\sqrt{2}}\,\bar{\imath}_{a\theta} \qquad . \quad . \quad . \quad (5.10)$$

and q is given in equation 2.22. The r.m.s. current i' is used when considering the current density in windings or rectifier ratings.

The efficiency of auto-excited transductors is higher than that of self-excited transductors wound on the same cores. This can be seen by comparing the equivalent circuits in *Figure* 5.1*a* and *b*. The winding *EF* in both cases contains the same weight of copper and occupies the same space since for *a* the turns are one-half and the resistance one-quarter that of *b*. The self-excited transductor has two such windings, however, the a.c. winding and the self-exciting winding, while the auto-excited transductor has only one. The latter therefore has a vacant space on the core which can be filled with wire resulting in an a.c. winding having one-half the resistance and hence half the copper losses for the same number of turns.

Originally the efficiency and therefore the losses in the two circuits were equal as shown by the similarity theorem, and

therefore if the auto-excited transductor were designed to make full use of the core, the copper losses would be halved. The required number of rectifier discs is proportional to the winding resistance (section 5.17), therefore the number of discs can also be halved, giving a complete unit which costs less and occupies a smaller space. The cost of winding and the winding space factor are also improved due to the smaller number of individual coils on the core making insulation problems easier.

5.11 Optimum Supply Voltage

The optimum supply voltage for the auto-excited transductor is one-half that for the self-excited transductor with equal a.c. turns ; this may easily be seen from the similarity theorem or alternatively from the fact that in the latter case two coils in series each having N_a turns are connected to the a.c. supply while in the former case each coil is across the supply. Thus for auto-excitation

$$\bar{E}_{as} = 4f N_a \Phi_s = 4af N_a \mathbf{B}_s \text{ volts} \quad . \quad . \quad . \quad (5.11)$$

If the supply voltage is varied the maximum flux swing also varies and the effect of this may be seen as follows. By the end of any half-cycle the flux in an unsaturated core is set from positive saturation Φ_s down to the minimum value ϕ_Q, and in the following half-cycle this flux is restored to positive saturation. When the supply voltage is maximum the flux ϕ_Q can be given any value between negative and positive saturation, depending upon the control voltage; the former corresponds to zero control voltage and the latter to maximum control voltage (section 5.10).

As the a.c. supply voltage is reduced, the minimum level to which the core flux can be set becomes less negative until for small supply voltages the minimum flux ϕ_Q may be only just below the positive knee of the magnetization curve. In these circumstances the full range of control only occurs over this knee, in which case the slope is small and the characteristic non-linear.

The effect of varying the supply voltage, and therefore the peak flux density, is shown in *Figure* 5.4 for an auto-excited transductor operating at 400 c.p.s. and using mumetal cores for which a suitable design value for the saturation flux density is equal to ·55 webers/M^2 (5,500 gauss).

Curve (*a*) shows the amplification characteristic obtained when the supply voltage is too low. The maximum load current and the operating range are very limited, the gain is also reduced

corresponding to operation on the knee of the magnetization curve. As the supply voltage is increased, the range and the mean amplification increase until when $K = 1$ curve (*c*), the optimum

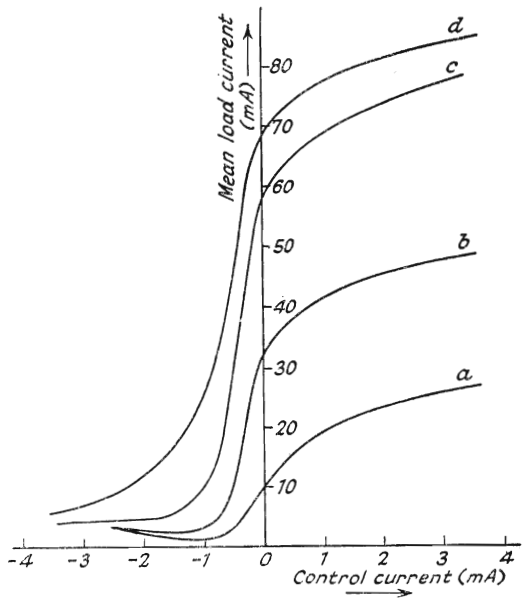


Figure 5.4. Curves showing the effect of various values of supply voltage for a mumetal cored transductor operating at 400 c.p.s. and using auto-excitation. (*a*) $K = 0{\cdot}4$. (*b*) $K = 0{\cdot}6$. (*c*) $K = 1{\cdot}0$ (peak flux density $= \mathbf{B}_s = 0{\cdot}7 \times$ ultimate saturation density). (*d*) $K = 1{\cdot}2$.

value of supply voltage $\bar{E}_{as}$ is reached. Curve (*d*) shows that if the supply voltage is increased further, the lower part of the characteristic becomes curved and the linear operating range therefore reduced as shown.

5.12 Effect of Variation of Supply Frequency

The most important effects of increasing the supply frequency of an auto-excited transductor result in an increased ratio of circuit power amplification to response time and simultaneously an increased output and efficiency for a given core size.

It is shown in section 8.3 that the time-constant of an auto-excited transductor is approximately equal to one-half the time-constant of the control circuit considered as a simple inductance thus : $\tau = \dfrac{L_{cw}}{R_c}$ where L_{cw} is the inductance of one control coil.

Let us now consider an auto-excited transductor which is designed to deliver the maximum permissible output without over-heating. As the supply frequency is increased, the mean supply voltage which can be applied to the core increases proportionally. The load resistance must be increased in proportion to the supply voltage in order to maintain the same maximum current output and thereby limit the heat generated in the windings. Thus the maximum output has increased in proportion to the supply frequency, and since the ratio of the load resistance to that of the windings has increased, the efficiency has also increased. The same control power is required to produce complete saturation, however, and therefore the mean power amplification has increased in proportion to the supply frequency. Furthermore, the time-constant equal to $\frac{L_{cw}}{R_c}$ has not changed. Thus, for a fixed time-constant, the output, the amplification and the efficiency have increased due to the increased frequency. Clearly, the increased amplification could be sacrificed for a correspondingly reduced time-constant by suitable redesign, e.g. by increasing the control circuit resistance. It is therefore evident that a high supply frequency is desirable. As the frequency increases, however, second order effects become prominent chiefly due to increased iron losses in the core which cause reduced amplification and increased control ratio. Effects such as these become important at frequencies above about 4 Kc/s when using cores of mumetal or permalloy *C*, but at considerably lower frequencies for silicon iron cores where losses are more pronounced.

Even at frequencies over 1 Kc/s, however, it becomes increasingly necessary to allow for the effect of iron losses, and it is usually advisable to derate the design values for the sensitivity permeability μ (Chapter 13) and saturation induction (Table 2, Chapter 13). Ferrites have been used at high frequencies, but their permeability and saturation induction are low compared with the nickel-iron alloys. The curves plotted in *Figure* 5.5 show the effect of increasing the supply frequency. These curves are generalized as described in the following section and apply to a core made of H.C.R. metal. At high frequencies, the characteristic can generally be improved by reducing the core area and increasing the a.c. turns to enable the transductor to operate with the same load and supply voltage.

If the turns are increased, this results in an increased impedance of the a.c. windings when the cores are saturated. When the frequency is high, although the permeability of the saturated core is comparatively low, the reactive component of this impedance is generally more significant than the resistance of the windings. It has been shown [3(d)] that the maximum range of

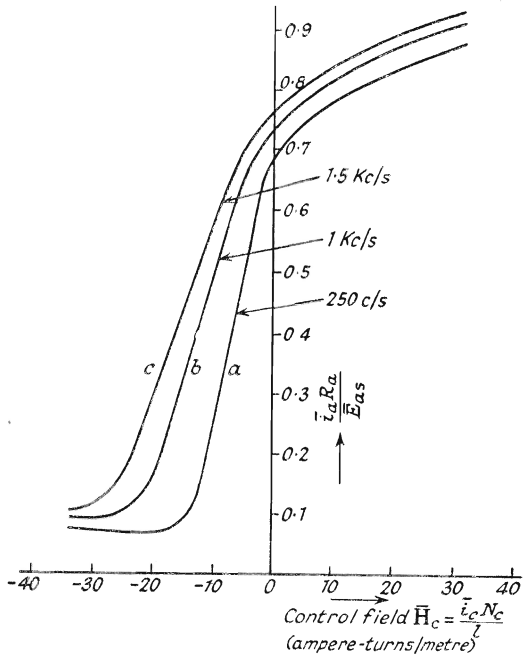


Figure 5.5. General control characteristics for an auto-excited transductor using H.C.R. laminated cores (lamination thickness = ·004 in.).
(*a*) 250 cycles/sec. (*b*) 1,000 cycles/sec. (*c*) 1,500 cycles/sec.

control is obtained when a balance is achieved between the iron losses and the saturated reactance of the a.c. windings, the optimum conditions are shown to apply when

$$\frac{\text{saturation reactance}}{\text{load resistance}} = \frac{\text{iron losses}}{\text{total power output}}.$$

Other effects which become prominent at higher supply frequencies arise due to the self-capacitance of rectifiers.[3(d)] This allows an appreciable reverse current to flow through the rectifier valves and as the frequency increases germanium or silicon

junction type rectifiers give better results than the selenium or copper oxide type.

5.13 Generalized Characteristics for Auto-Excited Transductors

A convenient method of plotting the control characteristics of auto-excited transductors is obtained if the curves are generalized and related to the magnetization characteristic of the core material. So far the amplification has been expressed in terms of the core permeability through the mutual inductance. This has been assumed constant below saturation, but in actual fact the characteristic is non-linear. If it is assumed that the magnetization curve of the core is expressed in the form $\mathbf{B} = \Psi(\mathbf{H})$ where $\mathbf{B}$ is the magnetic flux density and $\mathbf{H}$ the magnetizing field, then the control characteristic for an auto-excited transductor can be derived in terms of this relationship from first principles by considering the flux waveforms in *Figure* 5.2*f* where it is seen that $\phi_Q = \Phi_s - 2\hat{\phi}_t$ (approximately).

In order to generalize the result for any core of a given material, the expressions should involve flux density rather than flux so that using the appropriate suffixes the above relation may be written down

$$\mathbf{B}_Q = \mathbf{B}_s - 2\hat{\mathbf{B}}_t.$$

The flux density $\mathbf{B}_Q$ applies to the unsaturated core and remains appreciably constant during the conducting period. This flux density corresponds to the control field $\mathbf{H}_c$ acting at that instant. Provided the control circuit resistance is not too low (section 4.5), then by similarity with the self-excited transductor the control current will be smooth and therefore the instantaneous value of $\mathbf{H}_c$ is approximately equal to the mean value $\bar{\mathbf{H}}_c$, and since this is the only field acting on the core during the conducting period

$$\Psi(\bar{\mathbf{H}}_c) = \mathbf{B}_s - 2\hat{\mathbf{B}}_t \qquad . \quad . \quad . \quad (5.12)$$

The peak to peak flux change corresponding to $2\hat{\mathbf{B}}_t$ is proportional to the mean voltage $\bar{e}_t$ existing across the transductor windings giving

$$2\overset{\wedge}{\mathbf{B}}_t = \frac{\bar{e}_t}{2afN_a} = \frac{\bar{E}_{as} - \bar{\imath}_a R_a}{2afN_a} \text{ (see equations 1.23 and 3.37)}$$

also since $\bar{E}_{as} = 4afN_a\mathbf{B}_s$.
By substituting this gives

$$\bar{\imath}_a = \frac{2afN_a[\Psi(\bar{\mathbf{H}}_c)]}{R_a} + \frac{\bar{E}_{as}}{2R_a} \qquad . \quad . \quad . \quad (5.13)$$

Dividing by $\bar{E}_{as}/R_a$

$$\frac{\bar{\imath}_a}{(\bar{E}_{as}/R_a)} = \frac{\Psi(\bar{\mathbf{H}}_c)}{2\mathbf{B}_s} + 0.5 \qquad . \quad . \quad . \quad (5.14)$$

Equation 5.13 defines the control characteristic for a given transductor with a resistive load; this corresponds to the relation previously obtained in equation 4.20 for self-excited transductors in which the permeability was assumed to be constant; it will be noted that the term which previously involved the mutual inductance and gave a constant slope to the characteristic is now replaced by the more general term which can only be defined when the magnetization curve for the core is known. By rearranging the terms in equation 5.13 the relationship can be generalized so that it applies to any transductor employing a given core material as in equation 5.14. The characteristic of a transductor may be obtained in this form by plotting the mean load current as a fraction of its maximum possible value $\bar{E}_{as}/R_a$ and the control current as mean control ampere-turns per metre $\bar{\mathbf{H}}_c$.

If the magnetization characteristic is assumed linear, then when expressed in its non-dimensionalized form we have

$$\frac{\bar{\imath}_a}{(\bar{E}_{as}/R_a)} = \frac{\mu}{2\mathbf{B}_s}\left[\frac{\bar{\imath}_c N_c}{l}\right] + 0.5 \qquad . \quad . \quad . \quad (5.15)$$

in which case the slope is given by the permeability divided by the total flux range for the core.

Thus, to a first order, expressions of the form given in equation (5.14) and (5.15) define the control characteristic of any auto-excited transductor using a given core material and operating with resistive load at the maximum supply voltage $\bar{E}_{as}$.

Second order effects, however, usually influence the shape of this characteristic, and assuming that the rectifiers are being used under favourable conditions the two effects which are most significant arise due to iron losses and the variation of core materials from one batch to another. The former varies primarily with frequency as discussed in section (5.12), but the latter effect cannot be predicted and may be considerable, often amounting to a variation in slope of 30% or 40% for a given magnetizing field. This variation affects the maximum current amplification, and it is to be hoped that materials will become more uniform as time goes on, otherwise if close design tolerances are required one must resort to designing on the basis of the worst charac-

teristic and, after manufacture, introducing a small amount of negative feedback with trimming resistors as described in section 5.7.

5.14 Equivalent Circuit for Auto-Excited Transductor

As in the case of the self-excited transductor, the auto-excited transductor may be considered equivalent to a voltage generator of internal impedance r_x supplying an e.m.f. whose mean value when rectified is equal to $\left(\frac{1}{\pi}\omega M_{ca}\right)\bar{\imath}_c$, in series with a constant e.m.f. equal to $(K - \frac{1}{2})\bar{E}_{as}$ where K is the ratio of the actual supply voltage $\bar{E}_a$ to the optimum supply voltage $\bar{E}_{as}$. When K is made equal to unity as is normally the case, the e.m.f. of the equivalent generator is

$$\left.\begin{aligned}\bar{e}_a &= \left(\frac{1}{\pi}\omega M_{ca}\right)\bar{\imath}_c + \frac{\bar{E}_{as}}{2}\\ \text{or}\qquad \bar{e}_a &= \left(\frac{1}{\pi}\frac{\omega M_{ca}}{R_c}\right)E_c + \frac{\bar{E}_{as}}{2}\end{aligned}\right\} \quad . \quad . \quad . \quad (5.16)$$

This leads to a useful equivalent circuit for auto-excited transductors which is similar to the equivalent circuits which are often considered to apply to electronic valve amplifiers. If the constant terms are ignored, such circuits are of the form shown in *Figures* 5.6*a* and *c*. The circle around the voltage $\bar{e}_a$ represents the transductor and a.c. supply (both assumed to

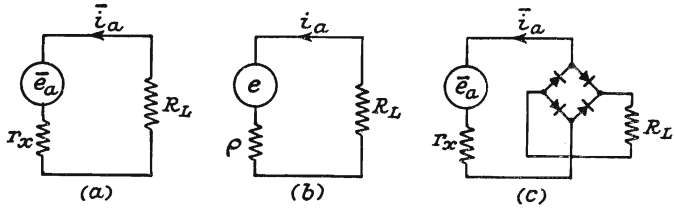


Figure 5.6. Equivalent circuits.
(*a*) Auto-excited transductor.
(*b*) Electronic valve.
(*c*) Transductor with external load rectifier.

(*a*) Transductor

$\bar{e}_a = \left(\frac{1}{\pi}\omega M_{ca}\right)\bar{\imath}_c$ volts

or $\bar{e}_a = (2af\,N_a)\,\Psi(\bar{\mathbf{H}}_a)$ volts

(*b*) Electronic valve

$e = mV_g$
i_a = anode current
ρ = internal resistance
m = amplification factor
V_g = grid voltage

have zero impedance); the internal resistance r_x is then considered to be in the external circuit.

In the presence of bias it is necessary to add another term to the voltage expression in equation 5.16. This will be a constant voltage and therefore the equivalent circuit e.m.f. including the signal and the constant terms are equal to

$$\bar{e}_a = \frac{1}{\pi}\omega M_{ca}\left(\bar{\imath}_c + \frac{\bar{\imath}_k N_k}{N_c}\right) + \frac{\bar{E}_{as}}{2}.$$

A more general expression for the e.m.f. of the equivalent voltage generator is given by the results in the previous section, in which case the generated voltage is shown to be non-linear with control current and is equal to

$$2\,afN_a[\Psi(\bar{\mathrm{H}}_c)] \text{ volts.}$$

For a given transductor the maximum transfer of power to the load occurs when the resistance is matched to the internal resistance r_x of the transductor. Under these conditions the thermal efficiency is only 50% and if the transductor is a large unit or operates at a high supply frequency the matched condition may lead to overheating (see Chapter 13).

For a given load, however, the power transfer increases as the internal resistance r_x is reduced. Reduction of the internal resistance involves an increase of core size which may not be permissible. Furthermore, since the load and required maximum output are usually fixed, the supply voltage $\bar{E}_{as}$ will be fixed also and thus for a given supply frequency and core size the a.c. turns will be predetermined by equation (5.11). In general, therefore, the minimum core size is usually chosen from thermal considerations, i.e. efficiency, and having done this the question of maximum power transferred does not arise. These considerations are further discussed in Chapter 13.

Limitations due to Rectifiers

It will by now be apparent that the rectifier plays a vital part in the operation of both self-excited and auto-excited transductors. It is therefore essential to know the more important limitations due to rectifier characteristics and in addition how the rectifiers should be used in order to have a minimum effect on the ultimate performance of the transductor.

5.15 The Load Rectifier

When considering the load rectifier the foremost requirements are a high efficiency and reasonable consistency from one rectifier element to another. The latter need arises when transductor pairs are used in push-pull when it is usually desirable to have available equally matched pairs of rectifier elements.

Rectifier efficiency is expressed as the ratio of mean volt-amperes input to d.c. volt-amperes output on maximum rating, and the chief cause of rectifier inefficiency is due to the forward resistance of the rectifying element. This will obviously cause a reduction of transductor efficiency in addition to that due to the transductor windings alone, but will not contribute to the heating of the transductor itself unless both transductor and rectifier are sealed in the same box. A low forward resistance could be obtained with any rectifier by placing a number of elements in parallel; this, however, reduces the reverse resistance and increases the self-capacitance, both of which have undesirable effects. In general, a rectifying element should have a high ratio of reverse resistance to forward resistance, and this is a function only of the material.

5.16 Excitation Rectifier

The rectifier reverse and forward resistances are far more important when the rectifier is used for self-excitation or auto-excitation, and it then becomes necessary to consider the rectifier as forming a part of the amplifier itself. In order to consider the effect of rectifier characteristics, a practical rectifier can be represented approximately as an ideal rectifier element in series with a resistance r_1 and the combination then placed in parallel with a resistance r_2. The resistances r_1 and r_2 represent the forward and reverse resistances respectively, provided $r_2 \gg r_1$.

When used for auto-excitation or self-excitation, the forward resistance merely causes a slight increase in the internal impedance of the transductor being added to the resistance of the windings. This results in a slight loss of efficiency as was found for the load rectifier.

The reverse resistance, however, tends to apply a negative feedback to the transductor, and if the reverse resistance is r_2 the feedback term denoted by h'_m is given from equation 5.6 by

$$h'_m = \frac{r_{aw} + r_1}{2(r_{aw} + r_1) + r_2}.$$

Also, since the reverse resistance is generally large compared with the winding resistance and the forward resistance, the above expression approximates to

$$h'_m = \frac{r_{aw} + r_1}{r_2}.$$

Now the ampere-turns ratio G_m is given by

$$G_m = \frac{g'_m}{1 + g'_m h'_m}$$

and for an auto-excited transductor with an ideal rectifier h'_m is zero, giving $G_m = g'_m$.

If, however, the value of the feedback term is not zero for reasons given above and in addition the ratio of reactance to resistance $\frac{\omega L_{aw}}{R_a}$ for the a.c. circuit is sufficiently large, then since

$$g'_m = \frac{1}{\pi} \frac{\omega L_{aw}}{R_a}$$

we have $$g'_m h'_m \gg 1$$

giving the m.m.f. ratio $G_m = \frac{1}{h'_m} = \frac{r_2}{r_{aw} + r_1}$.

Now if these resistances were stable as is usually arranged to be the case in negative feedback amplifiers, then the final transductor characteristic, although completely dependent upon the rectifier, would be both linear and free from drift ; unfortunately, however, this is not the case and the rectifier reverse resistance depends upon other variables such as temperature, voltage, ageing, etc. ; therefore it is usually advisable to avoid the above conditions in which the rectifier reverse resistance determines the transductor characteristics.

Thus it follows that the ratio of the reverse to the forward resistance lays down an upper limit for the m.m.f. ratio g'_m, beyond which the transductor would be too dependent upon the rectifier properties.

Taking a typical example of an auto-excited transductor whose winding resistance $r_{aw} = 20$ ohms, rectifier forward resistance 20 ohms, and reverse resistance 20,000 ohms, gives :

$$\frac{1}{h'_m} = \frac{20{,}000}{40} = 500.$$

Therefore the ampere-turns ratio cannot be greater than 500 no matter how large one makes the reactance/resistance ratio, and in order to make the transductor reasonably insensitive to rectifier drift, this ratio should be well below 500. Thus, when using selenium rectifiers, for example, it is not usual for the ampere-turns ratio of a single stage to exceed about 80 (section 4.8).

In addition to its effect on the gain, however, the reverse resistance causes the control characteristic to shift to the right, because the reverse current corresponds to a loss of d.c. excitation which must be provided by an increased control current. The effect shows up as a reduction of the constant term in equation 4.17 due to the increased value of h_i.

Thus the effect on the slope leads to a drift of gain and the effect on the constant term tends to cause zero drift. It is shown in Chapter 10 that by using a pair of transductors operating in push-pull, the latter drift tends to cancel provided a reasonable degree of matching can be attained between the rectifiers of each pair.

The reverse voltage has an important bearing on the rectifier performance when used for auto-excitation. In the case of a selenium rectifier the elements should be considerably under-run and a rating of one-third the normal is quite common. This tends to make the reverse resistance very high, so that slight variations due to temperature, voltage, etc., will be less significant.

5.17 Reverse Voltages on Rectifiers used in Auto-Excited Transductors

The reverse voltages applied to the rectifiers are governed by the same consideration as those given for the self-excited transductor (section 4.12).

Referring to *Figure* 5.1*b*, in the parallel (group 1) connection no reverse voltage appears across rectifier (2) until core A saturates, since both rectifiers conduct during unsaturated periods due to the circulating current $i_r/2$. After saturation the reverse voltage on rectifier (2) is equal to the voltage drop in the windings of the a.c. coil A.

In the bridge connection (group 2) the auto-excitation rectifiers are working under less favourable conditions (*Figure* 5.1*c*). During the conducting period a reverse voltage equal to the supply voltage appears across rectifiers (1) or (2). Simultaneously the voltage across the load appears as a reverse voltage across (3) or (4) respectively. The auto-excitation rectifiers (1) and (2) must

be considerably underrun to ensure linear performance and freedom from drift. Therefore, if these conditions are of primary importance, the parallel arrangement is preferable. Typical curves showing the effect of selenium rectifier characteristics in a practical transductor are given in *Figure* 5.7. The points shown on curve (*c*) represent the characteristic obtained with an auto-excited transductor in which the d.c. load is connected across a separate bridge rectifier as could be done in *Figure* 5.1*b*. In this

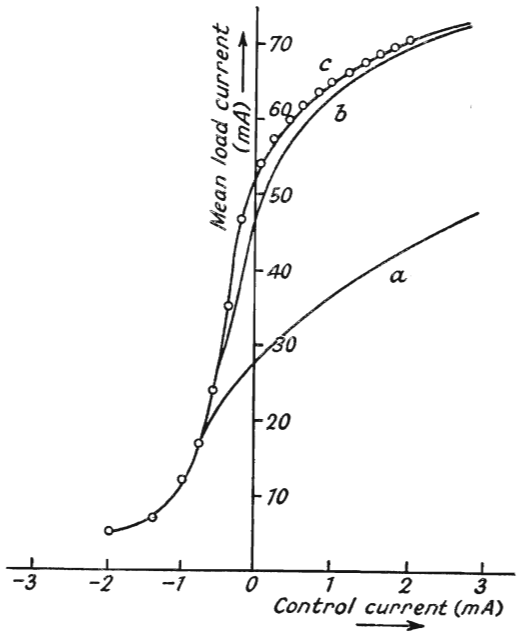


Figure 5.7. Curves showing the effect of overrunning the auto-excitation rectifier.

particular case the load resistance was 300 ohms, which was four times that of one a.c. winding. One disc per arm of the bridge was sufficient for the load rectifier, which was then being run at about three-quarters of its maximum rated value. The auto-excitation rectifiers also consisted of one disc each and under these conditions were being adequately underrun.

When the same circuit was connected up as a bridge transductor (*Figure* 5.1*c*), however, the auto-excitation rectifiers were being operated at almost their maximum rating as load rectifiers, and were supporting the full supply voltage. The reduced reverse

resistance with loss of gain and linearity is shown in curve (*a*). On increasing the number of discs in the auto-excitation rectifier to two, curve (*b*) was obtained and finally, with three discs, the original group (1) characteristic was reproduced as shown in the full line curve (*c*).

5.18 Types of Rectifier

Rectifiers used for auto-excitation are either of the metal type or crystal junction diodes. Valve rectifiers are generally unsuitable owing to the fact that a small cathode current flows for zero anode voltage giving rise to unwanted polarization of the core.

(*a*) *Copper Oxide Rectifier.* The copper oxide rectifier is chiefly used for instrument work, its only real advantage being a low forward resistance. The maximum peak reverse voltage is about 6 volts and the rectifier deteriorates at temperatures outside the range $-40°$ C. to $+40°$ C.

(*b*) *Selenium Rectifier.* The selenium rectifier is much more satisfactory in terms of both reverse voltage and temperature stability.

Reverse voltages up to 36 volts peak are possible and the rectifier will operate at disc temperatures of 75° C. With special processing, temperatures of 125° C. can be attained with satisfactory results for over 2,000 hours.

The ambient temperature range for these rectifiers is approximately $-55°$ C. to $+85°$ C. The current rating of the selenium rectifier is about 50 mA per cm.^{2} and the self-capacitance varies from 0·1 μF down to 0·02 μF per cm.^{2}, depending upon the applied voltage. In general, the capacitance of selenium rectifiers becomes important when the supply frequency exceeds about 1 Kc.

(*c*) *Germanium Crystal Junction Rectifiers.* These types of rectifier have a very low forward resistance and a very high reverse resistance and are therefore ideally suited to magnetic amplifier work. They are also much smaller than the metal rectifiers and withstand greater reverse voltages for a given forward resistance. The self-capacitance of germanium rectifiers is extremely small in comparison with that of selenium and no appreciable effects are apparent at frequencies as high as 2·4 Kc/s. Unfortunately, however, the maximum ambient temperature is somewhat lower than that applicable to selenium, being in the region of 60° C.

(*d*) *Silicon Junction Rectifiers.* Silicon junction diodes are

gradually superseding the germanium type of rectifier for magnetic amplifier work. Successful operation in ambient temperatures of the order of 150° C. is feasible with silicon junction diodes.

The curves in *Figure* 4.2 show the d.c. characteristics of some typical rectifiers.

5.19 Summary

It has been shown that auto-excited transductors are similar to transductors with complete self-excitation and that the ampere turns ratio can be changed by the use of various forms of feedback.

Iron losses limit the maximum permissible value of the supply frequency to about 4 Kc/s, but otherwise a high supply frequency is desirable on account of the improved ratio of power gain to response time.

The auto-excited transductor without feedback can be considered as a voltage generator in which the output voltage is a function of the control signal. As such the maximum power transfer occurs *for a given transductor* when the load is matched to the output impedance of the transductor, but these conditions can only apply in practice to small units operating at low frequencies.

The performance required of the auto-excitation rectifier is very high indeed, particularly in respect of its reverse/forward resistance ratio, otherwise the gain is limited if stable characteristics are required. Crystal junction diodes are the most satisfactory rectifiers for this purpose.

The load rectifiers do not require such a high performance and, provided the efficiency of rectification is adequate, the most important limitation is set by the temperature at which the rectifier ceases to operate satisfactorily.

The voltage or current gain of an auto-excited transductor is one-half that of a fully self-excited transductor with the same number of turns. The latter values are given in columns 2 and 3 of the table at the end of Chapter 4.

CHAPTER 6

TRANSDUCTORS WITH REACTIVE LOADS

So far the transductor has been considered for cases where the load is purely resistive. Very often, however, it is required to control a load device which may be appreciably inductive. It is also found that a smoothing capacitor across a d.c. load gives improved characteristics.

The first detailed study of transductor behaviour with reactive loading appears to have been made by Krabbe [3(b)]; papers have also been published by Milnes [6(a)] and Storm.[6(b)] This chapter will be devoted to a consideration of the transductor with various types of reactive loading under steady-state conditions.

6.1 Auto-Excited Transductor with A.C. Inductive Load

In many cases the transductor is required to deliver a.c. power to inductive loads such as a.c.-operated servo motors, a.c. torque motors, resolvers, etc. Often in such cases two transductors operate as a push-pull pair, and when the d.c. signal changes sign the load current undergoes a phase reversal of 180°. In this section we shall consider the transductor operating as a single unit. The same assumptions as those made previously will be applicable to the following cases. Thus when both cores are unsaturated it will be assumed that the transductor impedance in the a.c. circuit is large compared with that of the load, but when one core is saturated the whole supply voltage is available to drive current through the load circuit and that these conditions persist until the current in the a.c. windings decreases to a value which is insufficient to maintain the core saturated.

Let us first consider the circuit of the auto-excited transductor in *Figure* 6.1*a* and the waveforms in *Figure* 6.1*b*. Assuming a control signal has been applied and transients have died down,

then during any cycle up to the time t_q both cores are unsaturated and consequently the fluxes ϕ_A and ϕ_B are changing and the whole supply voltage is developed across the transductor. Assuming the

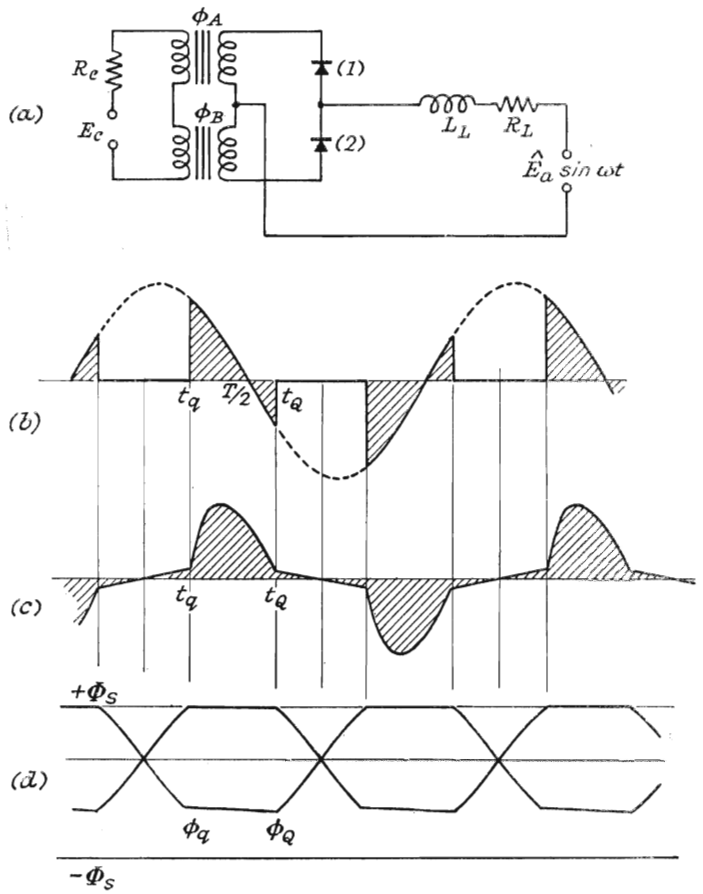


Figure 6.1. Waveforms of voltage, current and flux for transductor with a.c. inductive load.

(*a*) Circuit diagram of auto-excited transductor with a.c. inductive load. (*b*) Voltage across load. (*c*) Load current. (*d*) Core fluxes.

flux in core A is increasing, then at time t_q core A saturates and the whole supply voltage is available to drive current through the external circuit. The current cannot increase immediately, however, due to the inductance L_L and it therefore rises slowly, lagging behind the applied voltage as shown by the shaded area

over the interval following t_q (*Figure* 6.1*c*). When the supply voltage reaches zero at a time $T/2$, the current continues to flow due to the inertia of the inductance and it does not become small enough to cause desaturation until some time t_Q after the load voltage has become negative. The supply voltage will therefore continue to appear across the load after it has changed sign. The load current is less peaked than in the case of the transductor with resistive load due to the load inductance which tends to oppose the rapid changes of current which occur when a core becomes saturated.

The waveforms of flux are shown in *Figure* 6.1*d*. As previously (p. 127), the mean control m.m.f. is the only m.m.f. in the unsaturated core over the conducting period (e.g. t_q to t_Q). This m.m.f. corresponds to the final minimum flux ϕ_Q which, as before, is appreciably constant during conducting periods provided the referred value of the control circuit resistance $R_c\left(\frac{N_f}{N_c}\right)^2$ is sufficiently large. Therefore, when the mean load current is a minimum the flux ϕ_Q corresponds to the negative saturation level and the control ampere-turns are therefore equal to $-S\Phi_s$. When the transductor allows the full output current to flow, both cores are saturated throughout the cycle and the ampere-turns due to the control winding are equal to $+S\Phi_s$. Thus the total change of ampere-turns is $2S\Phi_s$, and this controls the transductor over its complete range from nearly zero up to the maximum output which corresponds to a mean load current equal to $\bar{E}_{as}/Z_a$ where

$$Z_a = \sqrt{(R_L + r_x)^2 + (\omega L_L)^2} \quad . \quad . \quad . \quad (6.1)$$

When the transductor is unsaturated throughout the cycle the mean load current may be considered negligibly small. The total change of load current divided by the total change of control current is therefore equal to $\frac{\bar{E}_{as}/Z_a}{2S\Phi_s/N_c}$, and on substituting for $\bar{E}_{as}$ and S, this gives a mean current amplification G_i equal to

$$\frac{1}{\pi}\frac{\omega M_{ca}}{Z_a} \text{ (for auto-excitation)} \quad . \quad . \quad . \quad (6.2)$$

or

$$\frac{2}{\pi}\frac{\omega M_{ca}}{Z_a} \text{ (for self-excitation)} \quad . \quad . \quad . \quad (6.3)$$

Thus, when the load is inductive, the mean amplification is given

by the same expression as when the load is resistive except that the impedance Z_a now replaces the resistance R_a.

The generalized curves in *Figure* 6.2 are for a mumetal-cored auto-excited transductor operating at 400 c/s into various a.c. loads of constant impedance Z_a. The load current is plotted as a fraction of its maximum possible value, and the control signal as mean magnetizing field in ampere-turns per metre. Curve (*a*) is for the case where Z_a is purely resistive, curve (*b*) for $\dfrac{\omega L_L}{(R_L + r_x)} = 1$, and curve (*c*) for $\dfrac{\omega L_L}{(R_L + r_x)} = 10$. The most noticeable effect due to the inductance is a reduction of the linear range compared with that obtained for a resistive load but substantially the same slope over the working part of the characteristic.

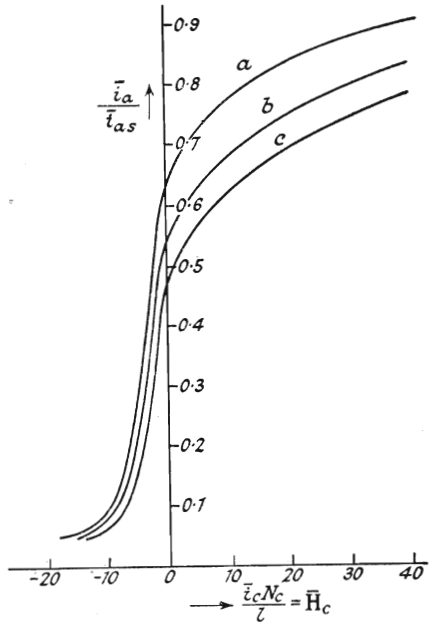


Figure 6.2. Generalized transductor control characteristics with a.c. inductive load of constant total impedance.

(*a*) $\omega L_L = 0$. (*b*) $\omega L_L = (R_L + r_x)$. (*c*) $\omega L_L = 10(R_L + r_x)$.

Transductors with D.C. Inductive Load

If the rectified load current from a transductor is smoothed, then in certain circumstances the current amplification is increased; this is particularly important at low signal levels, where the effect is most pronounced.

It has already been shown (section 5.14) that a transductor with auto-excitation or complete self-excitation is equivalent to a voltage generator whose mean output e.m.f. depends upon the control signal. Similarly, the transductor without self-excitation is equivalent to a current generator. In the former case the internal dynamic impedance is low whereas in the latter case it is high so long as the operation is confined to the linear range.

The transductor only allows energy to be delivered to the load over a fraction of the total half-cycle : this fraction is small, when operating at low output levels and the total energy transferred in this time depends upon the matching of the load impedance. If the load is purely resistive, load current will only flow during conducting periods and it will be limited by the total circuit resistance. By introducing an additional element capable of energy storage, however, it is possible to take in a comparatively large amount of energy from the source during conduction, and deliver this to the load during the following non-conducting period. The effect is therefore to lengthen the time over which load current flows and therefore, in certain circumstances, to increase its mean value for a given control signal.

6.2 Action of Load Rectifier with D.C. Inductive Load

When a rectifier-fed transductor load is resistive the current flowing in the transductor a.c. windings is the same as the current in the load (ignoring rectifier leakage) excepting that the former is alternating while the latter is routed in such a way as to be always unidirectional.

If the load is inductive, however, then the load rectifier will undergo periods of relaxation in precisely the same way as was described in section 4.2. During the periods when both transductor cores are unsaturated the current in the a.c. circuit is negligibly small, but if the load is inductive then an appreciable current may flow in the load, the circuit being completed through the two halves of the bridge rectifier acting in parallel.

Referring to the circuit in *Figure* 6.3*a*, the d.c. inductive load is considered to be supplied from a transductor which, as we have seen, delivers a pulse of voltage to the load circuit once in every half-cycle. The effect of the load inductance is primarily to smooth the load current and thereby tend to maintain it at a constant value throughout the cycle.

The resistance r_x shown on the a.c. side of the rectifier is the effective resistance in the a.c. circuit during a conducting period ; this includes the resistance of the transductor windings, the self- or auto-excitation rectifier and in addition the resistance of two load rectifier elements in series.* When the transductor is

* When the rectifier is relaxing all four elements are conducting and hence this contribution to r_x is halved. This effect is comparatively unimportant, however, since it arises primarily during unsaturated periods.

saturated, current is forced through this resistance and the load due to the applied voltage e_a which is the ungated supply voltage. As this decreases towards the end of the half-cycle, the load current is transferred to the closed circuit formed by the two halves of

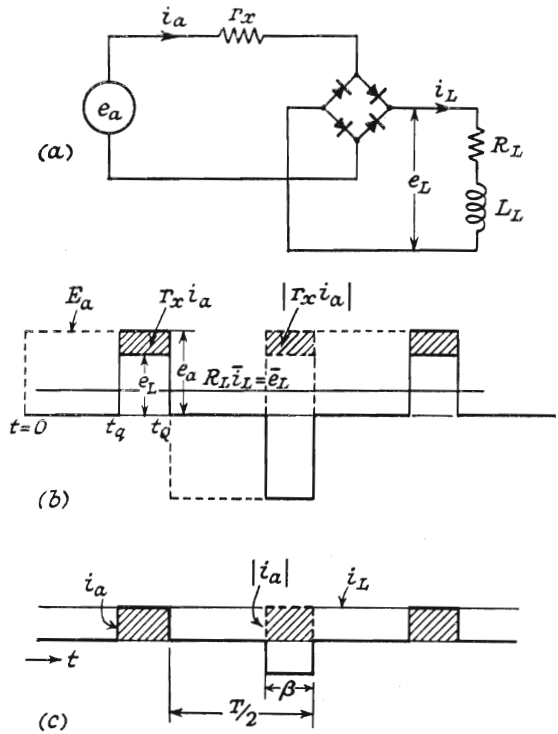


Figure 6.3. Diagrams illustrating effects of smoothing with a bridge rectified inductive load.

(*a*) Equivalent circuit.
(*b*) Waveforms of output voltage e_a and load voltage e_L.
(*c*) Waveforms of transductor current i_a and load current i_L.

the bridge rectifier acting in parallel; this current is maintained by the self-induced e.m.f. in the load inductance, and it continues to circulate through the bridge rectifier throughout the following unsaturated period until saturation occurs once more and the load current is again forced by the externally applied e.m.f.

When dealing with resistive loads in Chapters 4 and 5, it was explained that the ratio of mean output voltage to mean transductor current was equal to the total a.c. circuit resistance R_a,

but that this was not necessarily true for all types of load and hence the " mean resistance " was defined by

$$\bar{R}_a = \frac{\bar{e}_a}{\bar{i}_a} \qquad . \quad . \quad . \quad (6.4)$$

The need for this arises due to the relaxation of the rectifier when using inductive loads which in addition causes the load current i_L to differ at times from the transductor current i_a. The ratio of the mean output voltage and mean load current has therefore been defined as

$$\bar{R}_{aL} = \frac{\bar{e}_a}{\bar{i}_L} \qquad . \quad . \quad . \quad (6.5)$$

Now since the output voltage corresponds to the ungated portion of a sine wave its mean value depends upon the voltage amplification and the control voltage which was investigated in Chapter 4.

Knowing the voltage amplification, the mean load current or the mean transductor current are found by dividing the mean output voltage by either $\bar{R}_{aL}$ or $\bar{R}_a$ respectively. It therefore remains to investigate the nature of these quantities under the various conditions of operation.

Considerable simplification is achieved if, as in Chapter 3, the transductor is assumed to be excited with a square wave supply voltage, and while errors are likely to be introduced by doing this, the physical mechanisms are more easily understood. In addition it will be assumed that the load inductance is sufficiently large that the load current may be considered smooth.

The pulses of voltage e_a applied to the a.c. circuit by the transductor are rectangular in shape as shown in *Figure* 6.3*b*. Saturation occurs at t_q when the supply voltage is ungated and a current i_L is forced through the load ; desaturation occurs at t_Q, after which the load current i_L is maintained through the bridge rectifier. During conducting periods a voltage e_L is directly applied to the load as shown in *Figure* 6.3*a*, and the load current i_L will be equal to the transductor current i_a (*Figure* 6.3*c*). During relaxation, the applied voltage e_L is zero. Since the load current is smoothed it will always be constant, however, and therefore, provided the waveform is rectangular, the following equation may be written down for mean values considered over one half-cycle :

$$\bar{i}_a = (2\beta/T)\,\bar{i}_L. \quad \text{(See } \textit{Figure} \text{ 6.3}c\text{)} \qquad . \quad . \quad . \quad (6.6)$$

where β is the duration of a conducting period. Energy is only transferred to the load during conducting periods and therefore in the steady-state the total energy transferred during a conducting period is equal to the energy dissipated in the load over a half-cycle, thus

$$\int_{t_q}^{t_Q} e_L \bar{i}_L \, dt = R_L \bar{i}_L^2 \cdot \frac{T}{2}.$$

Multiplying both sides by $\frac{2}{T}$ and cancelling the constant term $\bar{i}_L$ gives

$$\bar{e}_L = R_L \bar{i}_L \quad . \quad . \quad . \quad (6.7)$$

This shows that although the voltage applied to the load circuit is of a different waveform from the actual resistive voltage drop in the load, the mean values of these two quantities are equal ; this result is obtained directly from energy considerations and is independent of the waveforms.

During the conducting period the voltage drop in the circuit resistance r_x will be $r_x i_a$ and therefore

$$| e_a | - e_L = | r_x i_a | \quad (\textit{Figure } 6.3b) \quad . \quad . \quad . \quad (6.8)$$

When both cores are unsaturated these variables are all negligible and during conducting periods they all have the same sign, therefore

$$\bar{e}_a - \bar{e}_L = r_x \bar{i}_a \quad \text{(see section 1.4)} \quad . \quad (6.9)$$

By combining equations 6.6, 6.7 and 6.9 it is easily shown that

$$\frac{\bar{e}_a}{\bar{i}_a} = \bar{R}_a = \frac{R_L\left(1 + (2\beta/T)\,\frac{r_x}{R_L}\right)}{2\beta/T} \quad . \quad . \quad . \quad (6.10)$$

also from 6·6

$$\frac{\bar{e}_a}{\bar{i}_L} = \bar{R}_{aL} = R_L\left(1 + (2\beta/T)\,\frac{r_x}{R_L}\right) \quad . \quad . \quad . \quad (6.11)$$

and

$$\bar{R}_a = \frac{\bar{R}_{aL}}{(2\beta/T)} \quad . \quad . \quad . \quad (6.12)$$

The quantity $\bar{R}_{aL}$ varies with the duration of the conducting period, from a minimum value equal to the load resistance R_L when $\beta = 0$ at low output levels to a maximum value equal to the total circuit resistance $R_a = R_L + r_x$ when $\beta = T/2$ at full output.

This does not represent an appreciable change, however, and it is therefore a reasonable approximation to regard the ratio

$\bar{e}_a/\bar{\imath}_L$ as being approximately constant as it would be for a purely resistive load. This does not apply, however, to the ratio $\bar{R}_a$, expressing the transductor current; this ratio which is inversely proportional to the duration of the conducting period.

Thus as the conducting period tends to zero $\bar{R}_a$ tends to infinity. This may be explained physically as follows. The current i_a reverses every half-cycle and if the conducting period is very small the pulses of the voltage are sharp and the current will be unable to build up due to the inertia of the inductance, therefore the ratio of $\bar{e}_a/\bar{\imath}_a$ will be infinite. The load current, on the other hand, is always unidirectional and therefore it must build up ultimately to a finite value in the steady-state, hence $\bar{e}_a/\bar{\imath}_L$ is finite.

6.3 Current Amplification with D.C. Inductive Load

Having obtained the ratios for mean voltage/mean current, it is now possible to estimate the voltage gain for any degree of self-excitation by substituting the value found for $\bar{R}_a$ into the term h_v (equation 4.15). It should be remembered at this stage that the basic voltage gain equations (4.13–4.15) are equally applicable to square wave excitation. The mean load current is then found by dividing the mean output voltage $\bar{e}_a$ by the " mean resistance " $\bar{R}_{aL}$.

Thus, considering the case of a fully self-excited transductor, the condition $h_v = 0$ gives a voltage gain $G_v = g_v$ as for a resistive load. The output is thus given by

$$\begin{aligned}\bar{e}_a &= g_v E_c \\ &= g_v R_c \bar{\imath}_c\end{aligned}$$

and therefore from equation 6.11

$$\bar{\imath}_L = \frac{g_v R_c \bar{\imath}_c}{\bar{R}_{aL}} \qquad . \quad . \quad . \quad (6.13)$$

and since

$$g_v = \frac{2}{\pi}\frac{\omega M_{ca}}{R_c}$$

$$\bar{\imath}_L = \frac{2}{\pi}\frac{\omega M_{ca}}{R_L\left(1 + (2\beta/T)\dfrac{r_a}{R_L}\right)} \cdot \bar{\imath}_c \qquad (6.14)$$

This shows that the current amplification of a fully self-excited transductor with an inductive load is not very different from that

of a resistively loaded transductor with complete self-excitation for which the current amplification $\frac{2}{\pi}\frac{\omega M_{ca}}{R_a}$ may be written

$$\frac{2}{\pi}\frac{\omega M_{ca}}{R_L\left(1+\frac{r_x}{R_L}\right)} \text{ since } R_a = R_L + r_x.$$

For an inductively loaded transductor the amplification tends to decrease as the condition of maximum output is approached due to the factor β, which increases with increasing control current. When auto-excitation is employed the same results apply excepting that the factor $2/\pi$ is replaced by $1/\pi$.

When there is no self-excitation or auto-excitation, the current amplification is obtained by considering the conditions of ampere-turn balance. The mean a.c. ampere-turns $\bar{i}_a N_a$ must be equal to the mean control ampere-turns $\bar{i}_c N_c$. But the mean load current $\bar{i}_L$ is not equal to the transductor alternating current and is given by equation 6.6. Combining these two results we have

$$\frac{\bar{i}_L}{\bar{i}_c} = \frac{N_c}{N_a}\frac{1}{2\beta/T} \quad . \quad . \quad . \quad (6.15)$$

or when using self-exciting turns $N_f < N_a$ and assuming the core reluctance is zero

$$\frac{\bar{i}_L}{\bar{i}_c} = \frac{N_c}{N_a - N_f}\cdot\frac{1}{2\beta/T}\,* \quad . \quad . \quad . \quad (6.15a)$$

Thus when the load is inductive the current amplification of the unself-excited transductor is no longer equal to $\frac{N_c}{N_a}$ but is non-linear, being infinite for small outputs and decreasing as the output increases. It should be noted that when the conducting period extends throughout the cycle the transductor output cannot increase any further and therefore although equation 6.15 indicates that at this stage the gain is still given by the turns ratio,

* This result may equally well be obtained from the basic equations :

Thus, $\bar{i}_L = (G_v/\bar{R}_{aL})\bar{i}_c R_c$

but since $G_v = \frac{1}{h_v}$ (for no self-excitation)

this gives (from table p. 122), $\frac{\bar{i}_L}{\bar{i}_c} = \frac{\bar{R}_a}{\bar{R}_{aL}}\cdot\frac{N_c}{N_a - N_f} = \frac{1}{2\beta/T}\cdot\frac{N_c}{N_a - N_f}$

it is obviously beyond the working range of the transductor, and in fact of course the gain has decreased to zero. In practice complete saturation does not occur discontinuously but gradually, with the result that as the output increases towards a maximum the current amplification does not tend to the value $\frac{N_c}{N_a}$ but approaches zero gradually due to gradual saturation of the magnetization curve.

These results are interesting in view of earlier remarks about the output impedance of the transductor. The transductor without self-excitation has been considered (section 4.11) as an amplifier with a high output impedance. If the load is resistive then for short conducting periods the energy transferred to the load is limited by the load resistance. If, however, the load contains series inductance then this will tend to match the transductor, being itself a high impedance to voltage pulses. Thus, although the conducting period is only of short duration, the inductance, being a favourable impedance match, stores a larger quantity of energy than could be dissipated in the resistance over the same period ; this energy is then transferred to the load during relaxation periods. Practical characteristics obtained with inductive loads are discussed in section 6.4.

6.4 D.C. Inductive Load with Sinusoidal Supply Voltage

In this section the waveforms will be considered for an auto-excited transductor operating with an inductive load and a sinusoidal supply voltage (*Figure* 6.4*a*). It will be assumed in what follows that complete smoothing of the load current occurs. Waveforms of the circuit voltages, currents and fluxes are shown in *Figure* 6.5*a*.

Up to the time t_q, both cores are unsaturated and the core fluxes are changing. The current flowing in the load is maintained at a constant value, *Figure* 6.5*c*, due to the self-induced e.m.f. in the inductance L_L ; this e.m.f. is such as to enable all four elements of the load rectifier to be conducting, and therefore the voltage drop across MN (*Figure* 6.4*a*) is due only to the forward resistance of the rectifiers.

At t_q core A saturates and the supply voltage is available to drive current through the circuit impedance which includes the load. Almost the whole supply voltage is therefore applied to the rectifier and is such that it makes J positive with respect to H.

The rectifier elements (3) and (5) therefore cut off and the load current flows through the a.c. windings on the saturated core via the rectifiers (4) and (6) and the auto-excitation rectifier (1). Over this period also the load current is constant due to the inductance of the load, and therefore a small constant voltage

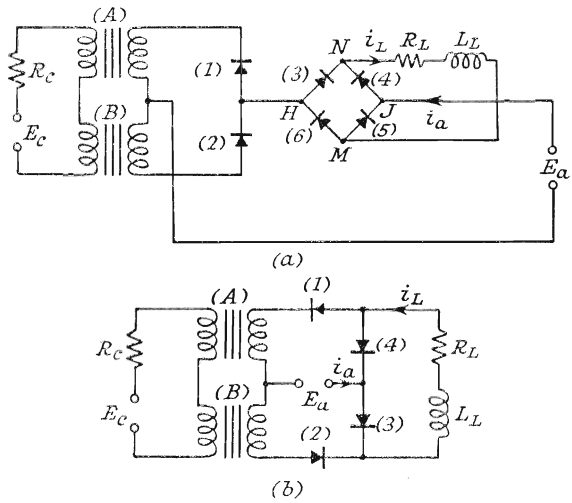


Figure 6.4. Auto-excited transductor circuits with d.c. inductive loads.
(*a*) Parallel connection (Group 1).
(*b*) Bridge connection (Group 2).

drop occurs in the resistance of the a.c. winding A as represented by the shaded area in *Figure* 6.5*a* and *c*. As the supply voltage decreases towards zero, it becomes equal to this voltage drop at a time t_Q, after which it is no longer able to maintain the saturation current in the external circuit. Since an applied voltage no longer exists across JH the rectifiers (3) and (5) are once again able to conduct and the load current is transferred back to the closed circuit through the rectifier. When this occurs the current in the a.c. winding A has become very small, and since the voltage has changed sign normal operation proceeds whereby the flux in core A decreases while that in core B increases. When core B saturates the same sequence of events occurs excepting that the constant load current is transferred from the rectifier circuit to the a.c. winding on core B.

When an inductive load is connected as in *Figure* 6.4*b*, the operation is similar to the arrangement in *Figure* 6.4*a*. During

non-conducting periods the constant load current flows round the circuit formed by the load rectifiers (3) and (4).

If a self-excited transductor is used with a d.c. inductive load, unsatisfactory operation results when the load is connected in

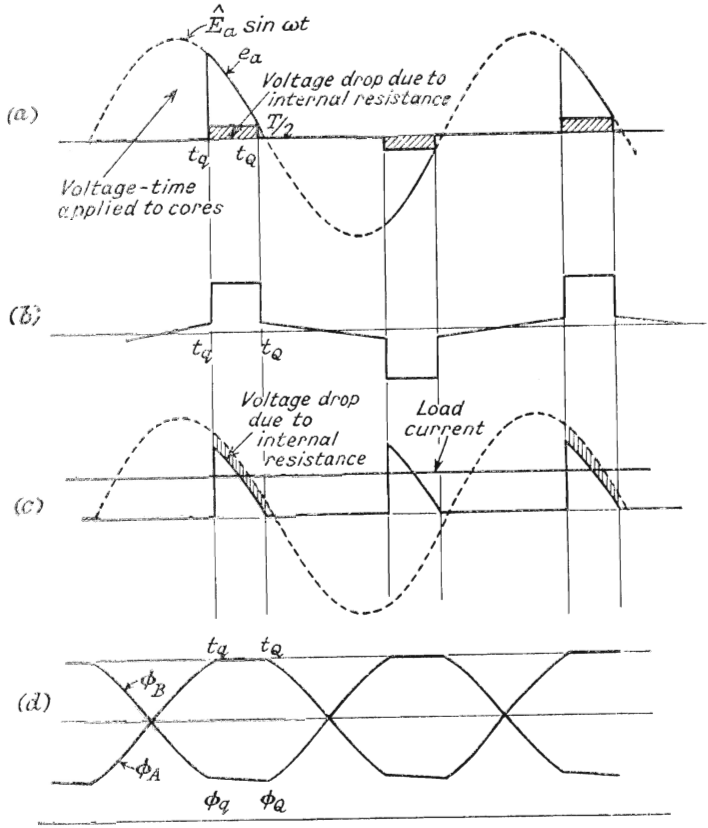


Figure 6.5. Voltage current and flux waveforms for transductor with d.c. inductive load.

(*a*) Voltage applied to a.c. terminals of rectifier and internal resistance (shaded area). (*b*) Current in a.c. circuit. (*c*) Total load voltage on d.c. terminals of rectifier and load current. (*d*) Core fluxes.

series with the self-exciting windings. This is because during non-conducting periods, the full load current flows through the self-exciting windings due to relaxation of the rectifier.

The effect can be minimized by placing a one-way rectifier element across the load. The rectifier provides a path for the load current during non-conducting periods. The point is compara-

tively unimportant, however, due to the fact that this type of connection is seldom used.

The curves in *Figure* 6.6 show the effect of inductive smoothing

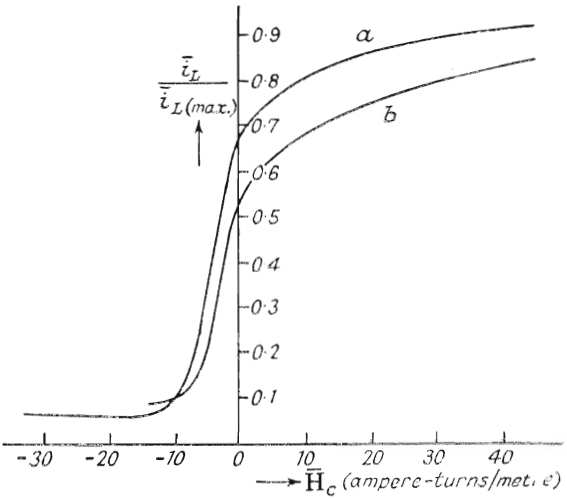


***Figure* 6.6.** Curves showing the effect of inductive smoothing on control characteristics for auto-excited transductor with mumetal cores and operating at 400 c/s.

(*a*) Zero inductance. (*b*) $\frac{L_L}{R_L} = 0{\cdot}1$ second.

on the generalized characteristic of an auto-excited transductor using mumetal cores and operating at 400 c/s. Curve (*a*) applies to a purely resistive d.c. load while curve (*b*) is for the same resistance but with an additional inductance, giving the load a time-constant equivalent to approximately 40 cycles of the supply frequency. As would be expected in this case, the inductance, having a high impedance to pulses, is not a suitable impedance match for the transductor and no improvement in sensitivity is

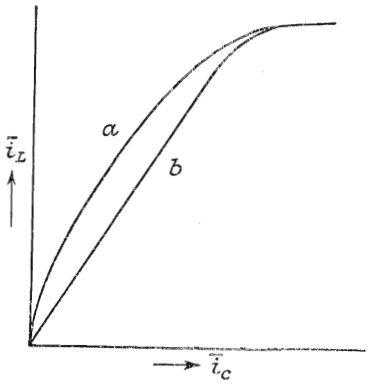


Figure 6.7. Control characteristics for unself-excited transductor, showing effect of inductive smoothing.

(*a*) Control characteristic for inductive load.

(*b*) Control characteristic for resistive load.

obtained as shown by the theoretical considerations in section 6.3.

A practical characteristic obtained without self-excitation is shown in *Figure* 6.7*a* for an inductive load, and *b* for a resistive load. When the output is small, the gain for an inductive load is large, as discussed in section 6.3, but as the output increases the slope of the curve decreases to the same order as that of the resistive load.

Transductors with D.C. Capacitive Load

6.5 Auto-Excited Transductor with Capacitor Smoothing of Load Current

When a smoothing capacitor is placed across a rectifier-fed d.c. load it tends to maintain a constant voltage across the load. The way in which this affects transductor operation will now be considered.

The waveforms of current and voltage applicable to a transductor with capacitor smoothed load current have been considered by Milnes,[6(a)] and his results are summarized in this section together with a description of the effects of smoothing upon the current amplification.

Turning to the auto-excited circuit in *Figure* 6.8, the load capacitor C_L is assumed to be very large. The total effective

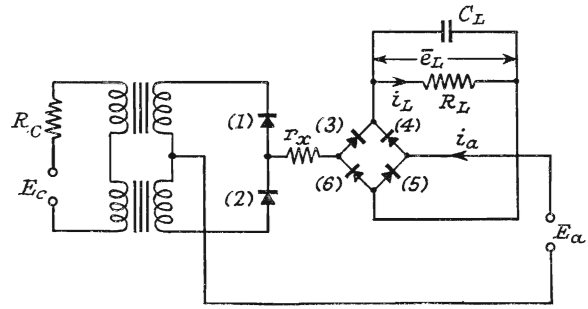


Figure 6.8. Auto-excited transductor with capacitive smoothing of load current.

internal resistance of the transductor has been lumped in r_x, which as before includes the resistance of one winding, the corresponding auto-excitation rectifier and two elements of the load rectifier. There are three points which are relevant to the operation of this circuit.

Firstly the capacitor charges up to an almost constant voltage $\bar{e}_L$, which acts in series with the sinusoidal supply voltage E_a. The

sign of $\bar{e}_L$, however, changes in relation to the a.c. circuit, depending upon which pair of elements in the load rectifier are conducting. Conduction of rectifier elements is dependent upon the current flowing in them (section 4.2), and when, for example, the current in the a.c. circuit changes from positive to negative, elements (4) and (6) stop conducting while (3) and (5) start conducting ; when this happens the direction in which the load voltage acts in the a.c. circuit changes sign also.

Secondly, as in the case of the transductor with resistive load, a core saturates when the total flux change has increased from the minimum value ϕ_Q to the saturation value. This will depend, as before, on the applied voltage-time integral, but in this case the applied voltage is modified by the existence of a constant voltage across the load.

Thirdly, the core desaturates when the total e.m.f. in the a.c. circuit has decreased to such a value that it can no longer maintain the saturation current through the internal resistance r_x. During conduction the a.c. circuit e.m.f. is the difference between the supply voltage and the constant load voltage and is given by the shaded area and the dark area in *Figure* 6.9*a*.

Referring to *Figure* 6.9*a*, the dotted line represents the sinusoidal supply voltage and the full line represents the voltage across the a.c. terminals of the load rectifier bridge. The latter voltage changes sign at times denoted by t', $T/2 + t'$, etc., when the current in the a.c. circuit (*Figure* 6.9*b*) changes sign, thereby causing commutation of the rectifier elements.

During non-conducting periods the load voltage does not remain truly constant due to discharge from the capacitor through the load which reduces slightly the voltage across the plates of the capacitor. The shaded area in *Figure* 6.9*a* up to the time t_q represents the voltage-time applied to the transductor a.c. windings; this voltage-time is causing the flux in core A to increase and the flux in core B to decrease. Core A saturates at t_q and the dark area corresponds to the circuit e.m.f. available to drive current through the internal resistance of the transductor. This current, shown in *Figure* 6.9*b*, flows through the auto-excitation rectifier (1) and the corresponding a.c. coil ; it also recharges the load capacitor via the rectifier elements (4) and (6). During the conducting period the coil current is proportional to the resultant applied voltage, corresponding to the dark area and therefore at a time t_Q it is no longer sufficient to maintain the core in the

saturated state; at this stage core A desaturates and the small alternating current now decreases out of phase with the supply voltage, which changes sign at $T/2$ while the magnetizing current is still positive (*Figure* 6.9*b*). Over this period the action of the auto-excitation circuit is precisely the same as that described in section 5.1; both rectifier elements (1) and (2) are conducting,

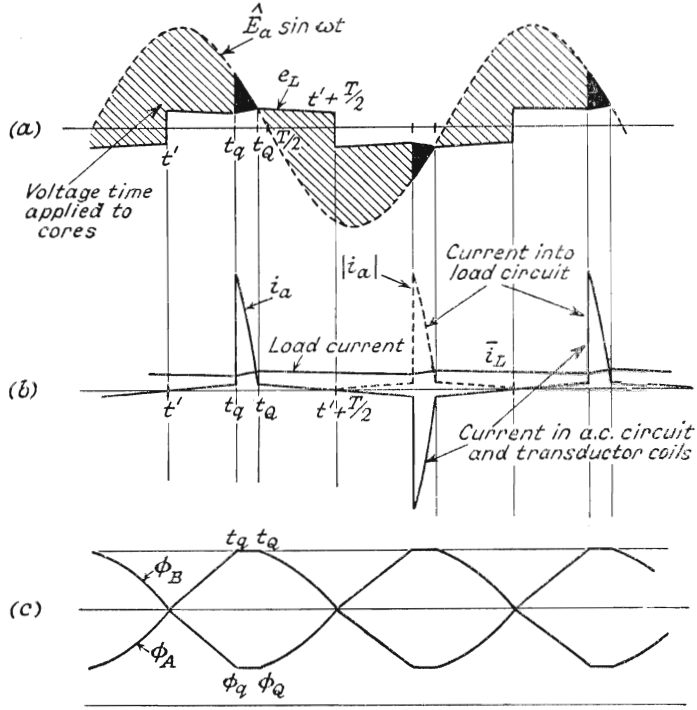


Figure 6.9. Waveforms of voltages, currents and fluxes for auto-excited transductor with capacitive smoothing.

and the flux in core A is decreasing while that in core B is increasing from the minimum value ϕ_Q. These flux changes are proportional to the shaded area which starts at time t_Q. At times t', $(t' + T/2)$, etc., the small magnetizing current in the a.c. winding changes sign, the load rectifier elements therefore commutate and the direct load voltage changes its direction of action in the a.c. circuit. This switching of the load voltage causes a sudden change in the magnitude of the voltage applied to the coils and consequently a discontinuity in the flux waveforms appears at these times as shown in *Figure* 6.9*c*.

It has been seen that the reverse voltage existing across the auto-excitation rectifier is equal to the voltage drop in the a.c. winding during the conducting period. When the load is resistive, the load current is the same as that flowing in the a.c. windings and therefore the reverse voltage is the voltage drop due to the load current flowing in the winding resistance. When capacitor smoothing is employed, however, although the mean load current is equal to the mean coil current, the latter only flows over a limited period while the former is nearly constant throughout the cycle. The coil current will therefore have a much greater peak value than the load current as shown in *Figure* 6.9*b* and it follows that both the peak reverse voltage on the auto-excitation rectifier and the r.m.s. current in the a.c. windings will be larger than that calculated from the load current. It is important to bear this in mind when determining the heating in the transductor and the current rating of the rectifiers.

The smoothing capacitor has important and beneficial effects on the control characteristics. These effects give rise to increased range of operation and greater current amplification, particularly at low signal levels. There is also a tendency in certain circumstances to improve the speed of response (section 8.12).

6.6 Increased Amplification Due to Capacitor Smoothing

The way in which a parallel capacitor affects the current amplification of a self-excited transductor will now be considered.

During conducting periods a large pulse of current is drawn from the supply through the transductor, part of this flows in the load, but most of the current flows into the capacitor which becomes charged. During the unsaturated period some of the charge in the capacitor flows into the load. Provided the charge on the capacitor is the same at the beginning of successive conducting periods, i.e. the system is in a steady-state, it must follow, that the total charge contained in the current pulse during the conducting period must be equal to the total charge which has passed through the resistor during the half-cycle, and, therefore, the mean value of the current pulse over one half-cycle is equal to the mean load current over the half-cycle, i.e.

$$\bar{i}_a = \bar{i}_L \qquad . \quad . \quad . \quad (6.16)$$

It is shown in Appendix 6, by similar methods to those used in

section 6.2, that where a parallel capacitor is used and square wave excitation is employed

$$\frac{\bar{e}_a}{\bar{i}_a} = \frac{\bar{e}_a}{\bar{i}_L} = r_x\left(1 + (2\beta/T)\,\frac{R_L}{r_x}\right)$$

i.e. $$\bar{R}_a = \bar{R}_{aL} = r_x\left(1 + (2\beta/T)\,\frac{R_L}{r_x}\right) \quad . \quad . \quad . \quad (6.17)$$

The mean resistance $\bar{R}_{aL}$ is dependent upon the duration of the conducting period and varies from the value r_x when $\beta = 0$ to the maximum value $R_L + r_x$ when $\beta = T/2$. In general this represents a greater variation than that obtained with an inductive load (equation 6.11), since the minimum value r_x corresponds only to the rectifier and winding resistances.

When no self-excitation is employed ampere-turn equality must exist between the mean control current and the mean transductor current $\bar{i}_a$. The latter is equal to the mean load current $\bar{i}_L$ (equation 6.16) and therefore the current amplification is not affected by a capacitor in parallel with the load, and is given by :

$$G_i = \frac{N_c}{N_a}$$

If the transductor is auto-excited or completely self-excited, then assuming that for small outputs the voltage amplification applicable to resistive loads is valid when a smoothing capacitor is used and is given by equation 4.16 thus :

$$G_v = g_v = \frac{2}{\pi}\,\frac{\omega M_{ca}}{R_c}$$

the mean current is then given by

$$\bar{i}_L = g_v\,\frac{R_c}{\bar{R}_{aL}} \cdot \bar{i}_c = \frac{2}{\pi}\,\frac{\omega M_{ca}}{r_x\left(1 + (2\beta/T)\,\dfrac{R_L}{r_x}\right)}\,\bar{i}_c \quad . \quad . \quad (6.18)$$

This leads to the important result that capacitor smoothing causes a marked increase in the current amplification for small values of β when complete self-excitation is used.

With a resistive load it has been seen that the current gain is inversely proportional to the total circuit resistance R_a. When the d.c. load consists of a shunt capacitor, however, the gain at low output levels tends to be inversely proportional to the internal resistance r_x which should be considerably less than R_a.

This result can also be considered on the basis of impedance matching. When energy is delivered to the load circuit over a short period the energy which can be dissipated in a pure resistance load is limited by the magnitude of the resistance. The capacitor, however, presents a low impedance to the output voltage pulses e_a and therefore when complete self-excitation is used it is a favourable impedance match to the transductor. It therefore stores a larger quantity of energy than could be dissipated in the load over short conducting periods. This energy is then transferred to the load during the longer non-conducting period. As would be expected, if no self-excitation is used the transductor output impedance is much higher and the matching between the transductor and the capacitor circuit is not favourable to a large energy transfer and therefore no marked increase in the amplification is obtained under these conditions.

The use of a load capacitor to obtain improved characteristics is quite common in transductor manufacture and corresponds to pulse lengthening. It is also found that when a capacitor is placed across an inductive load an increased current gain is generally obtainable. The required value of capacitance is usually best found experimentally, but in general the time constant $R_L C_L$ requires to be somewhat larger than the period of one half-cycle.

When the supply voltage is sinusoidal not only is an increased amplification observed but also an increased maximum output, and since this is a feature which is often utilized, the effects of capacitor smoothing will be considered for the case of a sine wave of supply voltage.

When the conducting period is very short, then in normal circumstances, without smoothing, the form factor of the load current is low and there is adequate scope for pulse lengthening as described for the square wave. In the case of a rectified sine wave of load current which applies when conduction occurs throughout the cycle, a small improvement in output is still obtainable by means of capacitor smoothing. The improvement obtained by smoothing in these circumstances is considered in Appendix 7 and the factor by which the maximum load current is increased is found to depend upon the ratio r_a/R_L as for the square wave. This factor is equal to $\pi/2$ when $r_a \rightarrow 0$ and unity when $r_a \rightarrow \infty$. The results are plotted in *Figure* 6.10*a*. It is not usually practicable to make r_a/R_L less than about 0·1 and this

imposes an upper limit of about 1·3 on the factor by which the maximum output is increased due to smoothing. This slight improvement applies also to the unself-excited transductor since at maximum output the internal impedance must be low since ampere-turn balance no longer exists and the supply voltage is virtually connected to the load rectifier.

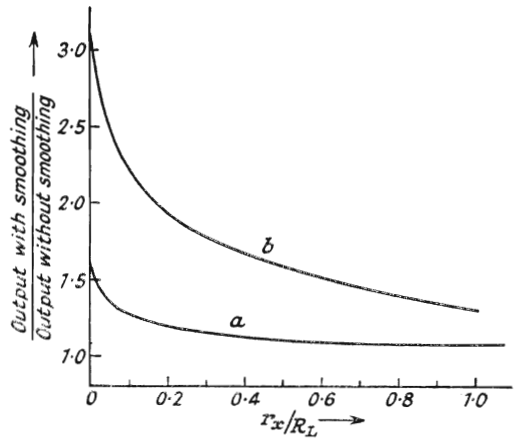


Figure 6.10. Curves showing effect of a smoothing capacitor on the output of an auto-excited transductor for two different control signals and sinusoidal excitation.

(*a*) 100% conduction ($\beta = T/2$).
(*b*) 50% conduction ($\beta = T/4$).

The maximum output of an auto-excited transductor occurs well beyond the limit of linearity, however, and a more realistic view is obtained by considering the improvement at some specific point lower down on the control characteristic. This is also calculated in Appendix 7 for the case where conduction occurs over one-quarter of a cycle, corresponding to a saturation time $t_q = T/4$. Under these conditions, the transductor with resistive load and no smoothing is delivering one-half its maximum output. From $t = 0$ to $t = T/4$ the supply voltage is supported by the changing fluxes in cores A and B, and from $T/4$ to $T/2$ the supply voltage is developed across the external circuit including r_x.

When capacitor smoothing is introduced a more marked improvement is noticed than that applying to 100% conduction ; this is to be expected as a result of the shorter conducting period.

The increased output due to smoothing is shown for the case of 50% conduction in curve (*b*) *Figure* 6.10.

For a ratio of $r_x/R_L = 0{\cdot}1$, the effect of smoothing is to increase the output by a factor well over two, thereby resulting in a corresponding increase in the mean amplification over this range.

When the conducting period is very short compared with the duration of a half-cycle, the square wave analysis is approximately applicable and the initial current amplification is limited only by the transductor resistance. Thus in the ideal case where the amplification without smoothing is given by

$$G_i = \frac{2}{\pi} \frac{\omega M_{ca}}{R_L + r_x},$$

when a large capacitor is introduced in parallel with the load the current gain at low output levels is given approximately by

$$G_i = \frac{2}{\pi} \frac{\omega M_{ca}}{r_x}$$

(for an auto-excited circuit the factor $\frac{2}{\pi}$ is replaced by $\frac{1}{\pi}$).

These conditions arise again when dealing with even harmonic transductors (Chapter 11).

The curves plotted in *Figure* 6.11*a* and *b* show the effect of capacitor smoothing on the control characteristic of an auto-excited transductor for which the ratio of the external resistance to the load resistance is equal to 0·43. The smoothing increases the linear range by a factor of about 1·6 as predicted from *Figure* 6.10*b*.

In addition the amplification for small outputs shows a considerable increase.

When the d.c. load is inductive, a smoothing capacitor has a similar effect on the characteristics, this is shown in *Figures* 6.12*a* and *b*.

Although the effects of capacitor smoothing are of a complex nature there is no doubt that in general a valuable improvement in the characteristics is realized, particularly when the load is inductive. It must be remembered, however, that when the transductor is used in control systems the smoothing may introduce undesirable time delays which can lead to instability or hunting. Sometimes, however, it is found that the dynamic

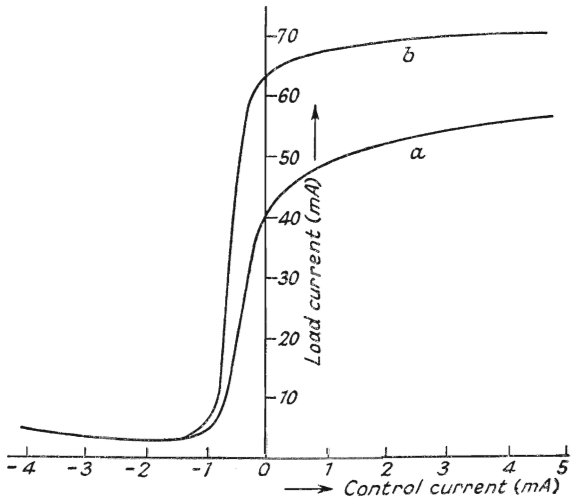


Figure 6.11. Control characteristic of auto-excited transductor showing effect of a large smoothing capacitor.

Curve (*a*) $R_L = 390\Omega$ $r_x = 170\Omega$ $C_L = 0$.
(*b*) $R_L = 390\Omega$ $r_x = 170\Omega$ $C_L = 8\mu F$.

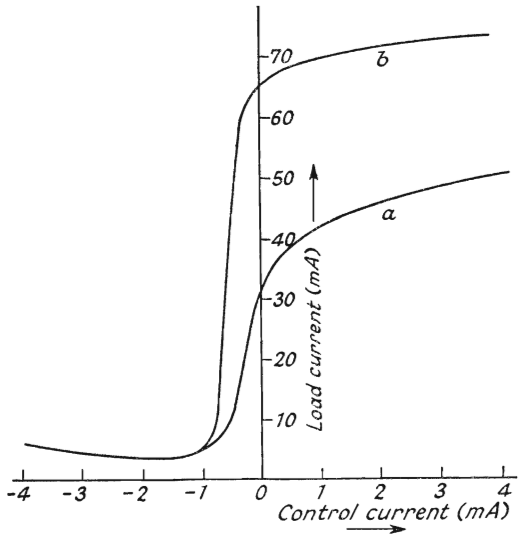


Figure 6.12. Control characteristics of auto-excited transductor showing the effect of a smoothing capacitor when the d.c. load is inductive.

(*a*) $R_L = 390\Omega$ $r_x = 170\Omega$ $L_L = 2{\cdot}3$ H $C_L = 0$.
(*b*) $R_L = 390\Omega$ $r_x = 170\Omega$ $L_L = 2{\cdot}3$ H $C_L = 8\mu F$.

characteristics are actually improved by the presence of a capacitor across the d.c. load. This tends to apply in particular to cases where the transductor time-constant is normally large and the self-excitation is incomplete—it is considered in Chapter 8.

6.7 Summary

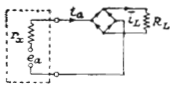
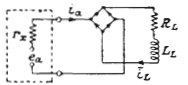
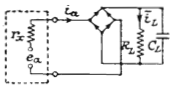
The presence of capacitance or inductance in a transductor load tends to modify the characteristics. In most cases this amounts to a slight reduction of current gain and a deterioration of the linearity. When a capacitor is placed in parallel with a d.c. load, however, considerable improvement is realized if the transductor is auto-excited or fully self-excited. The range of the transductor can increase by a factor between one and two and the amplification is considerably increased at low output levels.

These modifications depend upon the fact that the ratio of mean output voltage to the mean transductor current or the mean load current are no longer equal to the a.c. circuit resistance R_a but depend upon " mean resistances " which have been defined as $\bar{R}_a$ and $\bar{R}_{aL}$ respectively.

These mean resistances are dependent upon the output level and hence cause a slight non-linearity in the amplification characteristic. The extent of this is usually masked, however, by the greater non-linearity already present due to the curvature of the core characteristic.

The performance of transductors with various types of reactive loading is summarized in the table on page 178.

Summary of steady-state performance of transductors with various types of load

	(1) D.C. Resistive.	(2) D.C. Inductive.	(3) D.C. Capacitive.
Circuit			
Value of $\bar{R}_a = \bar{e}_a/\bar{i}_a$. (Assuming square wave excitation.)	$\bar{R}_a = R_a = (R_L + r_x) = R_L\left(1 + \frac{r_x}{R_L}\right)$	$\bar{R}_a = \frac{1}{(2\beta/T)} \cdot \left[R_L\left(1 + \frac{2\beta}{T}\frac{r_x}{R_L}\right)\right]$	$\bar{R}_a = \left[r_x\left(1 + \frac{2\beta}{T}\frac{R_L}{r_x}\right)\right]$
Value of $\bar{R}_{aL} = \bar{e}_a/\bar{i}_L$. Assuming square wave excitation.	$\bar{R}_{aL} = R_a = (R_L + r_x) = R_L\left(1 + \frac{r_x}{R_L}\right)$	$\bar{R}_{aL} = \left[R_L\left(1 + \frac{2\beta}{T}\frac{r_x}{R_L}\right)\right]$	$\bar{R}_{aL} = \left[r_x\left(1 + \frac{2\beta}{T}\frac{R_L}{r_x}\right)\right]$
Steady-state control characteristic for unself-excited transductor.	Linear current gain approximately equal to N_c/N_a.	Maximum output approximately the same as for resistive load. High gain at very low signal levels given by $\frac{N_c}{N_a} \cdot \left(\frac{1}{2\beta/T}\right)$ non-linear characteristic.	Increased maximum output but gain approximately the same as for resistive load.
Steady-state control characteristic for fully self-excited or auto-excited transductor.	Linearity determined by magnetization curve and gain approximately equal to : $\frac{k\omega M_{ca}}{(R_L + r_x)}$.	Approximately the same characteristic as for resistive load but slight deterioration of linearity.	Considerably increased gain at low signal levels where gain is approximately given by $\frac{k\omega M_{ca}}{r_x}$. Increased range of operation over that obtainable with resistive load.

CHAPTER 7

CLOSED LOOP CONTROL SYSTEMS

In many cases transductors are used as amplifiers in automatic control or servo-systems. In these circumstances the design of a suitable transductor to have a given steady-state and transient characteristic is largely determined by the nature of other time delays already present in the system.

Thus, for example, if two elements in a system each introduce a simple time delay then, under certain circumstances, the response of the system can be underdamped. The further addition of an integrating element such as a motor can cause maintained oscillating or hunting.

The design of transductors is therefore closely linked with some of the problems facing the servo engineer. The transient characteristics of transductors will be discussed in Chapter 8 and the response under various conditions will, where possible, be specified in terms of the notations commonly used in servo design. In addition closed loop theory is a convenient aid to understanding some of the more complex transient phenomena which arise in the case of transductors feeding reactive loads. For these reasons a brief discussion of the elementary aspects of servo theory is presented in this chapter. The subject is very extensive, however, and for anything more than the elementary ideas the reader is referred to more comprehensive works.[7(a, b)]

The chief problem confronting the servo engineer is to design a system having sufficient amplification to give adequate control, but at the same time sufficient damping to give overall stability of the system and reasonable response.

7.1 Requirements of Control Systems

A block diagram of a simple control system is shown in

Figures 7.1*a* and *b* where the object of the control is to maintain the output at a constant value determined only by the input. The output may be a shaft position as in a position controller, the temperature of an oven as in a temperature controller, or a voltage as in the case of a voltage regulator or negative feedback amplifier, etc.

A simple approach to the problem of controlling the output would be to actuate the controller by applying a constant input and thereby maintain the output at a constant value (*Figure* 7.1*a*). This, however, relies on the stability and linearity of the controller and the absence of external disturbances in the system. A more accurate method of control is achieved if the controller is

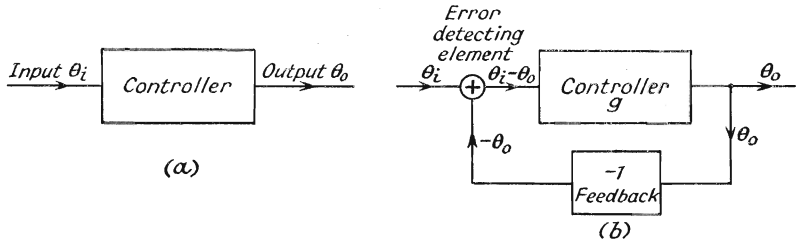


Figure 7.1. Block diagrams of simple control systems. (*a*) Open loop. (*b*) Closed loop.

actuated by the difference between the actual output and the required output (*Figure* 7.1*b*). This involves feeding back the output and comparing it with the input. The difference between the input and output is then amplified in the controller and used to maintain the output at the required level.

Thus a control system has three essential elements :—

(1) An error-detecting element which measures the difference between the required output and the actual output.

(2) A controlling element which provides amplification.

(3) A feedback network which measures the output and feeds it back to the error-detecting element.

If the required output is θ_i, the actual output is θ_0 and the gain of the controller is g then

$$g(\theta_i - \theta_0) = \theta_0$$

so that

$$\theta_0 = \frac{g}{1+g}\theta_i \qquad . \quad . \quad . \quad (7.1)$$

It therefore follows that if $g \gg 1$ the output will always be very nearly equal to the input.

The controller usually consists of a number of stages of amplification which will include the output stage comprising, for example, a motor if the control is a position servo, or a heater if the control is thermostatic. In all cases there are generally a number of time lags in the system and if the frequency response characteristics of the controller are examined with the loop open it will be found that as a result of the time lags both the gain and the phase shift between input and output vary as the input frequency is increased. On closing the loop, the output should be subtracted from the input, but if, at a certain frequency, the phase shift in the controller is 180° then the feedback will no longer oppose the input, but will become positive and assist it, so that unless the total gain around the loop is less than unity *at this frequency* the energy fed back will be sufficient and correctly phased to cause the build-up of self-maintained oscillations around the loop.

The chief problems are therefore to design the system :

(*a*) To give accurate control, i.e. to make the loop gain as large as possible.

(*b*) To be stable.

These two requirements are conflicting, however, because if the loop gain is made large at low frequencies it is unlikely to be sufficiently attenuated at the frequency for which the phase shift is 180°. This will result in oscillations at the latter frequency being superimposed on the output whatever the frequency of the input. These oscillations tend to build up until their amplitude is limited by the saturation of some element.

In practice the system must not only be stable but adequately damped, and this usually involves the need for an adequate margin over the critical conditions for stability.

There are a number of useful criteria for stability (see references), and in general it is possible to improve the response by the provision of suitable networks and/or feeding suitable time derivatives of the output back to the input, thereby modifying the open-loop frequency response characteristics of the system.

Nevertheless, before the frequency response of the system can be estimated and suitable stabilizing means employed, the response of all elements within the system must be known.

7.2 Frequency Response of Servo Elements

The following discussion is related to linear elements and we

will start by considering an element for which the input e and the output V_0 are related by an equation of the form

$$\tau \frac{dV_0}{dt} + V_0 = ge \qquad . \quad . \quad . \quad (7.2)$$

The input e is some function of time and τ is a constant. Equation 7.2 specifies the behaviour of many physical systems. For example, if a voltage e is applied to a circuit consisting of a capacitor C in series with a resistor R, the voltage V_0 across the capacitor varies with time in a manner defined by the above equation in which $g = 1$ and $\tau = RC$. Similarly, if a mass m is moving in a viscous fluid under the action of an impressed force e the velocity V_0 of the mass will be defined by the equation

$$m \frac{dV_0}{dt} + KV_0 = e$$

where $\tau = \frac{m}{K}$, $g = \frac{1}{K}$ and $K =$ viscous resistance per unit velocity.

If the symbol D replaces d/dt in equation 7.2, then the equation can be written in the operational form

$$V_0 = \frac{g}{(1 + \tau D)} e \qquad . \quad . \quad . \quad (7.3)$$

where $\frac{g}{1 + \tau D}$ is termed the transfer function and specifies the relation between the input e and the output V_0, both in the transient and steady-states.

The input is usually some known function of time of which the most commonly considered are the sinusoidal function and the step function. In the latter case, if an input $e = E$ is suddenly applied at time $t = 0$, we have already seen that the output follows an exponential law governed by a time-constant τ and has a steady-state value of $V_0 = gE$.

Thus $$V_0 = gE\,(1 - e^{-t/\tau}) \qquad . \quad . \quad . \quad (7.4)$$

This is plotted as a curve of V_0 against t in *Figure* 7.2*a*.

If the input e is a sinusoidal function given by $e = \hat{E} \sin \omega t$, then the solution of equation 7.2 after initial transients have died away is

$$V_0 = \frac{g\hat{E}}{\sqrt{1 + \omega^2\tau^2}} \sin (\omega t - \psi) \qquad . \quad . \quad . \quad (7.5a)$$

where $\psi = \tan^{-1} \omega\tau \qquad . \quad . \quad . \quad (7.5b)$

Equations 7.4 and 7.5 both apply to the same system operating under different conditions. The former equation specifies the actual response of the device to a constant input which is applied suddenly, while the latter gives information about the amplitude

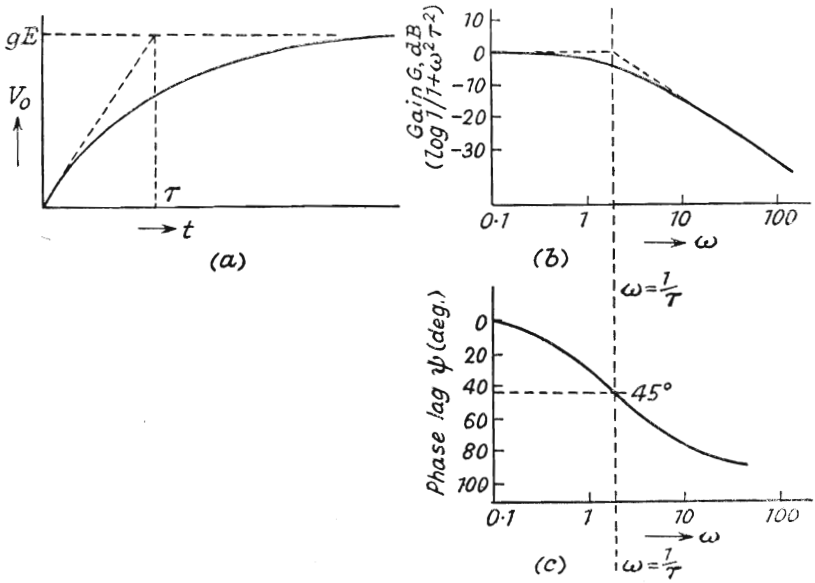


Figure 7.2. (*a*) Transient response due to a simple time delay. (*b*) and (*c*) Frequency response due to a simple time delay.

and phase shift ψ of the output in the steady-state for various values of the signal frequency.

If the gain of the element is given by $\frac{\hat{V}_0}{\hat{E}}$ then

$$\text{gain, } G = \frac{g}{\sqrt{1 + \omega^2\tau^2}} \qquad . \quad . \quad . \quad (7.6)$$

This quantity varies from g to zero as the signal frequency increases from zero to infinity. Since g is the gain at zero frequency it refers to the d.c. conditions.

The gain is conveniently plotted in decibels * against angular frequency plotted to a logarithmic scale. The constant and variable components of the gain may be separated thus :

$$\log G = \log \frac{1}{\sqrt{1 + \omega^2\tau^2}} + \log g.$$

* Gain measured in dBs $= 20 \log_{10}$ (gain ratio).

The frequency dependent term can then be plotted independently of the steady-state gain which may be added by shifting the curve up or down by the number of dBs corresponding to g. The frequency response applicable to this particular transfer function is shown in *Figure* 7.2*b*, where the steady-state component is omitted ; it is seen that a close approximation to the actual response is given by two straight lines (shown dotted) ; a horizontal line signifying a constant gain of unity (zero dB) up to a frequency $\omega = 1/\tau$, and thereafter a line which defines a uniform decrease of gain with frequency at 6 dBs per octave.

Simultaneously the phase angle between the input and the output will decrease from zero to $-90°$ over the same frequency range, the phase angle being given by *Figure* 7.2*c* and equation 7.5*b* ; the phase lag is equal to 45° when $\omega = \frac{1}{\tau}$.

Thus the element represented by a relation of the form given in equation 7.2 may be defined either by the time-constant τ or by the gain/frequency and phase/frequency relationships ; in either case the system is completely specified by the transfer function $\frac{g}{(1 + \tau D)}$.

Other commonly met systems are those governed by an equation of the form :

$$\frac{1}{\omega_0^2}\frac{d^2V_0}{dt^2} + \frac{2\zeta}{\omega_0}\frac{dV_0}{dt} + V_0 = ge \quad . \quad . \quad . \quad (7.7)$$

This equation applies to devices containing two energy storage elements and viscous damping.

The transfer function for the second order equation may be written down as before and by putting D for d/dt we have

$$V_0 = \frac{ge}{\frac{D^2}{\omega_0^2} + \frac{2\zeta}{\omega_0}D + 1} \quad . \quad . \quad . \quad (7.8)$$

The output V_0 is given by the complete solution of equation 7.7. There are three possible forms for this solution, however, depending upon the value of ζ.

Considering the case of a constant input E applied at time $t = 0$, the following conditions are possible :

(1) $\zeta < 1$. Under these conditions the output is underdamped with the result that it consists of an oscillatory transient which

ultimately dies away, leaving the steady-state condition $V_0 = gE$.

The solution of equation 7.7 for $\zeta < 1$ is

$$V_0 = gE\left[1 - \frac{e^{-\zeta\omega_0 t}}{\sqrt{1-\zeta^2}} \cdot \sin\left(\omega_0\sqrt{1-\zeta^2} \cdot t + \psi'\right)\right] \quad . \quad (7.9)$$

and the corresponding response is shown as a function of non-dimensionalized time $\omega_0 t$ for various values of ζ (*Figure* 7.3*a*).

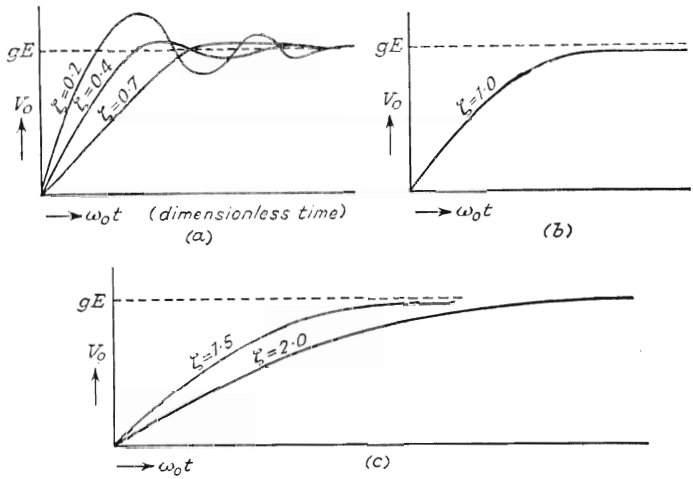


Figure 7.3. Transient responses of a system containing two time delay elements. (*a*) Lightly damped $\zeta < 1$. (*b*) Critically damped $\zeta = 1$. (*c*) Overdamped $\zeta > 1$.

(2) $\zeta = 1$. This corresponds to the critically damped condition which is plotted in *Figure* 7.3*b*.

(3) $\zeta > 1$. This case corresponds to a large amount of damping and it is possible to factorize the quadratic in D so that equation 7.8 may be written in the form

$$V_0 = \frac{ge}{(1+\tau_1 D)(1+\tau_2 D)}$$

$$= \frac{g}{(\tau_1 - \tau_2)}\left[\frac{\tau_1}{(1+\tau_1 D)} - \frac{\tau_2}{(1+\tau_2 D)}\right]E.$$

By comparing this with 7.3 it is seen that the solution may be represented as the sum of two exponentials, typical transient curves of which are reproduced in *Figure* 7.3*c*.

If ζ is negative, the system is unstable and self-maintained oscillations build up until limited by saturation of the amplifier.

The present discussion is confined to positive values of the damping factor.

The transfer function of equation 7.8 can no longer be defined by a simple time-constant τ, as could that of equation (7.3), but by the two constants ω_0 and ζ. The constant ω_0 is the natural angular frequency of oscillation of the undamped system and is known as the undamped natural angular frequency. The constant ζ is the damping factor.

In general, the slightly underdamped condition is of most interest due to the fact that this tends to give the most desirable response; typical practical values for the damping factor lie between 0·5 and 0·8 (*Figure* 7.3*a*).

When the input is sinusoidal, the steady-state solution is

$$V_0 = \frac{g\hat{E}\sin(\omega t - \psi)}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + 4\frac{\zeta^2\omega^2}{\omega_0^2}}} \quad . \quad . \quad . \quad (7.10a)$$

where

$$\psi = \tan^{-1}\frac{2\zeta\frac{\omega}{\omega_0}}{1 - \frac{\omega^2}{\omega_0^2}} \quad . \quad . \quad . \quad (7.10b)$$

In this case the frequency response can be plotted as for the first order system. This is done in *Figure* 7.4*a* and it is seen that the gain which is unity at low frequencies increases to a maximum when the signal frequency is equal to the damped natural frequency. At this stage the amplitude is limited only by the damping term. As the signal frequency increases still further the amplitude quickly decreases, ultimately at 12 dB per octave. Over the same frequency range the phase lag of the output behind the input varies from 0 to 180°. The phase angle is equal to $-90°$ at a signal frequency equal to the undamped natural frequency (*Figure* 7.4*b*).

Finally we shall consider integrating and differentiating elements.

An integrating element is one in which the output at any instant is proportional to the time integral of the input, thus

$$V_0 = \frac{1}{\tau}\int_0^t e\,dt,$$

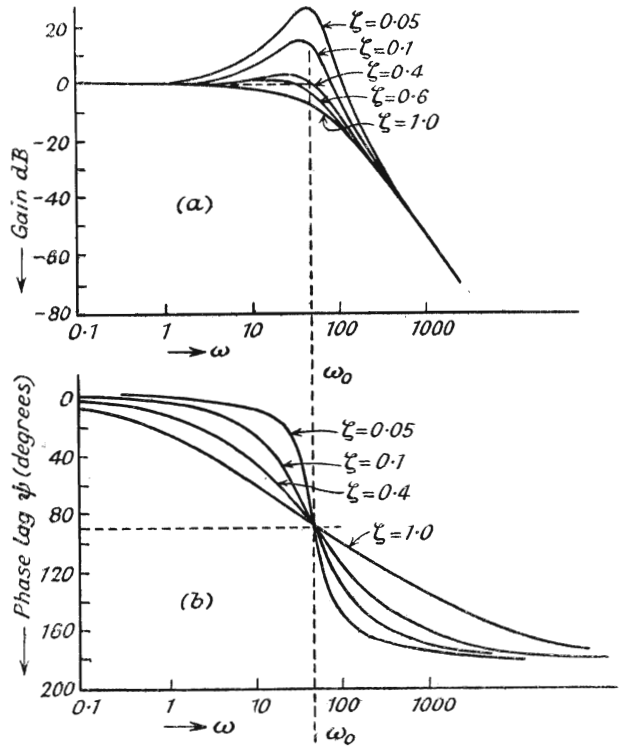


***Figure* 7.4.** Frequency response of a system with two time delays and various degrees of damping.
(*a*) Amplitude ratio against frequency.
(*b*) Phase angle against frequency.

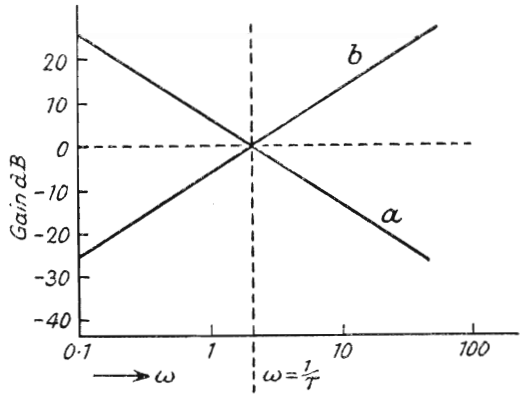


***Figure* 7.5.** (*a*) Frequency response of integrating element.
(*b*) Frequency response of differentiating element.

the transfer function being given by

$$\frac{V_0}{e} = \frac{1}{\tau D}$$

and

$$V_0 = \frac{e}{\tau D} \quad . \quad . \quad . \quad (7.11)$$

If the applied input e is equal to $\hat{E} \sin \omega t$ then the solution is

$$V_0 = \frac{\hat{E}}{\omega\tau} \sin (\omega t - \pi/2) \quad . \quad . \quad . \quad (7.12)$$

Variation of the gain $\frac{1}{\omega\tau}$ with signal frequency is plotted in *Figure* 7.5*a*. This is very large at low frequencies and becomes equal to unity (0 dB) at an angular frequency $\omega = 1/\tau$; the logarithmic gain decreases uniformly at 6 dBs per octave and the phase shift corresponds to a constant lag of 90° at all frequencies.

A differentiating element gives an output proportional to the time derivative of the input, thus

$$V_0 = \tau \frac{de}{dt}$$

so that

$$V_0 = \tau De \quad . \quad . \quad . \quad (7.13)$$

In this case the solution to a sinusoidal input $\hat{E} \sin \omega t$ is

$$V_0 = E\omega\tau \, (\sin \omega t + \pi/2) \quad . \quad . \quad . \quad (7.14)$$

and the resulting gain/frequency response diagram is shown in *Figure* 7.5*b*. The gain increases at 6 dBs per octave and the phase angle between input and output is constant at 90° leading.

7.3 Block Diagrams and Overall Frequency Response

A complete system of elements may be represented in the form of a block diagram where the transfer function of each element is assumed to be a function of D of the form $g_n y_n(D)$ where g_n is the steady-state gain and $y_n(D)$ defines the frequency dependent part of the transfer function as described in section 7.2.

Provided the input of one element does not appreciably load the output of the preceding element, then the overall response may be obtained as was done for the simple control system in *Figure* 7.1*b* by multiplying the transfer functions. Thus we

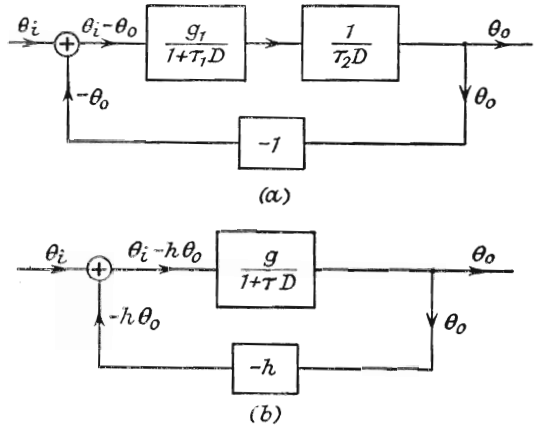


Figure 7.6. Closed loop control systems.
(*a*) Simple delay and integrator.
(*b*) Simple delay with negative feedback factor < 1.

will consider the system in *Figure* 7.6*a*, in which the controller contains a first order element having a transfer function $\frac{g_1}{1+\tau_1 D}$ followed by an integrating element, $\frac{1}{\tau_2 D}$. If the feedback element has a constant transfer function equal to unity at all frequencies, we have

$$\theta_0 = \frac{g_1}{\tau_2 D(1+\tau_1 D)}(\theta_i - \theta_0)$$

thus

$$\theta_0 = \frac{1}{\frac{(\tau_1\tau_2)}{g_1}D^2 + \frac{\tau_2}{g_1}D + 1}\theta_i.$$

By inspection it is seen that the undamped natural frequency and the damping factor are given by equating coefficients from equation 7.8.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g_1}{\tau_1\tau_2}}$$

and

$$\frac{2\zeta}{\omega_0} = \frac{\tau_2}{g_1}, \text{ i.e. } \zeta = \frac{1}{2}\sqrt{\frac{\tau_2}{g_1\tau_1}}.$$

Therefore, the natural frequency increases and the damping decreases with increasing gain g_1. The system is always stable since the damping is never negative and the maximum possible

phase lag is 180° which occurs at infinite frequency. The system could be underdamped, however, and would then exhibit oscillatory tendencies when subjected to sudden disturbances at the input.

Alternatively, let us consider the system in 7.6*b* for which the open loop transfer function is that of an element having a simple exponential time lag, i.e. $\frac{g}{1+\tau D}$.

If a proportion h of the output is fed back to the input ($h < 1$), then the output θ_0 is related to the input θ_i by the equation

$$\theta_0 = \frac{g}{1+\tau D}(\theta_i - h\theta_0) \quad . \quad . \quad . \quad (7.15)$$

if gh is $\gg 1$ this becomes

$$\frac{\theta_0}{\theta_i} = \frac{1/h}{1+\frac{\tau}{gh}D} \quad . \quad . \quad . \quad (7.16)$$

In the present case, therefore, the steady-state gain is reduced to the value $1/h$ and is therefore primarily dependent upon the feedback network while the time-constant has been reduced from τ to τ/gh.

These methods of specifying the elements in a closed loop system bring out an important consideration often overlooked in transductor design.

It is sometimes inferred that by connecting two amplifiers in cascade the response time for a given amplification is reduced ; this of course is perfectly true, but two time-constants in cascade correspond to a second order system with more than adequate damping. If these two amplifiers are included in a closed loop, however, then the damping is reduced as the loop gain increases and the response may become oscillatory.

On the other hand one time-constant, however large, can never cause instability when connected in a closed loop, and hence it may be that this alternative would be preferable in some cases. The problem must be carefully considered in relation to other time delays in the system. Thus if a short time-constant is inevitably present the two-stage amplifiers would cause hunting, while by using the single stage having a large time-constant the system would tend to be stable. If a large time-constant is inevitably

present, however, then the two-stage amplifiers would be preferable, since the two short delays could be regarded as negligible compared with the long delay already present. The former method of stabilizing a system results in a servo having a sluggish response, and therefore in high-speed servos it may be preferable to shape the open loop frequency response characteristics such that stability is achieved without losing speed of response.

CHAPTER 8

TRANSIENT AND FREQUENCY RESPONSE OF TRANSDUCTORS

In most applications the transient response of an amplifier is as important as the steady-state performance. This is particularly true when dealing with automatic control and servo systems where it has already been seen that an exact knowledge of the transient characteristics is necessary in order to design a stable control system.

In certain types of amplifier the time lags are negligible compared with other time delays in the system ; this is seldom the case when dealing with transductors, however, and therefore it is essential that the transient characteristics of the magnetic amplifier should be understood.

8.1 Energy Storage

In most amplifying devices energy is drawn from the signal source and is stored in the amplifier in some form which permits the output to be suitably controlled.

In the case of a transductor the input energy defines the mean flux level in the cores, which in turn determines the voltage which shall be applied to the load and hence the power dissipation in the load.

Energy cannot be stored instantaneously, however, and the actual energy available for amplifying depends upon two factors, firstly the rate at which energy is supplied to the amplifier from the signal source, and secondly the time over which energy flow takes place. The rate of energy input is the power input and the time taken to store this energy is usually the chief cause of lag between the application of an input and the appearance of the amplified output. In addition to the energy storage which is directly associated with a change in the output power level there

is a certain rate of energy dissipation in the input circuit due to the copper losses, and although this energy dissipation appears to play no part in the actual process of amplification, it is closely related to the time-constant, as may be seen from the following elementary considerations.

The present problem may be considered in terms of an inductance L in series with a resistance R. The flux level in the inductance corresponds to the output of an amplifier, and when a voltage is applied to this series circuit, the current i, and therefore the flux, increases exponentially with a time-constant $\tau = L/R$. If the conditions of the circuit are considered after a time τ has elapsed the following relations may be written down :

$$\begin{aligned} &\text{Rate of energy dissipation} &&= Ri^2 \\ &\text{Total energy stored } W &&= \tfrac{1}{2}Li^2 \\ &\text{Time constant } \tau &&= L/R \end{aligned}$$

Therefore :

$$\tau = \frac{2 \times \text{energy stored}}{\text{rate of energy dissipation}}$$

$$= \frac{2 \times \text{energy stored}}{\text{power input}} \qquad . \quad . \quad . \quad (8.1)$$

This shows that for a given time-constant the power continuously dissipated in the input is proportional to the stored energy. If a given power output can be obtained from the amplifier for a certain energy storage, then by considering w to be the power output per unit of energy stored we have :

$$w = \frac{\text{power output}}{\text{energy stored}} \qquad . \quad . \quad . \quad (8.2)$$

and by combining this with equation 8.1 it is seen that

$$w \propto \left[\frac{\text{power output}}{\text{power input}}\right]\frac{1}{\tau} \qquad . \quad . \quad . \quad (8.3)$$

A useful figure of merit is thus proportional to the power output per unit of stored energy and is the ratio of the power amplification to the amplifier time-constant, the best conditions applying when the figure of merit is large.

It is shown in Chapter 3 (equation 3.44) that for a given supply frequency the time-constant is proportional to the circuit power

amplification divided by the ampere-turns ratio, and it follows that the figure of merit will be directly proportional to the ampere-turns ratio G_m which should therefore be as large as is permitted by considerations of zero stability.

Magnetic amplifiers thus tend to fall naturally into two classes, namely those having a slow response and a high power gain per stage and those having fast response and a low gain per stage. Within certain limitations there is no reason why one type of circuit should not be capable of exhibiting either of the above characteristics by a simple adjustment of the circuit parameters. In fact, however, better performance is obtained if the basic circuits are modified according to the required operating characteristics.

This has given rise to the two types of circuit, the conventional full-wave circuits having large gains and comparatively slow response and the high-speed circuits based on the half-wave mode of operation which are characterized by a comparatively low gain per stage. The transient response of the former type of circuit is dealt with in this chapter, while the latter types are discussed in Chapter 9.

Note that there are two ways of defining the response of a transductor amplifier. In certain circumstances it may be defined by the time-constant, i.e. the time required for the output to reach a certain fraction $\left(1 - \frac{1}{e}\right)$ of its maximum value, having applied a voltage suddenly to the input.

In other cases the time-constant definition is of limited value and more useful information is obtained by considering the transfer function (Chapter 7) and the response of the amplifier to a sinusoidal control voltage of various frequencies.

In Chapter 3 the transient response was considered for a transductor without self-excitation, and in this case it was shown that for a resistive load the control flux increases by a small amount in each half-cycle until it eventually reaches a final value which is limited by the amount the control current can rise. It was also shown in Chapter 3 that the response when considered over a large number of cycles is approximately given by a smooth exponential increase of load voltage, once again the important limitation being that the load is resistive. Provided the overall performance of the transductor is defined by a first order equation (e.g. equation 7.2), the response will be exponential and completely defined by the time-constant.

If the load is also capable of energy storage, however, then another source of time delay is introduced ; this arises in the case of inductive or capacitive loads and often has the effect of completely modifying the transient characteristics which can in certain circumstances give an underdamped response or even give rise to maintained oscillations.

8.2 Factors Causing Time Delays

There are three sources of time lag all of which may arise in a transductor :

(1) An exponential time lag between the control voltage and the control flux. This has already been discussed in Chapter 3 for certain limiting conditions. In practice it is usually somewhat longer than the value calculated on the basis of the assumptions made in Chapter 3.

(2) A fundamental time delay due to the fact that the device is behaving essentially as a switch, and that sampling does not take place continuously but at predetermined times thereby giving rise to a time delay of at least one half-cycle.

(3) A time lag between the application of a voltage to the load and the growth of current in the load ; this time lag will depend upon the nature of the load, and it may interact with the flux time-constant to give underdamped response of the transductor, particularly when the self-excitation is incomplete. Whether or not the response of the transductor is oscillatory, however, it is most important to recognize the existence of two time lags and to realize that however well damped they may be on open loop they are liable to cause instability when used in a control system on closed loop under conditions where one time lag, although much longer, could never cause instability. It is therefore a serious fallacy to assume that the overall response due to two exponential time lags can be expressed in terms of a single time-constant. These facts follow from the discussion in section 7.3. The various time lags will now be considered in greater detail.

Transductor Flux Time Lag

8.3 Time Lag Due to Control Winding Only

The time delay due to the back e.m.f. set up by the growth of control flux is the only appreciable time lag in a transductor when

its load is resistive. In the case where no other windings are coupled to the control winding, this time-constant has already been evaluated for self-excited transductors in terms of the voltage amplification (equation 3.43) or the circuit power amplification and the ampere-turns ratio (equation 3.44). The corresponding expressions for auto-excited transductors can be obtained immediately from the similarity theorem.

Thus, considering an auto-excited transductor having a.c. turns N_a, it was shown in sections 5.2 and 5.4 that the equivalent self-excited transductor has a.c. turns $N_a/2$ and also half the ampere-turns ratio.

Therefore for auto-excited circuits the respective time-constants corresponding to those given in equations 3.43 and 3.44 are

$$\tau = \frac{1}{2f}\frac{G'_p}{G_m} \quad . \quad . \quad . \quad (8.4)$$

and

$$\tau = \frac{1}{2f} G_v \cdot \frac{N_c}{N_a} \quad . \quad . \quad . \quad (8.5)$$

Now since $G_m = \frac{1}{\pi}\frac{\omega L_{aw}}{R_a}$ (equation 5.4)

and $G'_p = \left[\frac{1}{\pi}\frac{\omega M_{ca}}{R_a}\right]^2 \frac{R_a}{R_c}$ (equation 5.8)

Substitution in 8.4 shows that

$$\tau = L_{cw}/R_c$$

where L_{cw} is the inductance of one control coil, and R_c the total control circuit resistance. It is seen that this is half the time-constant of the control circuit when operated under the conditions of no a.c. excitation.

8.4 Time Lag Due to Additional Windings Coupled to the Control Winding

In practice other closed circuits are magnetically coupled to the control circuit and e.m.f.s induced in these circuits produce currents which also tend to slow down the growth of the average flux ; the control circuit, however, is generally the predominant factor in determining the flux time-constant. Nevertheless, the existence of induced e.m.fs. elsewhere can place a lower limit on the time-constant however fast acting the control circuit is made.

The following closed circuits, in addition to the control circuit, tend to delay the growth of flux.

(*a*) *Eddy Current Paths in the Core.* When the mean flux changes, induced e.m.f.s are set up in the core as a result of which eddy currents circulate in the core material itself. The eddy currents produce a component of flux which opposes the original change in the mean flux level and thereby introduces extra damping associated with the growth of flux. This problem is of course a well-known one and is minimized by forming the core of a large number of thin laminations insulated on one side. For a given core the time-constant due to eddy currents is proportional to the permeability divided by the specific resistance of the core material, and in most cases the effect of eddy currents is sufficiently small to be neglected.

(*b*) *Bias Windings.* Bias windings are coupled to the control windings and are therefore liable to cause flux damping. The time-constant of the bias circuit is usually reduced to negligible proportions by placing a large resistance in series with the bias windings ; the necessary current is then forced through the bias coils from a comparatively high voltage source.

(*c*) *Extra Control Windings.* When there is more than one control winding on a transductor, the overall flux time-constant depends upon the damping due to each individual winding although at any given time only one of the applied inputs may be changing. At any instant the resultant control m.m.f. acting on the core is given by the sum of the individual m.m.f.s due to all circuits linking the control circuit. Let us now, therefore, consider the case of an auto-excited transductor having several control windings 1, 2 . . . n and corresponding control voltages E_{c_1}, E_{c_2}, etc. (*Figure* 8.1). Each of these windings are coupled to the same pair of cores and it is shown in Appendix 8 that, provided the transductor operates over its linear range, the mean flux increases exponentially with a time-constant τ_t.

Where
$$\tau_t = \frac{L_{c_1}}{R_{c_1}} + \frac{L_{c_2}}{R_{c_2}} + \ldots + \frac{L_{c_n}}{R_{c_n}} \quad . \quad . \quad . \quad (8.6)$$

Therefore the time-constant due to any one pair of control windings is equal to one-half its time-constant considered as a simple inductance consisting of two coils in series having a total circuit inductance $2L_{c_n}$ and circuit resistance R_{c_n}. The total time-constant for the control flux is then given by the sum of the contributions due to each circuit.

The final value of the mean flux will be due to the total m.m.f.

from the steady-state currents flowing in each of the circuits. Closed circuits to which no control voltage is applied will not influence the steady-state value but will contribute to the delay nevertheless during the transient period. It should be noted that, contrary to the remarks made in section 8.2, the addition of time-constants in this case is permissible because the complete relationship between control voltage and flux is a first order equation which defines a simple exponential, whereas two time-constants in cascade cannot be so represented (Chapter 7) and therefore in the latter case the overall time-constant cannot be obtained by adding the individual time-constants.

(*d*) *Closed Circuit Due to Self-Exciting Windings.* Of all the possible circuits coupled to the control circuit the one which exerts the most important influence on transients is the closed circuit formed by the self-exciting windings through the self-excitation rectifier from M to N (*Figure* 5.1*b*). A similar closed circuit of course exists in the case of auto-excited connections. Unfortunately a realistic analysis of this effect is very difficult due to the complicated action of the auto-excitation rectifier. The extra damping due to this circuit has been investigated by Krabbe [3(b)] on the basis of rectifiers for which the resistance changes discontinuously from zero to infinity at the origin. Even with these limiting assumptions, however, the mode of operation is still obscure and the results of Krabbe's analysis do not agree well with practical experience. Despite this, however, the analysis certainly has some value, firstly because it indicates the existence of a time-constant which depends upon the magnitude and sign of the signal, a result which is borne out in practice, and secondly because it is probably correct in principle but rendered inaccurate by the uncertainty regarding the exact operation of the rectifier when the current flowing in it is small.

Krabbe considers the mode of operation for several types of auto-excited transductor loading, and in all cases the predicted contribution to the time-constant is greater than that found in practice. One case will be considered in the following section.

8.5 Total Flux Time Lag of Auto-Excited Transductor with Smooth Control Current

The circuit about to be considered is shown in *Figure* 8.2, in which the load is external and may be either an a.c. load or fed through a separate rectifier. The currents flowing in the a.c.

windings are, as previously, divided into two components, a circulating component $i_f/2$ and the components of external current $i_a/2$. It has already been shown that the only essential difference between this circuit and the one employing complete self-excitation is that in the latter case the two components of current flow in separate windings, namely the self-exciting windings and the a.c. windings, the former being coupled magnetically to the control windings. In the auto-excited connection a similar coupling occurs. This coupling is such that e.m.f.s induced from the control circuit flow along the same path as the circulating component $i_f/2$ but do not influence the resultant component which flows in the external a.c. circuit.

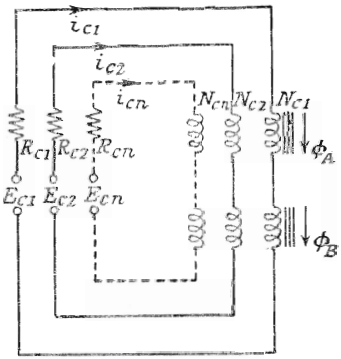


Figure 8.1. Transductor control with several isolated control windings.

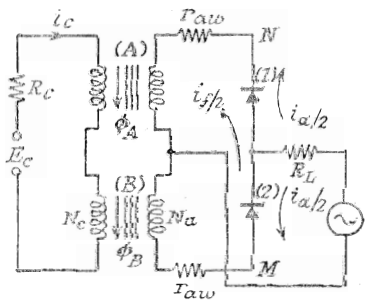


Figure 8.2. Auto-excited transductor to show extra flux damping due to a.c. windings.

If the total resistance round the closed circuit through MN is very large, then the time-constant of this closed circuit will be small as a result of which the circuit will not influence appreciably the flux time-constant. If the resistance is small, however, large currents will flow as a result of the induced e.m.f.s and the resulting effect on the time-constant will be appreciable. The resistance round this closed circuit is almost entirely governed by the rectifiers and on the basis of the idealized rectifier, there will be two possible values of the resistance. If both rectifiers are conducting the resistance round the closed circuit will be that of the a.c. windings $2r_{aw}$, but if either rectifier is non-conducting the resistance becomes infinite. It is therefore necessary to investigate the behaviour of the rectifiers in the auto-excitation

circuit. This is done on the basis of the reverse voltages existing across the rectifier valves and has already been considered in Chapter 5 for the steady-state conditions.

When considering transients the rectifier operation is also influenced by e.m.f.s induced in the closed circuit from the control circuit and it is therefore necessary to consider the various possible modes of operation which are likely to arise during a half-cycle. These will differ essentially according to the magnitude and sign of the applied control voltage.

Case 1. *One Core Saturated.* When one of the cores is saturated (core A say). It has already been shown that a reverse voltage exists across the rectifier (2), the magnitude of this reverse voltage being equal to the voltage drop in the a.c. windings on core A. This voltage drop will have a mean value equal to $\bar{\imath}_a r_{aw}$ where $\bar{\imath}_a$ is the mean load current all of which flows in the a.c. winding on core A during this conducting period. The rectifier (2) can only become conducting if an e.m.f. induced from the control circuit via the a.c. coil B is equal and opposite to this voltage drop.

If the control voltage is suddenly increased, tending to send an additional control current in a clockwise direction (*Figure* 8.2), the induced e.m.f. in the a.c. coil B will attempt to force a component of current in such a direction as to oppose the attempted increase of control flux. This e.m.f. will act against the rectifier (2), however, and will therefore merely increase its reverse voltage. Thus an increase in the control voltage cannot make the non-conducting rectifier become conducting. By the same reasoning, however, a decrease in the control voltage could cause an induced e.m.f. sufficiently large to cancel out the reverse voltage across the rectifier (2) and hence make it conduct. In order to do this the induced voltage would need to be greater than the voltage drop in coil A.

Let us now consider the likelihood of this occurring. The voltage induced in coil B from the control circuit cannot be greater than $\dfrac{E_c N_a}{N_c}$. Therefore :

$$\frac{\text{voltage induced in a.c. coil } B}{\text{voltage drop in a.c. coil } A} < \frac{E_c N_a / N_c}{\bar{\imath}_a r_{aw}}.$$

If this ratio is less than unity then the rectifier valve (2) will continue to experience a reverse voltage during the conducting period. In order to obtain the order of magnitude of this ratio,

consider the conditions when the load resistance R_L is equal to the internal resistance r_{aw}. Under these circumstances the above ratio is equal to

$$\frac{1}{\text{voltage amplification}} \times \frac{N_a}{N_c},$$

which is approximately equal to $\frac{1}{2f\tau}$ (equation 8.5). The quantity $2f\tau$ is the number of half-cycles which elapse over an interval equal to the time-constant. This is usually much greater than unity and hence the reciprocal is less than unity so that the rectifier cannot conduct over this period.

It therefore follows that the auto-excitation circuit cannot contribute to the flux damping during the period when one core is saturated since one of the rectifiers is cut off and the circuit is therefore open.

During these periods, setting of the flux from ϕ_q to ϕ_Q (*Figure* 5.2*f*) takes place purely due to the control voltage and changes in the mean flux level will be governed by the control circuit alone.

Case 2. ***Both** Cores Unsaturated.* During the period when both cores are unsaturated it has already been shown that both rectifiers tend to conduct due to the circulating current $\bar{\imath}_f/2$ which is maintained round the closed circuit (section 5.1). It follows that for small changes of control voltage in either direction this closed circuit will represent an additional winding coupled to the control winding.

If a large increase of control voltage takes place the rectifiers will tend to cut off during non-conducting periods so that once again the time-constant would be governed primarily by that of the control circuit alone. Throughout the cycle, therefore, the control windings are not coupled to any other windings. This case corresponds to the simple one investigated in Chapter 3.

If the control voltage is suddenly decreased by a large amount, however, an e.m.f. is induced in the closed auto-excitation circuit which causes a current to flow through the rectifier in the forward direction from M to N, thereby causing the extra damping due to the a.c. windings.

From the preceding discussion it is seen that, in general, for increasing signals, the flux time-constant depends only upon the control circuit, but for decreasing signals it is governed by two possible values of time-constant, one which applies when one of

the cores is saturated and the other when both cores are unsaturated, and since the relative times over which they apply depend upon the duration of the conducting period, the time-constant depends to some extent upon the output level.

If the conducting period occupies a large fraction of the half-cycle, then the time-constant will tend to be governed primarily by that of the control circuit. But at low output levels the time-constant is appreciably longer due to the greater period over which the extra damping is effective.

In practice, the response time is less than that predicted by the above results ; this is probably due to the fact that the

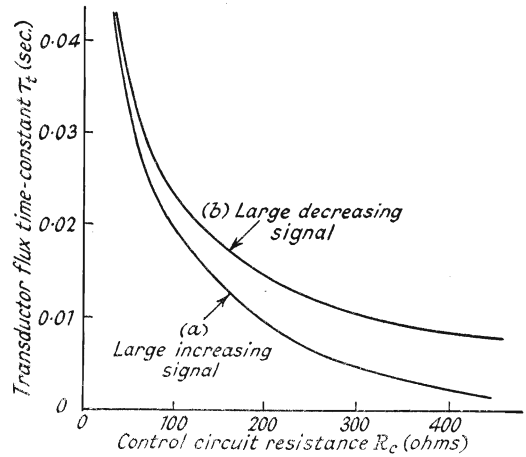
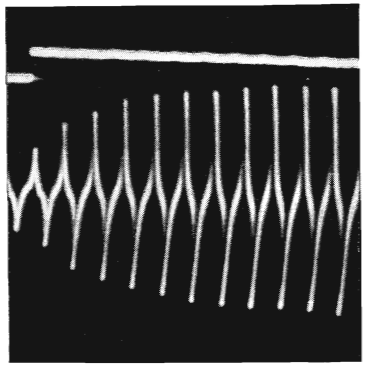


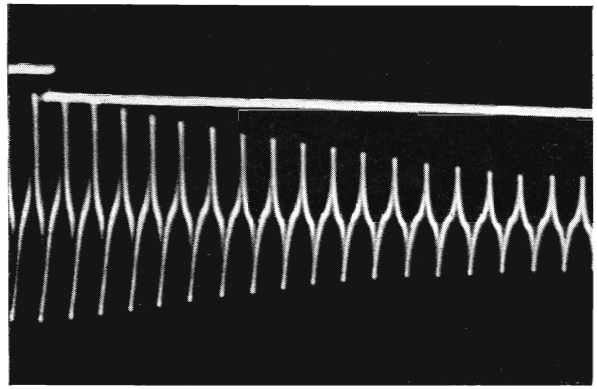
Figure 8.3. Typical curves showing flux time-constant for an auto-excited transductor as a function of control circuit resistance.

forward resistance of the rectifier cannot easily be taken into account. This resistance varies considerably for changes of current near zero, and such variations which are taking place due to the small magnetizing components of current are quite likely to mask the effect of the winding resistance.

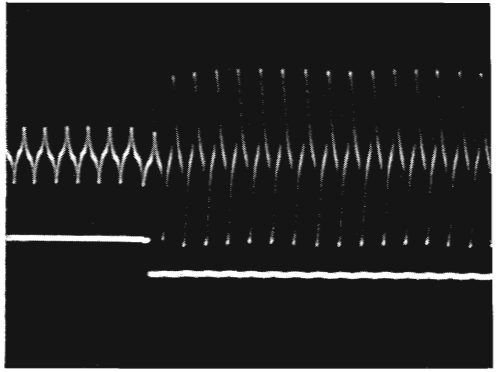
Figure 8.3 shows a typical variation of time-constant with control circuit resistance for a large change of output level : curve (*a*) applies to an increasing control voltage. In this case, there is no damping from the a.c. windings due to the blocking action of the rectifier, and the transductor response time, being governed only by the control circuit, approaches the limiting value of one half-cycle as the control circuit resistance increases



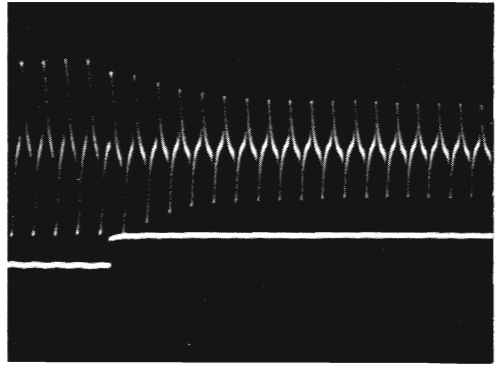
(*a*) Moderate value of R_c
E_c increasing.



(*b*) Moderate value of R_c
E_c decreasing.



(*c*) Very large value of R_c
E_c increasing.



(*d*) Very large value of R_c
E_c decreasing.

Figure 8.4. Oscillograms showing transductor output transients for a step-function change of control voltage, when using auto-excitation.

[*Facing p.* 203.

indefinitely. Curve (*b*) applies to a decreasing control voltage, and here the time lag approaches a finite value which is that due to the closed circuit formed by the a.c. windings.

Oscillograms taken for these two conditions, i.e. increasing and decreasing control voltage, are shown in *Figures* 8.4*a* to *d*. The traces (*a*) and (*b*) record the output transient for increasing and decreasing step inputs respectively. The control circuit resistance in this case has a moderate value. When the control circuit resistance is very large (*c* and *d*) it is seen that the response to an increase of control voltage is practically instantaneous, but that an appreciable delay is still present for decreasing signals.

In cases when it becomes important to reduce the time lags to a minimum this can be achieved by placing the load in the closed circuit formed by the two a.c. windings (*Figure* 5.1*c*). This has the effect of reducing the extra damping and therefore speeding up the response of the transductor. Unfortunately this method of connection leads to correspondingly higher reverse voltages across the auto-excitation rectifiers as described in section 5.17.

Due to the identity existing between the auto-excited and the completely self-excited connections, the preceding remarks apply also to self-excited transductors, in which case improved response is obtained by placing the load in series with the self-exciting windings.

The time delay due to the closed circuit formed by the auto-exciting windings cannot be completely eliminated in these conventional full-wave circuits, even by sacrificing power amplification, and this is the chief reason why it has been found necessary to evolve the special circuits discussed in Chapter 9 in order to get a high speed of response.

8.6 Frequency Response of Auto-Excited Transductor

It is evident from the previous section that the response of a transductor with auto-excitation is non-linear and dependent upon the magnitude and direction of the signal ; it therefore cannot strictly be represented by a linear transfer function.

Useful results are often obtained in servo-engineering, however, by making approximations which allow the use of the transfer function in order to facilitate the design of control systems. Therefore, although the problems discussed in this chapter are non-linear in varying degrees, there is little doubt that advantages

are gained by making approximations in order to achieve a result.

If, for example, the flux time-constant is estimated as being approximately equivalent to some value τ_t, allowance being made for the various spurious effects discussed in sections 8.4 and 8.5. Then the transient relationship between the control voltage and the mean output voltage could be written as the transfer function for a first order system, i.e.

$$\frac{\bar{e}_a}{E_c} = G_v \overline{Y}_v(D) = \frac{g'_v}{1 + \tau_t D} \quad . \quad . \quad . \quad (8.7)$$

This, however, does not completely specify the response to variations of signal frequency. This is because the change in

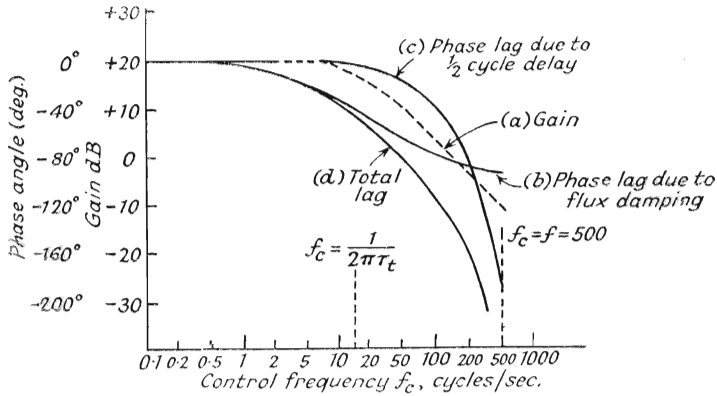


Figure 8.5. Frequency response of auto-excited transductor.

output following a step increase (or decrease) of input is characterized by a number of discreet pulses of output each of which is slightly larger (or smaller) than the foregoing one. Thus there is liable to be a fixed time lag of one half-cycle of the supply in addition to the time-constant due to flux damping.

When referred to a signal frequency response diagram this time lag does not produce attenuation of the output but does give rise to a phase shift. If the signal frequency is very low compared with the supply frequency this phase shift is negligible and the frequency response therefore approximates to that implied by the transfer function of equation 8.7. As the signal frequency increases, however, the constant time lag produces an appreciable phase shift and therefore modifies the phase-

frequency characteristic, but does not alter the amplitude-frequency characteristic.

Since the maximum possible value of the lag is one half-period of the supply frequency it follows that this lag will give a phase shift of 180° when the signal frequency is equal to the supply frequency. If the signal frequency is a quarter of the supply frequency, then the phase shift due to this lag is 45°. The latter signal frequency is quite a likely one to occur in practice and the phase shift correction is appreciable. It is therefore important to take this into account when designing a fast magnetic amplifier for servo applications.

Example. The estimated time-constant of a 500 c/s transductor is ·01 secs. and the voltage gain is 10 (20 dB). It is required to know the gain/phase response of this unit as a function of signal frequency.

Figures 8.5*a* and *b* show the gain and phase shift respectively corresponding to a first order time delay of ·01 sec.

Figure 8.5*c* shows the additional phase shift, due to a half-cycle delay, as a function of signal frequency where the additional

$$\text{phase lag} = \frac{f_c}{500} \times 180° \qquad . \quad . \quad . \quad (8.8)$$

Figure 8.5*d* shows the overall phase frequency response diagram obtained by adding (*b*) and (*c*).

Time Lags Due to Load

8.7 Transients Due to the Load Circuit

In the previous section it was shown that a simple time lag exists between the control voltage and the mean output voltage of an auto-excited transductor. When the load contains reactive elements, however, there is a further time delay between the mean output voltage $\bar{e}_a$ and the resulting currents, before a steady-state is reached. There are two currents which require consideration, firstly the load current and secondly that flowing in the a.c. windings of the transductor. The former is of interest since it defines the output directly, and the latter because it may react back on the flux time-constant in such a way as to modify the overall response of the transductor. This occurs when the self-excitation is incomplete or when feedback is used with an auto-excited transductor. In Chapter 6 we considered the steady-state-values of mean load current $\bar{\imath}_L$ and mean transductor

current $\bar{i}_a$ in terms of mean resistances $\bar{R}_a$ and $\bar{R}_{aL}$ respectively for inductive and capacitive loads. Interest is now centred on the transients which occur prior to the steady-state condition.

For a circuit consisting of linear elements it is usually a comparatively simple matter to determine the transients which occur as a result of changing the voltage applied to the circuit. When considering the transductor, however, two difficulties arise, first the load circuit normally includes a rectifier which is a non-linear element, and second the applied voltage e_a is of a form which cannot be expressed as a simple function of time. The voltage consists of pulses corresponding to the ungated portions of a sine wave and the mean value of these pulses considered over a half-cycle of the supply frequency is the mean output voltage $\bar{e}_a$ applied to the external circuit. This circuit includes the internal resistance of the transductor and that of the load rectifier and the resistance of the load circuit. Our present interest is centred on the way in which the mean load current and the mean current in the a.c. windings change following a sudden change in the mean applied voltage.

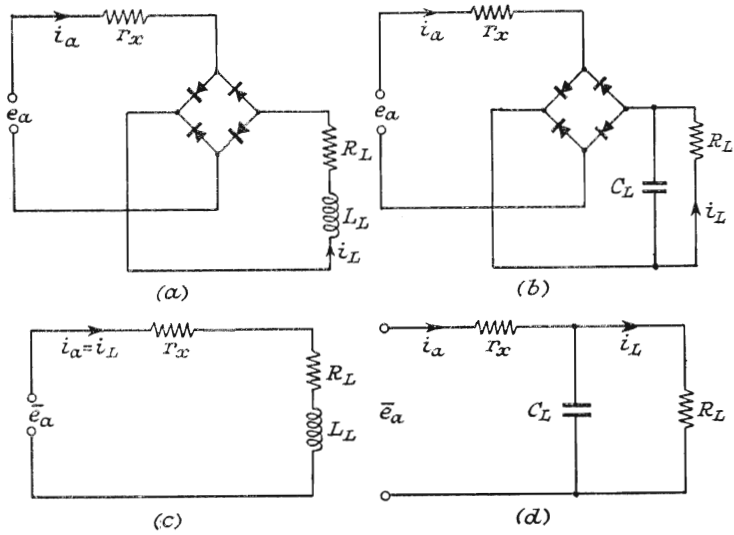


Figure 8.6. Inductive and capacitive loads.

(*a*) Rectifier fed (series inductive). (*b*) Rectifier fed (shunt capacitive). (*c*) Linear inductive circuit. (*d*) Linear capacitive circuit.

As in previous cases the present problem is simplified by assuming that the supply voltage is a square wave and therefore the ungated portion of this, i.e. the output from the transductor consists of rectangular pulses.

When a core saturates this voltage is applied to the rectifier fed load circuit through the internal resistance of the transductor. Two types of load will be considered as shown in *Figure* 8.6; (*a*) corresponds to a load consisting of series inductance while (*b*) corresponds to a load with shunt capacitance. It is assumed in the following discussion that the load transients settle down over a period which is large compared with the duration of one half-cycle.

8.8 Transients Due to Inductive Load

Considering the circuit in *Figure* 8.6*a*, at a time $t = 0$ a train of pulses of peak value $\hat{E}_a$ is suddenly applied to this circuit (*Figure* 8.7*a*). Although these pulses are of alternate sign,

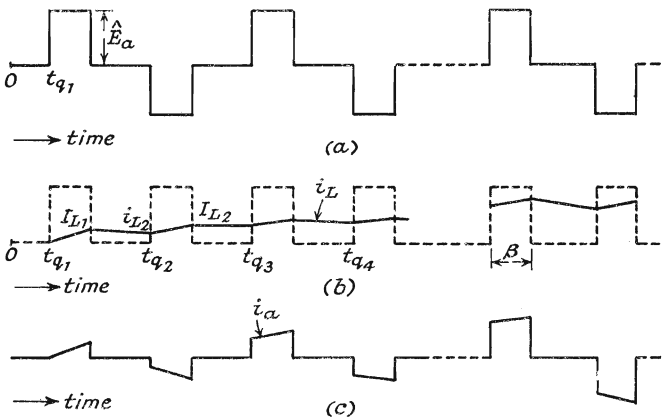


Figure 8.7. Load circuit transients due to a sudden change in transductor output (inductive load).

(*a*) Output voltage. (*b*) Output voltage rectified and load current i_L. (*c*) Transductor current.

they appear to the load circuit as unidirectional due to the routing action of the bridge rectifier. The sign of the pulses applied to the load circuit is therefore always positive as shown dotted in *Figure* 8.7*b*. Diagrams showing the growth of load current and transductor current are shown in *Figures* 8.7*b* and *c* respectively.

Up to the time t_{q1}, no voltage is applied because the cores are unsaturated, but at t_{q1} the full voltage $\hat{E}_a$ is applied to the external

circuit. The current cannot increase suddenly, however, due to the inductance and therefore during the first pulse almost the whole applied voltage appears across the inductive part of the load and the rate of increase of current is a maximum and approximately equal to $\hat{E}_a/L_L$. Over the duration of this pulse the current builds up to a value I_{L_1} by the end of the first half-cycle when desaturation occurs and the applied voltage decreases to zero. The load current now tends to be maintained round the closed circuit formed by the two halves of the bridge rectifier acting in parallel. Over this period the load current decays to a value i_{L_2} due to the resistance of the load. The rate of increase of load current during conducting periods is proportional to the voltage applied to the inductance L_L, and the rate of decay during non-conducting periods is proportional to the magnitude of the current. Thus in each conducting period the rate of increase of current becomes successively smaller due to the increased potential drop across the resistance. Similarly the rate of decay during non-conducting periods gets greater as the load current increases. Ultimately this results in a steady-state of operation when the total increase of load current during conducting periods is equal to the decrease during non-conducting periods.

Thus the load current gradually builds up by a small amount in each half-cycle but approaches a final steady-state mean value. It is shown in Appendix 9 that the time-constant governing this build-up is given by

$$\tau_L = \frac{L_L}{R_L\left(1 + (2\beta/T)\,\dfrac{r_x}{R_L}\right)} = \frac{L_L}{\bar{R}_{aL}} \quad . \quad . \quad . \quad (8.9)$$

This result shows that τ_L depends slightly upon the duration of the conducting period and furthermore that the result is governed by the mean resistance already defined for the steady-state performance. In practice the errors involved by considering the value of τ_L to be constant even for a sine wave supply voltage are not unduly large. Since the transductor current is equal in magnitude to the load current during conducting periods (*Figures* 8.7*b* and *c*), it must be governed by the same time-constant as that applying to the growth of load current.

The steady-state values of mean load current and mean transductor current for a given applied voltage have also been

calculated in terms of $\bar{R}_a$ and $\bar{R}_{aL}$ (see equations (6.10) and (6.11)). Since they are also both governed by the same exponential time lag, the transfer impedances may be written down thus :

$$\frac{\bar{\imath}_L}{\bar{e}_a} = \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}} = \frac{1}{\bar{R}_{aL}}\left[\frac{1}{1+\tau_L D}\right] \quad . \quad . \quad . \quad (8.10)$$

$$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{\overline{Y}_a(D)}{\bar{R}_a} = \frac{1}{\bar{R}_a}\left[\frac{1}{1+\tau_L D}\right] \quad . \quad . \quad . \quad (8.11)$$

and for an inductive load

$$\tau_L = L_L/\bar{R}_{aL} \quad . \quad . \quad . \quad (8.11a)$$

$$\bar{R}_{aL} = R_L\left[1 + (2\beta/T)\,\frac{r_x}{R_L}\right]$$

and

$$\bar{R}_a = \frac{1}{(2\beta/T)}\,R_L\left[1 + (2\beta/T)\,\frac{r_x}{R_L}\right]$$

where $\overline{Y}_{aL}(D)$ and $\overline{Y}_a(D)$ are the frequency-dependent parts of the transfer functions and in this case are equal. The mean applied voltage $\bar{e}_a$ may be varied sinusoidally with time by applying a low-frequency signal voltage to the control circuit. This will cause a corresponding sinusoidal variation of mean load current. The amplitude ratio $\dfrac{\bar{\imath}_L \text{ (peak)}}{\bar{e}_a \text{ (peak)}}$ of these two low-frequency variations and the phase shift between them may be plotted logarithmically for various signal frequencies as described in section 7.2 where it was shown that the response is constant up to a frequency equal to $\dfrac{1}{2\pi\tau_L}$ after which the amplitude of the load current decreases at 6 dBs per octave and the phase shift approaches 90°.

Since the assumed transfer function for the mean transductor current is of the same form, its frequency response diagram will be the same as that for the load current apart from the steady-state term.

It should be remembered that, although the load current i_L is unidirectional, the transductor current i_a is alternating and the transfer impedance given in equation 8.11 refers to its mean value considered over a positive half-cycle of the supply frequency. If we compare the circuit in *Figure* 8.6*a* excited from alternating pulses with that in *Figure* 8.6*c* excited from a smooth applied voltage the following points are worth noting.

In the latter case, since there is no rectifier, i_L and i_a are the same and the steady-state value is $\frac{\bar{e}_a}{R_a}$. The transfer function is given by

$$\frac{\bar{i}_a}{\bar{e}_a} = \frac{1}{R_a} \frac{1}{(1 + \tau_L D)} \quad . \quad . \quad . \quad (8.12)$$

In the former case the steady-state mean values of transductor current and load current are not equal but differ by the factor $\bar{R}_{aL}/\bar{R}_a = 2\beta/T$ which takes into account the fact that the applied voltage occurs in the form of alternating pulses switched by the rectifier. In both cases, however, transients are governed by the same time-constant which gives rise to a lagging phase angle between both currents and the applied voltage.

8.9 Transients Due to Load with Parallel Capacitance

When a capacitor is placed in parallel with the load (*Figure* 8.6*b*) a time delay is again introduced ; in this case it is because the voltage across the capacitor cannot change suddenly and since the capacitor voltage exists also across the load it follows that the load current experiences a similar time lag. It will be shown, however, that the mean transductor current does not behave in this way and that if the mean applied voltage is suddenly increased the current $\bar{i}_a$ is initially very large, eventually decaying to a much smaller value in the steady-state.

The transients due to a capacitive load will now be considered for the case where the mean applied voltage consists of a train of rectangular pulses applied to the load circuit at a time $t = 0$ as in *Figure* 8.8*a*. The full-line curve in *Figure* 8.8*b* represents the load voltage and is therefore proportional to the load current. During the first conducting period, starting at a time t_{q1}, charge flows rapidly into the capacitor raising its voltage to a value E_{L1} by the end of the conducting period. The rapid flow of charge corresponds to a large current drawn from the supply during the first conducting period (*Figure* 8.8*c*). During the following unsaturated period the coil current is appreciably zero due to the gating action of the transductor, and a small quantity of the accumulated charge in the capacitor flows into the load resistor thereby slightly reducing the capacitor voltage, i.e. the load voltage to the value e_{L2}. During succeeding conducting periods the charge in the capacitor increases progressively, but at a steadily diminishing rate as the load voltage gradually approaches

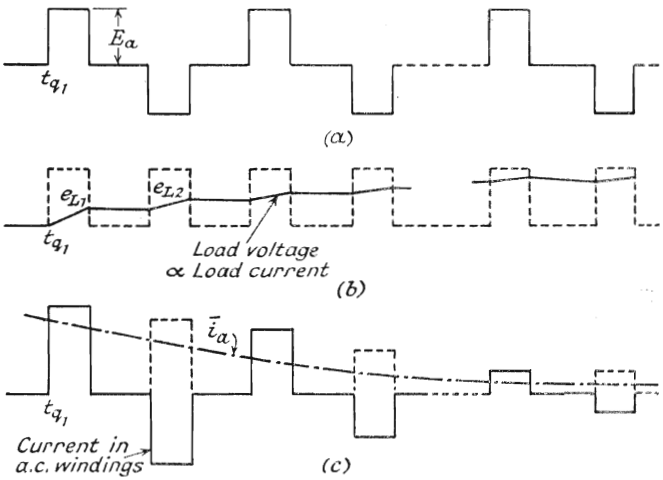


Figure 8.8. Transients with capacitive load due to sudden change of transductor output.

(*a*) Output voltage. (*b*) Output voltage rectified and load voltage. (*c*) Transductor current.

its steady-state value. At the same time the current drawn from the supply decreases progressively in each conducting period until ultimately the mean coil current has decreased to its normal steady-state value. The coil current during conducting periods contains two components, one due to the load current and the other due to the flow of charge into the capacitor. The latter component is proportional to the rate of flow of charge and therefore is a maximum when the rate of charge flow is maximum, i.e. when the voltage e_a is first applied (*c*).

It is shown in Appendix 9 that for the same limiting assumptions as those applying to an inductive load, the frequency response of the mean load current and mean transductor current are respectively given by

$$\frac{\bar{i}_L}{\bar{e}_a} = \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}} = \frac{1}{\bar{R}_{aL}}\left[\frac{1}{1+\tau_L D}\right] \quad . \quad . \quad . \quad (8.13)$$

and

$$\frac{\bar{i}_a}{\bar{e}_a} = \frac{\overline{Y}_a(D)}{\bar{R}_a} = \frac{1}{\bar{R}_a}\left[\frac{1+\tau'_L D}{1+\tau_L D}\right] \quad . \quad . \quad . \quad (8.14)$$

where

$$\tau_L = \frac{C_L R_L r_x}{\bar{R}_{aL}} \quad . \quad . \quad . \quad (8.15)$$

$$\tau'_L = C_L R_L \quad . \quad . \quad . \quad (8.16)$$

and from Chapter 6 we have for a capacitive load

$$\bar{R}_{aL} = \bar{R}_a = r_x\left[1 + (2\beta/T)\frac{R_L}{r_x}\right].$$

This circuit with capacitor smoothing may be considered analogous to the circuit in *Figure* 8.6*d* to which the input $\bar{e}_a$ does not consist of pulses but a steady voltage applied at a given time, or a low frequency sine wave. The following linear transfer function may be written down for this circuit :

$$\frac{\bar{i}_L}{\bar{e}_a} = \frac{1}{R_a}\left[\frac{1}{1 + \tau_1 D}\right] \quad . \quad . \quad . \quad (8.17)$$

$$\frac{\bar{i}_a}{\bar{e}_a} = \frac{1}{R_a}\left[\frac{1 + \tau_2 D}{1 + \tau_1 D}\right] \quad . \quad . \quad . \quad (8.18)$$

where $\tau_1 = \dfrac{C_L R_L r_x}{R_a}$

$$\tau_2 = C_L R_L$$

and $$R_a = R_L + r_x.$$

The earlier equations (8.13 and 8.14) for the non-linear circuit are seen to be identical to these (8.17 and 8.18) for the linear case, provided the resistance $\bar{R}_a$ in the former replaces R_a in the latter.

It is interesting to note the close similarity in form between the non-linear and the linear cases, however. The theoretical frequency response for the mean transductor current corresponding to linear transfer functions of the form given by equation

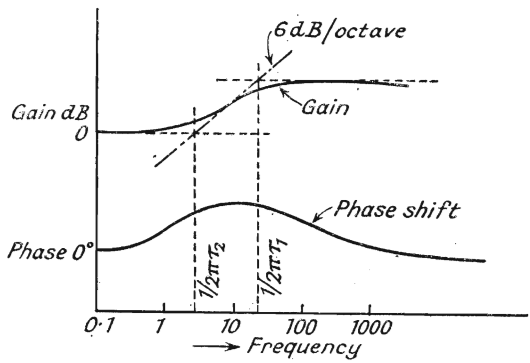


Figure 8.9. Frequency response diagram showing typical log gain and phase response plotted against frequency for a linear capacitive circuit (*Figure* 8.6*d*).

(8.17) and (8.18) is shown in *Figure* 8.9. At very low frequencies and very high frequencies the phase shift is zero, but over a range of signal frequencies the phase angle is leading and the amplitude ratio increasing. Therefore, to summarize the foregoing results, it has been shown that when the transductor is operating into a rectifier fed load, the mean load current and the mean transductor current are both approximately determined by transfer impedances which are of the same form as those governing linear circuits excited by continuous voltages rather than alternating pulses.

There are, of course, a large number of other load circuit configurations, many of which would have extremely complex transfer impedances—the two already discussed are of a relatively simple nature and the relationships evolved for many purposes represent an over simplification of the problem.

It should be remembered, however, that the transfer impedances expressed in the general form $\overline{Y}(D)$ need not be expressed analytically, but rather by means of the frequency response characteristic which gives a more accurate representation if it is obtained experimentally in the form of a curve such as that shown in *Figure* 8.9.

Overall Response Characteristics of Transductors on Load

Having considered the transductor flux time lag and the possible nature of additional time lags due to energy storage in the load, it is now possible to determine the combined effects of these two time delays on the overall response characteristics.

The type of response obtained with a transductor depends very much on the conditions of operation and in particular the degree of self-excitation. We will therefore start by considering the simplest and commonest case, namely that of a transductor operating with complete self-excitation or auto-excitation without feedback.

8.10 Auto-Excited Transductor with Reactive Loading

In order to consider the dynamics of the auto-excited transductor with a reactive load, it is necessary to assume that the auto-excitation rectifiers have infinite reverse resistance otherwise

the action would be similar to that of a partially self-excited transductor. In the latter case interaction occurs between the load time-constant and the flux time-constant which might cause an oscillatory response—this is considered in section 8.11.

In the present case this cannot occur, and therefore, as shown in Chapters 4 and 5, the transductor has a low output impedance so that the mean voltage applied to the load is ideally dependent only upon the control voltage and related to it by a simple first order transfer function $G_v\overline{Y}_v(D)$ (see section 8.6)

where $$G_v = g'_v = \frac{1}{\pi}\frac{\omega M_{ca}}{R_c}.$$

The mean output voltage, in turn, produces a current in the load according to the general relationship

$$\frac{\bar{\imath}_L}{\bar{e}_a} = \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}}.$$

Thus, considered as a block diagram, the auto-excited transductor with any type of load may be represented as in *Figure* 8.10.

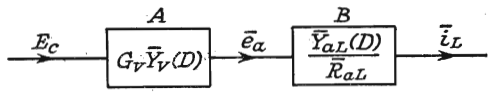


Figure 8.10. General block diagram, for an auto-excited transductor with any type of load.

The element A corresponds to the transductor amplifier and the element B the load, to which the input is a voltage and the output is the mean load current.

Since in this case the transductor has a low impedance output, it can be considered that the voltage $\bar{e}_a$ is independent of the load impedance and in these circumstances the overall transfer function is the product of the individual transfer functions (section 7.3).

Thus $$\bar{\imath}_L = G_v\overline{Y}_v(D)\frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}}E_c \qquad . \quad . \quad . \quad (8.19)$$

The dynamic response is fixed by the product $\overline{Y}_v(D)\overline{Y}_{aL}(D)$, and if each of these functions is known either in an analytical form or as frequency response characteristics, then the overall response can easily be found by combining the two as shown in Chapter 7. As a simple example let us consider the case of a transductor with

auto-excitation feeding a capacitive load such as that described in section 8.9. In this case we have

$$G_v = \frac{1}{\pi}\frac{\omega M_{ca}}{R_c}$$

$$\overline{Y}_v(D) = \frac{1}{1+\tau_t D}$$

$$\overline{Y}_{aL}(D) = \frac{1}{1+\tau_L(D)}$$

$$\bar{R}_{aL} = r_x\left[1+(2\beta/T)\frac{R_L}{r_x}\right].$$

Therefore $\bar{\imath}_L = \dfrac{1}{\pi}\dfrac{\omega M_{ca}}{r_x\left[1+(2\beta/T)\dfrac{R_L}{r_x}\right]}\left[\dfrac{1}{(1+\tau_t D)(1+\tau_L D)}\right]\dfrac{E_c}{R_c}.$

The overall frequency response of an auto-excited transductor with a d.c. reactive load is governed by the product $Y_v(D)$, $\overline{Y}_{aL}(D)$ and typical response characteristics depicting gain and phase shift are shown in *Figures* 8.11 and 8.12 respectively. These curves refer to a transductor for which

$$f = 400 \text{ c.p.s.}$$
$$\tau_t = \cdot 065 \text{ secs.}$$
$$R_L = 100 \text{ ohms.}$$
$$r_x = 100 \text{ ohms.}$$
$$C_L = 52\ \mu F$$

and the conducting period β is, at all times, less than $T/8$.

The curves (*a*) in *Figures* 8.11 and 8.12 are estimated from the above numerical data while the experimental points are also shown for comparison.

The estimated curves are obtained by the following steps :

(1) *Determine Cut-off Frequencies.* In the first instance both the transductor flux lag τ_t and the load time lag τ_L are considered to give rise to the familiar type of response associated with a single time delay each having a nearly constant gain up to the cut-off frequency $\dfrac{1}{2\pi\tau}$ and thereafter an attenuation of 6 dB per octave ; the phase lag being 45 degrees at the cut-off frequency and increasing to 90 degrees as the signal frequency increases indefinitely.

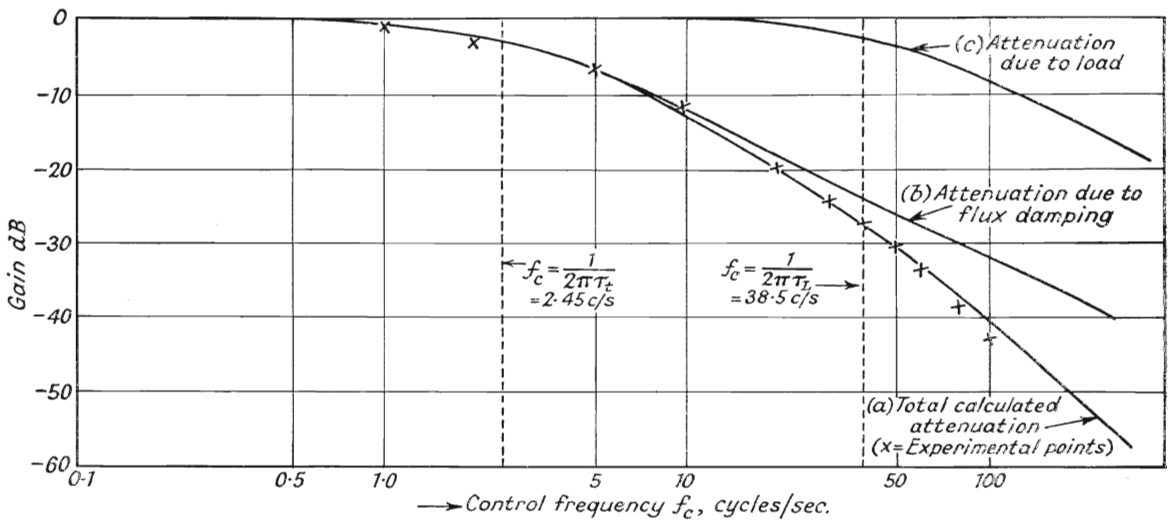


Figure 8.11. Attenuation-frequency response for a capacitively loaded transductor.
(*a*) Overall response. (*b*) Response of transductor. (*c*) Response of load.

In the case of the transductor time lag τ_t the signal frequency at cut-off is equal to

$$\frac{1}{2\pi \times \cdot 065} = 2{\cdot}45 \text{ c/s.}$$

The load time lag is not known exactly, but it may be estimated approximately on the basis of a square wave supply voltage and a conducting period equal to $T/8$, thus from equation (8.15)

$$\tau_L = \frac{C_L R_L r_x}{\bar{R}_{aL}},$$

and for a capacitive load

$$\bar{R}_{aL} = r_x\left[1 + (2\beta/T) \cdot \frac{R_L}{r_x}\right].$$

Therefore

$$\tau_L = \frac{C_L R_L}{1 + (2\beta/T)\dfrac{R_L}{r_x}}$$

$$\doteqdot \frac{52 \times 100 \times 10^{-6}}{1 + \frac{1}{4} \cdot 1}$$

$$= \frac{52}{1{\cdot}25} \times 10^{-4}$$

$$= \cdot 0041 \text{ seconds}$$

giving a cut-off signal frequency equal to

$$\frac{1}{2\pi \times \cdot 0041} = 38{\cdot}5 \text{ c/s.}$$

(2) *Determination of Overall Gain/Frequency Response.* The gain/frequency characteristics for the transductor and the load may now be plotted separately (*Figures* 8.11*b* and *c* respectively). The result of superimposing these curves gives the overall gain referred to unity (zero dB) at zero frequency (*Figure* 8.11*a*).

(3) *Determination of Phase/Frequency Response.* The usual phase/frequency characteristics appropriate to a simple time lag are also plotted for the two cut-off frequencies determined in (1) above. These are shown in curves (*b*) and (*c*), *Figure* 8.12, for the transductor and load respectively.

The result of superimposing these gives the curve (*d*), to which must be added the phase shift due to a half-cycle delay as described

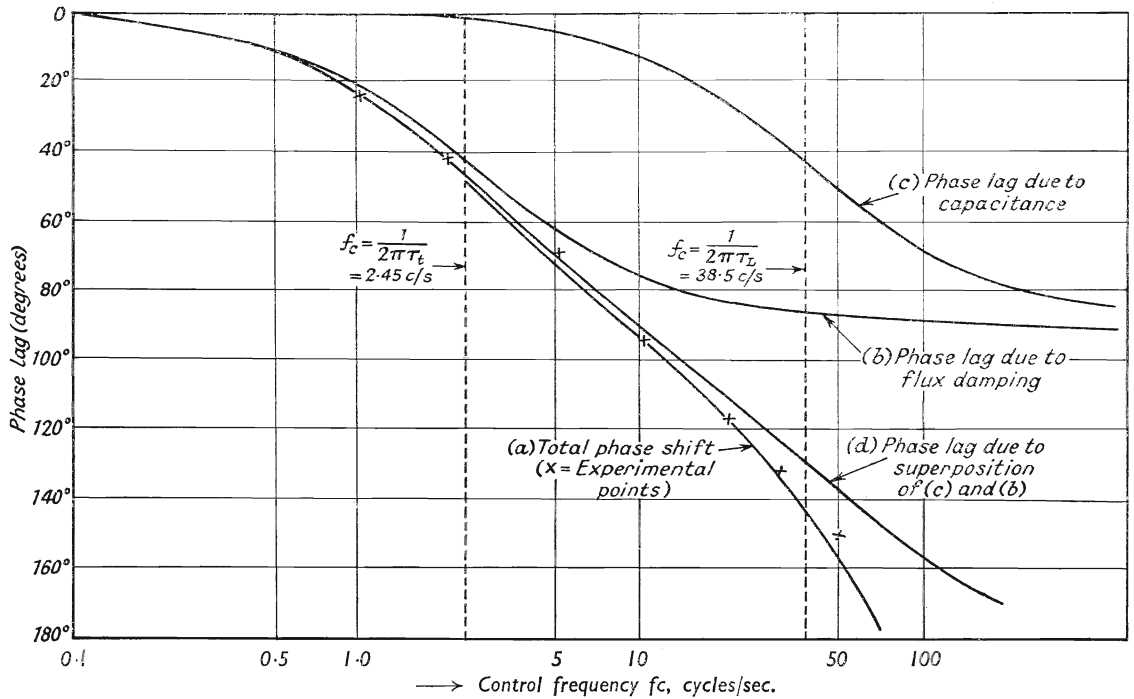


Figure 8.12. Phase-frequency response for a capacitively loaded transductor.

(*a*) Overall response allowing for inherent half-cycle delay. (*b*) Phase lag due to flux damping. (*c*) Phase lag due to load. (*d*) Resultant of (*b*) and (*c*).

in section 8.6. At a signal frequency f_c this phase shift is equal to $\frac{f_c}{400} \times 180$ degrees and the result of adding this correction is shown in the final curve (*a*).

Although there is some discrepancy between calculated and theoretical results, the calculated result is pessimistic rather than optimistic with the result that these calculations are on the safe side when estimating for design purposes.

When the load is inductive, the same type of result is obtained, the load time-constant having been defined in equation 8.11*a*.

It should be noted that in the case of a capacitive load the leading phase angle between the transductor current and the output voltage (section 8.9) does not influence the operation ; this is because the transductor current cannot react back on the core flux due to the auto-excitation ; for this reason the function $\overline{Y}_a(D)$ is irrelevant to the output, which is governed by $\overline{Y}_{aL}(D)$, a simple exponential lag.

8.11 Transient Response of a Transductor with any Proportion of Self-Excitation

The amplification due to a transductor with any proportion of self-excitation was considered in Chapter 4 for the steady-state condition. The analysis in Chapter 4 may be extended to cover the dynamic operation ; before considering this, however, let us first consider the problem on a physical basis.

This problem relates to the way in which a phase displacement between the *mean* output voltage and the *mean* transductor current $\bar{\imath}_a$ can cause an oscillatory response to an applied control voltage. In order to simplify the discussion it will be related to a transductor which has no self-excitation.

If the control voltage is considered to vary at a low frequency, the *mean* output voltage will vary at the same frequency but with a lagging phase angle due to the flux lag τ_t. As a result of this output voltage a certain mean current flows in the transductor windings.

If the load is resistive this mean current will be in phase with the output voltage. At zero signal frequency the transductor current flows in the windings in such a direction as to oppose the effect of the control voltage thereby giving rise to the negative feedback action discussed in Chapter 4. As the control frequency increases the phase angle between the control voltage and the output

voltage increases up to a maximum of 90°, and therefore the transductor current has always got a component which opposes the action of the control voltage.

If the load is reactive, however, then, as we have seen, an additional time lag is introduced. This causes a phase shift between the mean output voltage applied to the a.c. circuit and the mean value of the resulting transductor current so that, although in the steady-state, the voltage-time due to the a.c. windings opposes that due to the control windings; there may be certain control signal frequencies for which the total phase shift will be sufficient to cause this action to assist the control voltage with the result that at these frequencies the sensitivity is increased.[8(a)]

The increased sensitivity at certain frequencies is characteristic of systems which are defined by second order equations and which in addition are only slightly damped.

Such properties can be attributed to the transductor without self-excitation or to one with a limited amount of self-excitation.

When the self-excitation is complete, however, there are no a.c. components magnetizing the core during conducting periods and therefore the mean flux level in the core is determined by the control voltage acting alone.

In this case the system is still defined by a second order equation, but the damping is much greater than critical and the response is given by two exponentials in cascade as shown in section 8.10.

In order to examine these effects on a semi-quantitative basis it will be assumed that the load network can be defined by a linear transfer function of the forms given in sections 8.8 or 8.9.

It was shown from the m.m.f. and flux diagrams in *Figure* 4.4 that at any instant,* assuming the control current does not contain high-frequency components (i.e. $\lambda = 1$), the mean m.m.f.s and fluxes may be related by the equation

$$\bar{F}_c + S\Phi_s = \bar{\imath}_a(N_a - N_f) + 2S\phi_0 \quad . \quad . \quad . \quad (8.20)$$

(see equation 4.12)

Also the mean output voltage applied to the external circuit is

* It may be noted that the term "instantaneous value" has two distinct meanings when considering the transductor. It can either be applied to the rapid changes of voltage, current, etc., which are recurring at the supply frequency or, alternatively, as in this case, to the slower changes which are occurring at the control signal frequency. These latter changes refer essentially to mean values integrated over one half-cycle.

related to the mean transductor current and to the mean core flux ϕ_c by the respective equations

$$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{\overline{Y}_a(D)}{\bar{R}_a} \qquad . \quad . \quad . \quad (8.21)$$

$$\phi_c = \frac{\bar{e}_a}{8fN_a} \qquad . \quad . \quad . \quad (8.22)$$

In addition the mean control m.m.f. $\overline{F}_c = \bar{\imath}_c N_c$ may be supplied from only one winding or from several control windings linking the core, in either case if a voltage is applied to one of these windings we have (Appendix 8)

$$\frac{E_c N_c}{R_c} = \overline{F}_c + \frac{d\phi_0}{dt} \sum \frac{2N_{cn}^2}{R_{cn}} \qquad . \quad . \quad . \quad (8.23)$$

Equations 8.20 to 8.23 can be combined as shown in Appendix 10 to give the following general result in terms of the input and output voltages thus :

$$\frac{\delta \bar{e}_a}{\delta E_c} = G_v \overline{Y}_v(D)$$

$$= \frac{g_v}{(1 + \tau_t D) + g_v h_v[\overline{Y}_a(D)]} \qquad . \quad . \quad . \quad (8.24)$$

where

$$g_v = \frac{2}{\pi} \frac{\omega M_{ca}}{R_c}$$

$$h_v = \frac{R_c}{\bar{R}_a} \frac{N_a - N_f}{N_c}.$$

$\overline{Y}_a(D)$ is the load transfer function and τ_t is the transductor flux lag which is equal to L_{cw}/R_c if no damping is present due to other windings.

Equation 8.24 gives the voltage amplification transfer function for a transductor with any degree of self-excitation and with any type of load. Usually the ultimate output is considered to be the current in the load, and this is obtained by considering the $\frac{\text{load current}}{\text{control voltage}}$ ratio thus :

$$\frac{\delta \bar{\imath}_L}{\delta E_c} = \frac{\delta \bar{e}_a}{\delta E_c} \cdot \frac{\delta \bar{\imath}_L}{\delta \bar{e}_a} = G_v \overline{Y}_v(D) \cdot \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}} \qquad . \quad . \quad . \quad (8.25)$$

All these results define the transient and steady-state characteristics, the latter being obtained by inserting the condition

$D = 0$ in the expression for the transfer function. By putting the appropriate conditions into equation (8.24) or (8.25), the transient performance can be determined for certain special conditions as follows :

(*a*) *No Self-Excitation Resistive Load.* In this case the following particular values are applicable :

$$\bar{R}_a = R_a$$

$$\bar{Y}_a(D) = 1$$

$$N_f = 0$$

so that

$$h_v = \frac{R_c}{R_a}\frac{N_a}{N_c}.$$

Also assuming that

$$\tau_t = \frac{L_{cw}}{R_c}$$

we have from (8.24)

$$\frac{\delta\bar{e}_a}{\delta E_c} = \frac{\dfrac{2}{\pi}\dfrac{\omega M_{ca}}{R_c}}{\left[1 + \dfrac{L_{cw}}{R_c}D\right] + \dfrac{2}{\pi}\dfrac{\omega M_{ca}}{R_c}\cdot\dfrac{R_c N_a}{R_a N_c}}.$$

Dividing by $\dfrac{2}{\pi}\dfrac{\omega M_{ca}}{R_a}\dfrac{N_a}{N_c}$ and remembering that

$$\frac{M_{ca}}{L_{cw}} = \frac{N_a}{N_c} = \frac{L_{aw}}{M_{ca}}$$

gives

$$\frac{\delta\bar{e}_a}{\delta E_c} = \frac{(R_a N_c)/(R_c N_a)}{\left[\dfrac{\pi}{2}\dfrac{R_a}{X_{aw}} + 1\right] + \dfrac{1}{4f}\left[\dfrac{R_a}{R_c}\dfrac{N_c^2}{N_a^2}\right]D}.$$

Since $X_{aw} \gg R_a$, the term involving these quantities is negligible compared with unity, and if it is neglected we have

$$\frac{\delta\bar{e}_a}{\delta E_a} = \frac{(R_a N_c)/(R_c N_a)}{1 + \dfrac{1}{4f}\left[\dfrac{R_a}{R_c}\cdot\dfrac{N_c^2}{N_a^2}\right]D}.$$

This represents the transfer function of an unself-excited transductor. As found previously, the steady-state voltage gain is $\dfrac{R_a N_c}{R_c N_a}$ and the time-constant is given by

$$\tau = \frac{1}{4f}\frac{R_a}{R_c}\cdot\frac{N_c^2}{N_a^2}$$

$$= \frac{1}{4f}\text{ (circuit power gain).}$$

(*b*) *Complete Self-Excitation Resistive Load.* In this case $h_v = 0$ which gives the immediate result from equation 8.24 that

$$\frac{\delta\bar{e}_a}{\delta E_c} = \frac{2}{\pi}\frac{\omega M_{ca}}{R_c}\left[\frac{1}{1+\tau_t D}\right] \text{ (cf. equation 8.7).}$$

(*c*) *Complete Self-Excitation Inductive Load.* As before, $h_v = 0$ and therefore

$$\frac{\delta\bar{e}_a}{\delta E_c} = \frac{2}{\pi}\frac{\omega M_{ca}}{R_c}\left[\frac{1}{1+\tau_t D}\right].$$

In order to determine the mean load current this is combined with the load current transfer function as in equation (8.25) thus

$$\frac{\delta\bar{i}_L}{\delta E_c} = \frac{2}{\pi}\frac{\omega M_{ca}}{R_c}\left[\frac{1}{1+\tau_t D}\right]\frac{\bar{Y}_{aL}(D)}{\bar{R}_{aL}}$$

and in this case

$$\bar{Y}_{aL}D = \left[\frac{1}{1+\tau_L D}\right]$$

Thus

$$\bar{i}_L = \frac{2}{\pi}\frac{\omega M_{ca}}{\bar{R}_{aL}}\frac{1}{(1+\tau_t D)(1+\tau_L D)}\frac{E_c}{R_c}.$$

where

$$\tau_L = \frac{L_L}{\bar{R}_{aL}}$$

and

$$\bar{R}_{aL} = R_L\left[1 + (2\beta/T)\frac{r_x}{R_L}\right].$$

8.12 Underdamped Transients and Sustained Oscillations in Transductors

If the self-excitation of a transductor is incomplete and the load introduces a time lag between the mean output voltage from the transductor and the mean current in the a.c. windings, then the response may be underdamped, in which case the transductor exhibits increased sensitivity to certain control signal frequencies. This problem is first considered for an inductively loaded transductor.

It has already been shown that for any system or element the transient characteristics are specified by the expression in the denominator of the transfer function, and if this expression can be written in the form

$$k_1 D^2 + k_2 D + 1$$

where $k_2^2 < 4k_1$

then the undamped natural frequency $\omega_o = \sqrt{1/k_1}$ and the damping ratio $\zeta = \dfrac{k_2}{2\sqrt{k_1}}$ } cf. equation 7.8.

If the load is assumed to have a time-constant τ_L and the transductor a time-constant τ_t, then putting

$$\bar{Y}_{aL}(D) = \bar{Y}_a(D) = \left[\frac{1}{1 + \tau_L D}\right] \text{ (for inductive load)}$$

we have by combining equations 8.24 and 8.25, dividing by $g_v h_v$ and ignoring $\dfrac{1}{g_v h_v}$ compared with unity :

$$\bar{\imath}_L = \frac{E_c/(h_v \bar{R}_{aL})}{\left[\dfrac{\tau_t \tau_L}{g_v h_v}\right] D^2 + \left[\dfrac{\tau_t + \tau_L}{g_v h_v}\right] D + 1} \quad . \quad . \quad . \quad (8.26)$$

The steady-state condition is obtained by putting $D = 0$, giving

$$\bar{\imath}_L = \frac{N_c}{N_a - N_f} \frac{1}{(2\beta/T)} \frac{E_c}{R_c} \text{ (cf. equation 6.15}a\text{).}$$

It may be seen by inspection of equation 8.26 that the undamped natural frequency and damping ratio of the transductor are respectively given by

$$\omega_o = \sqrt{\frac{g_v h_v}{\tau_t \tau_L}} \quad . \quad . \quad . \quad (8.27)$$

and

$$\zeta = \tfrac{1}{2}\sqrt{\frac{1}{g_v h_v} \frac{(\tau_t + \tau_L)^2}{\tau_t \tau_L}} \quad . \quad . \quad . \quad (8.28)$$

If ζ_n is much less than unity, the transductor will be particularly sensitive to control signals varying at the damped natural frequency which is approximately equal to $\dfrac{\omega_o}{2\pi}$ (Chapter 7). The response to transients will also exhibit a tendency to be oscillatory at this frequency.

When the damping ratio is very small then changes in the output will be governed by a prolonged oscillatory transient. The conditions for the production of transients are therefore most favourable when $g_v h_v$ is large and $(\tau_t + \tau_L)^2/\tau_t \tau_L$ is a minimum, i.e. when $\tau_t = \tau_L$. The term $g_v h_v$ is large when there is no self-excitation and the ratio of reactance to resistance for the a.c. circuit is large (see p. 106). In practice, transients due to an inductive load rarely consist of more than one or two overshoots, and a typical example is shown by the oscillogram in *Figure* 8.13, which applies to an unself-excited transductor operating a rectifier fed inductive load.

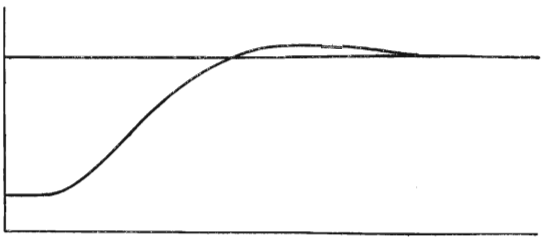


Figure 8.13. Oscillogram of transient response of unself-excited transductor with rectifier-fed inductive load.

By the same methods, the overall response due to a smoothing capacitor may be considered. In this case it has been shown that the mean transductor current $\bar{i}_a$ may have a leading phase angle on the mean output voltage $\bar{e}_a$. This will be considered by using the results already obtained for the transfer impedances of a capacitive load and proceeding as for the inductive load. It may be seen physically, however, that the effect of a smoothing capacitor could lead to improved transient performance in a transductor which is not self-excited.

Let us consider a control voltage suddenly applied to the control circuit ; during the following conducting period the supply voltage is ungated ; when the load is resistive the mean transductor current increases by a small amount during each conducting period. When the load is capacitive, however, it has been shown (section 8.9) that there is a tendency for the mean transductor current to be large during the early part of the transient, *Figure* 8.8*c*, so that although the mean output voltage $\bar{e}_a$ is small in the first conducting period, the transductor current is greater than it would be if the load were purely resistive, due to the fact that the

capacitor at this stage is empty of charge and is therefore able to draw a large current. This effect occurs during each conducting period, but to a lesser extent as the capacitor charges up to the steady-state conditions of operation. At any time the mean control current is proportional to the mean transductor current due to the ampere-turn balance relations, therefore a smoothing capacitor increases the initial rate of build-up of mean control current. The ultimate value of the control current is fixed at the value E_c/R_c, however, and it must follow that the time taken to reach the steady-state is reduced since at all times the rate is increased although the final value is the same. These last two conditions do not necessarily imply a satisfactory response, however, since they would be fulfilled by an oscillating transient which although rapidly reaching the steady-state value, oscillates about this value as discussed for the case of an inductive load. Improved response is generally obtained in systems of this nature, firstly by making the natural frequency as high as possible to ensure rapid build-up, and then by introducing sufficient damping independently to damp out the oscillations to the required extent. In servo systems this is achieved by phase advance or velocity feedback. Equations 8.27 and 8.28 show that for an inductive load it is theoretically possible to make the natural frequency large by increasing the gain ; this, however, simultaneously reduces the damping to give a type of transient which is generally undesirable.

Equation 8.14 shows that when the load is capacitive the mean output voltage and mean transductor current are related by a transfer impedance of the form

$$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{1}{\bar{R}_a}\left[\frac{1+\tau'_L D}{1+\tau_L D}\right].$$

If this expression is applied to the general results (equations 8.24 and 8.25) it may be shown that

$$\omega_o = \sqrt{\frac{g_v h_v}{\tau_L \tau_t}} \qquad . \quad . \quad . \quad (8.29)$$

and

$$\zeta = \tfrac{1}{2}\sqrt{\frac{1}{g_v h_v}\,\frac{(\tau_t + \tau_L + g_v h_v \tau'_L)^2}{\tau_L \tau_t}} \qquad . \quad . \quad . \quad (8.30)$$

In this case it can be seen that the natural frequency once

again depends on the gain. This time however the damping depends upon the additional time constant τ'_L.

Underdamped response with a capacitor in parallel with the load is often observed under practical conditions, and at first sight it might be thought that the above results explain this fully. It has been conclusively proved, however, that this effect also depends upon there being self-capacitance present in the load rectifier discs.[8(a)] By using low capacitance germanium rectifiers in place of selenium ones it is not easy to obtain an oscillatory transient with a simple shunt capacitance across the load although some improvement in response is noted for the reasons given above. By introducing a small capacitor in parallel

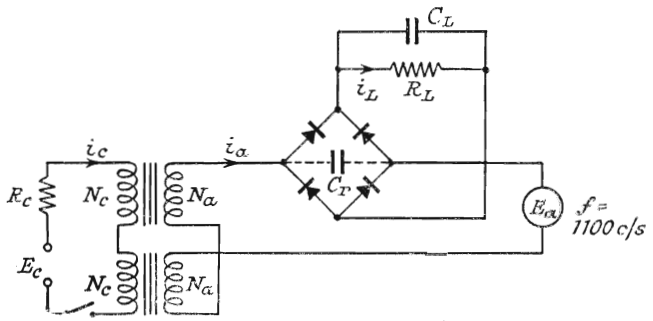


Figure 8.14. Circuit of unself-excited transductor with capacitive smoothing of load current and rectifier capacitance C_r introduced.

with the a.c. terminals of the rectifier a considerable change can be made to the transient response. There is little doubt that the system is still behaving along the lines just described, but an analytical approach to this problem is difficult. The improvement to the response is considerable, however, although the capacitance across the a.c. terminals of the rectifier may be small. A complete circuit is shown in *Figure* 8.14 for the case of an unself-excited transductor operating at 1,100 cycles per second. The oscillograms in *Figures* 8.15*a* to *e* indicate typical response curves of load current following a suddenly applied voltage to the control circuit. *Figure* 8.15*a* represents the transient response of the mean load current with no smoothing capacitor and no rectifier capacitance ; the response is approximately exponential and governed by the transductor time-constant τ_t. Oscillogram (*b*) shows that by using a smoothing capacitor a

distinct improvement is noted. In this case, for a load resistance of 1,100 ohms, the optimum smoothing capacitor is about 2μF. *Figures* 8.15*c*, *d* and *e* show the effects of adding rectifier capacitance C_r. It is seen that a considerable modification is made by using a rectifier capacitance C_r equal to only about 0·5% of the smoothing capacitor while a very small further increase of C_r produces self-maintained oscillations.

One important point is that neither of these two capacitors are noticeably effective when used alone. A second point which is even more significant arises from the fact that although self-maintained oscillations are possible there is no deliberate feedback loop in the system, as may be seen from *Figure* 8.14. The implication is that the feedback, which must be present to support the oscillations, is inherent in the coupling between the coils. This result was anticipated in Chapter 4 and is now confirmed by the

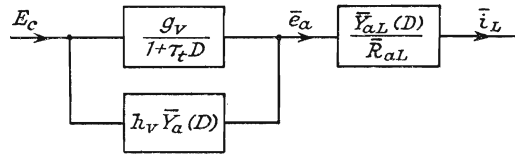
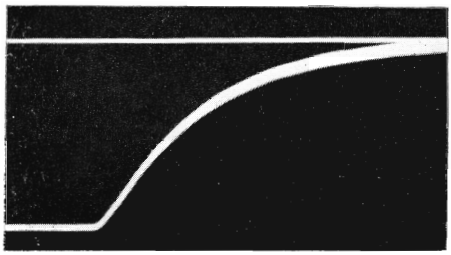


Figure 8.16. General block diagram for transductor as a feedback system.

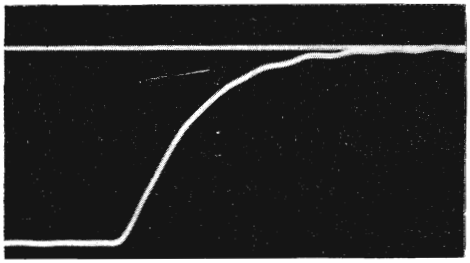
transient behaviour. A possible representation of such a system for transients with an inductive load has been given by Storm,[8(b)] and a more general representation is given in *Figure* 8.16. It is interesting to compare the performance of a transductor with capacitor smoothing under the two extreme conditions of complete self-excitation and zero self-excitation. In the former case the steady-state amplification is increased as described in Chapter 6, while in the latter the steady-state performance is unaffected but the response time tends to be reduced.

8.13 Summary

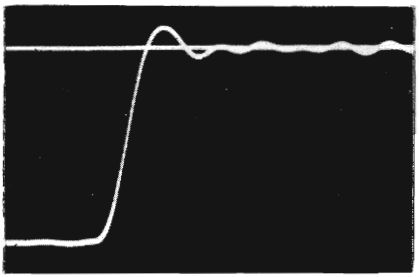
It has been shown that the essential cause of the transductor time lag arises due to the fact that an energy interchange between the transductor and the signal source is inevitably associated with changes in the mean flux level. If there are no closed windings coupled to the control windings this time lag can be expressed in terms of the power amplification or of the voltage amplification. Thus



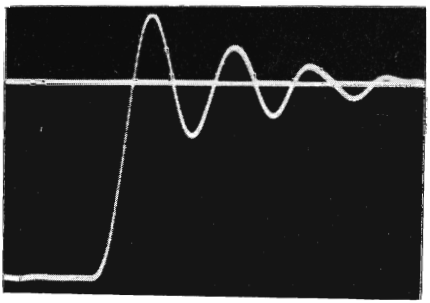
(*a*) $C_L = 0$. $C_r = 0$.



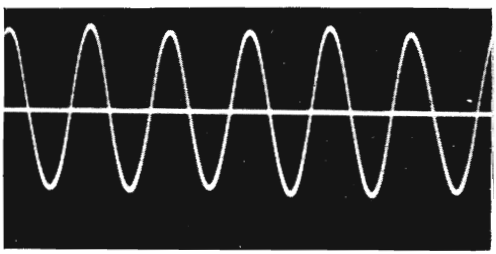
(*b*) $C_L = 2\mu$F. $C_r = 0$.



(*c*) $C_L = 2\mu$F. $C_r = \cdot 011\mu$F.



(*d*) $C_L = 2\mu$F. $C_r = \cdot 014\mu$F.



(*e*) $C_L = 2\mu$F. $C_r = \cdot 016\mu$F.

Figure 8.15. Oscillograms of transients taken for circuit of *Figure* 8.14 with various values of load capacitance and rectifier capacitance.

[*Facing p.* 228.

$$\tau = \frac{1}{nf} \times \frac{\text{power amplification}}{\text{ampere-turns ratio}}$$

or
$$\tau = \frac{1}{nf} \times \text{voltage amplification} \times \frac{N_c}{N_a}.$$

The ratio of power amplification to time-constant can be improved by increasing the permeability of the core material which increases the ampere-turns ratio. In the limit, for complete self-excitation and an ideal core of infinite permeability, the ratio of power gain to time-constant tends to infinity ; typical values for this ratio in practice vary from about 5,000 at a supply frequency of 50 c.p.s. up to 10^5 at supply frequencies of the order of 1 Kc.

The ratio of voltage amplification to time-constant is fixed by the supply frequency and the turns ratio, and it cannot be increased even by increasing the core permeability indefinitely.

Taking the auto-excited transductor as the basic element, it has been shown that the flux time-constant τ_t in this case is approximately equal to one-half that of the control circuit operating as a simple d.c. inductance. The damping effect of the auto-exciting windings has been shown to increase the time-constant slightly over the value L_{cw}/R_c, particularly for decreasing signals.

Additional time delays due to reactive loads have also been considered. When the load consists of series inductance or parallel capacitance the load time-constants are of the same form as those obtained for the simple equivalent d.c. circuits. The load time-constant has been shown to react back on the transductor time-constant to give a response which may be underdamped when there is no self-excitation. In the most usual case of a fully self-excited or auto-excited transductor no such effect occurs, however, and the transient performance may be represented by two time lags in cascade.

The transfer function treatment has been stressed throughout this chapter primarily in order to provide a direct link with servo design problems.

The following approximate transfer functions have been established :

(1) General expression for voltage transfer function for transductor

$$\frac{\delta \bar{e}_a}{\delta E_c} = G_v \bar{Y}_v(D) = \frac{g_v}{(1 + \tau_t D) + g_v h_v [\bar{Y}_a(D)]}$$

TABLE 8.1

Type of load (rectifier fed)	$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{\bar{Y}_a(D)}{\bar{R}_a}$	$\frac{\bar{\imath}_L}{\bar{e}_a} = \frac{\bar{Y}_{aL}(D)}{\bar{R}_{aL}}$	τ_L	τ'_L	$\bar{R}_a$	$\bar{R}_{aL}$
Resistive . .	$1/R_a$	$1/R_a$	0	0	R_a	R_a
Series inductive .	$\frac{1}{\bar{R}_a}\left[\frac{1}{1+\tau_L D}\right]$	$\frac{1}{\bar{R}_{aL}}\left[\frac{1}{1+\tau_L D}\right]$	$L_L/\bar{R}_{aL}$	—	$\frac{R_L}{(2\beta/T)}\left[1+(2\beta/T)\frac{r_x}{R_L}\right]$	$R_L\left[1+(2\beta/T)\frac{r_x}{R_L}\right]$
Parallel capacitive	$\frac{1}{\bar{R}_a}\left[\frac{1+\tau'_L D}{1+\tau_L D}\right]$	$\frac{1}{\bar{R}_{aL}}\left[\frac{1}{1+\tau_L D}\right]$	$\frac{C_L R_L r_x}{\bar{R}_{aL}}$	$C_L R_L$	$r_x\left[1+(2\beta/T)\frac{R_L}{r_x}\right]$	$r_x\left[1+(2\beta/T)\frac{R_L}{r_x}\right]$

where g_v and h_v have been defined for the steady-state operation, in Chapter 6, and $\overline{Y}_a(D)$ is the frequency dependent part of the load transfer function.

(2) General expression for load current of transductor

$$i_L = \frac{g_v}{(1 + \tau_t D) + g_v h_v [\overline{Y}_a(D)]} \frac{\overline{Y}_{aL}(D)}{R_{aL}} E_c$$

(3) Load transfer functions are defined in terms of

$$(a) \quad \frac{\overline{Y}_a(D)}{\overline{R}_a} = \bar{i}_a/\bar{e}_a$$

and

$$(b) \quad \frac{\overline{Y}_{aL}(D)}{\overline{R}_{aL}} = \bar{i}_L/\bar{e}_a$$

Values of these are given in the Table on p. 230 for inductive and capacitive loading.

CHAPTER 9

HIGH SPEED TRANSDUCTORS AND MAGNETIC AMPLIFIERS

9.1 General Considerations

THE existence of appreciable time delays tends to limit the application of transductors as amplifiers in high-speed servos. In many such cases, however, the transductor is ideally suited in other respects and for this reason every effort has been made to develop transductor circuits which have a high speed of response but at the same time retain sufficient gain to render the devices useful as power amplifiers. A few of the circuits and techniques which have been developed will be described briefly in this chapter.

It has already been shown that in theory there is a basic relationship between power or voltage amplification and time-constant, and that by reducing the power amplification the response time can be made to approach a minimum value. In such cases adequate amplification can generally be regained by cascading two or more amplifiers. The overall amplifier response is then characterized by a number of short time-constants (one for each stage) instead of one long one. The cascading of short time-constants is often a more desirable arrangement, particularly when one large time-constant (e.g. a motor field) is already present in a servo loop.

With orthodox circuits such as those described in Chapter 4, the minimum response time for one stage may correspond to perhaps three or four cycles of the supply frequency, due to the flux damping caused by the auto-exciting windings, and no matter how much power amplification is sacrificed, the time-constant cannot be reduced below this value.

Broadly speaking, therefore, the high-speed circuits employ techniques which make it possible to sacrifice power amplification

in such a way as to produce the minimum possible time delay of one half-cycle of the supply frequency.

It has already been shown that for the auto-excited transductor working with the optimum a.c. excitation, the output voltage is proportional to the mean flux level ϕ_c in the two cores (section 3.13), and that this latter is raised or lowered by varying the control voltage. If there were no resistive voltage drops in the control circuit the mean flux level in any half-cycle would depend upon the magnitude of the control voltage and the time for which it had been applied. In these circumstances the transductor would behave as an integrator and the output would build up by equal increments in each half-cycle, following the application of a control voltage, until the transductor had become saturated throughout the gating half-cycle and the full output had been obtained.

These conditions would apply either if the control circuit resistance were zero or if the core permeability were infinite. Neither of these possibilities can arise, however, and in practice the resistive voltage drop in the control circuit increases as the control current builds up. It follows that the voltage actually applied to the control windings diminishes progressively in every half-cycle so that although a constant voltage may be applied continuously to the input terminals of the control circuit only a finite voltage-time integral is applied to the cores. Thus the mean output voltage increases at a diminishing rate until eventually it tends to the final steady-state value.

If, however, the output is considered at intervals immediately following the application of a control voltage, then over these early intervals the device will appear to be behaving as an integrator since the control current will not have had time to build up appreciably. Thus the output will be given very nearly by the time integral of the control voltage and in any one half-cycle can be determined for a given control voltage without any knowledge of the control current, control circuit resistance, or core permeability, provided they all fall within reasonable limits.

The various techniques employed to obtain a high speed of response achieve this by limiting the integration time of the control voltage to one half-cycle. It therefore follows that the steady-state mean output voltage of a high-speed transductor is comparable with the mean output voltage of a similar low-speed

transductor in the conducting period following the first half-cycle for which control is applied. The difference is that the latter continues to build up in subsequent half-cycles while the former is reset back to the same initial conditions at the end of every half-cycle.

It was shown in Chapter 3 that the gradual build-up of the output current extending over several half-cycles arose because the initial conditions at the beginning of any half-cycle depended upon the mean output in the previous conducting period. This in turn was shown to be due to the fact that flux changes in the unsaturated core were dependent upon the duration of saturation of the adjacent core.

By using only one core and arranging for this to be saturated at the end of every conducting half-cycle, the initial conditions at the beginning of every cycle would be the same, namely saturation. Thus, if a step voltage signal is applied at the beginning of one cycle, the next and subsequent cycles must be like that one since all start from the same initial conditions. Therefore the final mean output must be the same as the mean output of the first cycle.

9.2 Setting and Sampling of Flux in a Transductor

Referring back to *Figure* 3.3*e*, let us now consider the flux change occurring in one of the two cores throughout a cycle. At time t_{Q1} the flux in core A starts at the saturation value and the negative applied voltage-time is such as to cause the flux to decrease, eventually reaching the value ϕ_{Q2}. This may be termed the setting period. The core is now able to absorb a positive voltage-time integral before it saturates again. This is done in the following half-cycle when the total voltage applied to the core is larger than that during the setting half-cycle so that saturation is reached in less than one half-cycle at a time t_{q3}. This is the sampling period.

Thus the complete cycle consists of three distinct operations : firstly the setting, when the core flux is set by the total applied negative voltage-time ; secondly, the sampling when the flux is returned to saturation by applying positive voltage-time ; and, thirdly, the conduction when the supply voltage is ungated and applied to the external circuit. The duration of conduction depends upon the time taken to sample, which in turn depends upon the total flux set.

When using two cores, as discussed so far, the setting operation in one core coincides with sampling and conduction in the other and it has been shown how this modifies the setting process in the unsaturated core at time t_{q1}, t_{q2} . . . t_{qn}, thereby leading to a growth of flux which takes place over several cycles. By using a single core, however, the setting of flux may be completely confined to one half-cycle while sampling and conduction occur in the following half-cycle.

If only one core were used, as in a transformer, however, coupling would always exist between the a.c. and control windings (p. 19). This would result in the impedance of the latter always appearing across the a.c. windings, so disturbing the operation.

By using rectifiers and appropriate core materials it is possible to overcome this effect, as will be shown in the following sections.

It will be assumed that the hysteresis loop for the cores is such that the remanent flux is equal to the saturation flux as shown by the curve in *Figure* 3.1*d*. In this case the core flux will increase on applying positive voltage-time to the windings, saturation occurring for an m.m.f. F_s and flux Φ_s. On removal of the magnetizing field, however, the m.m.f. falls to zero but the flux remains at the saturation value and will not decrease until the m.m.f. has become slightly negative.

Such a characteristic is obtained with square loop materials such as HCR or Deltamax.

Thus, considering a coil wound on a square loop core, if the core is saturated due to an applied voltage, then on decreasing the voltage to zero the current also decreases to zero with the voltage. Such a core is incapable of energy storage and should be distinguished from one in which the remanence is low, in which case a positive current would continue to flow after the voltage had changed sign. The importance of this point will become evident after we have considered the operation of the rectifiers in the high-speed circuits.

Half-Wave Transductors

9.3 Single Core Transductor with Half-Wave Rectified Excitation

The circuit in *Figure* 9.1*a* shows a single core half-wave transductor consisting of a single mutual inductor, one winding of which is connected to the a.c. supply through the load and a

half-wave rectifier, the other winding being connected to the control source. When the core is unsaturated and the rectifier conducting, the circuit may be represented by that shown in *Figure* 9.1*b* provided the turns on the two coils are equal. When the core is saturated the equivalent circuit is as in *Figure* 9.1*c*.

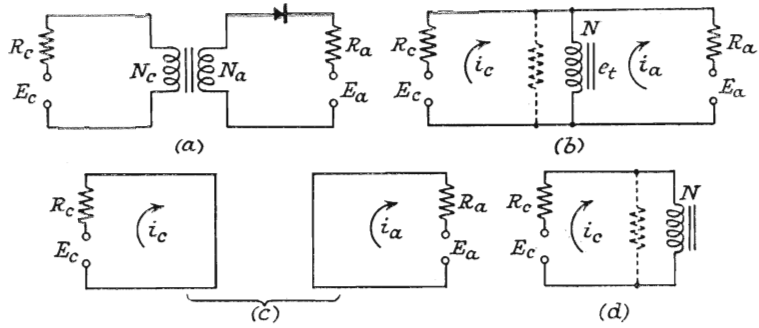


Figure 9.1. Transductor with half-wave rectified excitation.

(*a*) Basic circuit.
(*b*) Equivalent circuit when core is unsaturated and rectifier conducting (condition applies to $N_a = N_c = N$).
(*c*) Equivalent circuit when core is saturated.
(*d*) Equivalent circuit when core is unsaturated and rectifier non-conducting.

Referring to *Figures* 9.1*b* and *c*, if it is arranged that during positive half-cycles of the supply voltage the rectifier conducts and the core flux increases, then during negative half-cycles the supply is isolated from the coil, and flux changes which occur are due only to the control voltage (see *Figure* 9.1*d*).

In order to investigate this circuit there are three conditions to consider :

Case (*a*). *Core Unsaturated, Rectifier Conducting* (*Figure* 9.1*b*). Since the reactance of the coil is large compared with the circuit resistances we have :

$$i_a = i_c = \frac{E_c + E_a}{R_c + R_a} \qquad . \quad . \quad . \quad (9.1)$$

and writing $E_a = R_a i_a + e_t$

gives $$e_t = N\frac{d\phi}{dt} = \left[\frac{E_a R_c}{R_a + R_c} - \frac{E_c R_a}{R_a + R_c}\right] \quad . \quad . \quad (9.2)$$

This mode of operation applies to the sampling period.

Case (*b*). *Core Saturated, Rectifier Conducting* (*Figure* 9.1c).

$$i_a = \frac{E_a}{R_a} \qquad . \quad . \quad . \quad (9.3)$$

$$i_c = \frac{E_c}{R_c} \qquad . \quad . \quad . \quad (9.4)$$

This applies to the conducting period.

Case (*c*). *Core Unsaturated, Rectifier Non-conducting* (*Figure* 9.1*d*). Once again the impedance of the coil is large compared with the resistance and hence

$$E_c = -N\frac{d\phi}{dt} \qquad . \quad . \quad . \quad (9.5)$$

This applies to the setting period.

Let us now consider this circuit on the basis of a square wave excitation voltage of peak value $\hat{E}_a$ as used in section 3.4.

In the absence of any control voltage the supply voltage is assumed to be sufficient to cause the flux to change from negative to positive saturation when applied for a time $T/2$. Therefore, when the supply voltage is switched on it causes saturation in the first positive half-cycle. During negative half-cycles the rectifier is cut off, therefore the core remains saturated the whole time, the flux returns to the remanent point at the end of every positive half-cycle, and a load current $i_a = \frac{\hat{E}_a}{R_a}$ flows throughout each positive half-cycle. This state of affairs applies so long as the control voltage is zero, and is shown over the period from 0 to $T/2$ in *Figure* 9.2. If a control voltage is applied at time $T/2$ say, then during the negative half-cycle of supply voltage a flux is set. The flux decreases linearly with time at a rate E_c/N (equation 9.5 and *Figure* 9.2*c*) until the end of the half-cycle, at which point the total flux set is ϕ_{Q1}, having departed from positive saturation by an amount $\frac{E_c}{N} \cdot \frac{T}{2}$. This corresponds to the end of the first setting period. The flux change over this period is proportional to the negative shaded area (*Figure* 9.2*a*), which corresponds to the induced voltage e_t in the coil during the setting period. The unshaded area in *Figure* 9.2*a* over this period is numerically equal to $|\hat{E}_a| - |e_t|$ which is the reverse voltage on the rectifier. During the following positive half-cycle the flux reverses and once again proceeds towards saturation at a rate

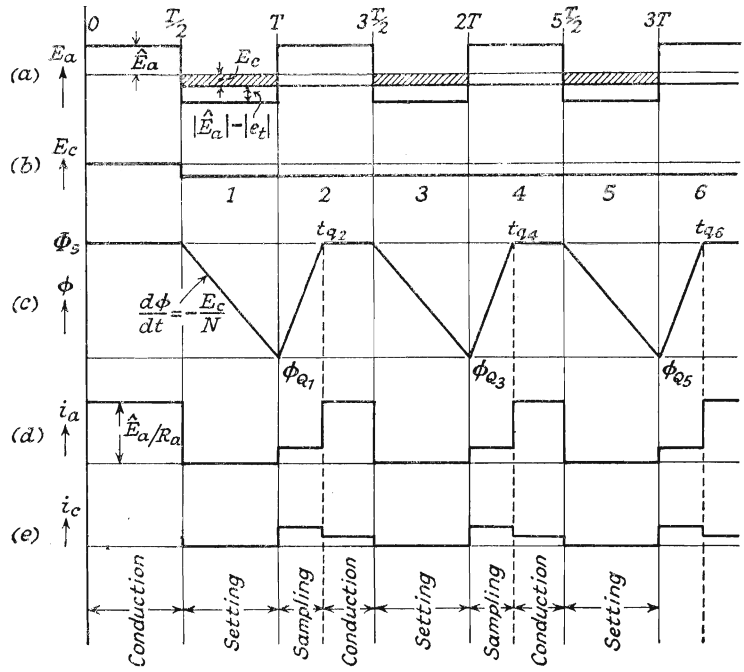


Figure 9.2. Waveforms of voltage, flux and current for a transductor with half-wave rectified excitation and an idealized core.

(*a*) Supply voltage. (*b*) Control voltage. (*c*) Flux changes. (*d*) Load current. (*e*) Control current.

given by equation 9.2. During this sampling period, coupling occurs between the control circuit and load circuit and currents flow in these circuits according to equation 9.1 and *Figures* 9.2*d* and *e*. When the core saturates at t_{q2}, the values of load current and control current are then determined solely by the respective circuit resistances (equations 9.3 and 9.4).

Thus, the rate of flux setting is proportional to the signal voltage ; this determines the total flux set ϕ_Q, which in turn fixes the time of saturation in the sampling period and therefore the duration of conduction. Therefore, by varying the signal voltage applied to the core, the mean load current flowing in the a.c. circuit is subjected to a corresponding variation.

It is immediately evident that this arrangement settles down to a steady-state much more quickly than the double cored transductor. This is because the initial flux at the beginning of every setting period is always at the saturation value Φ_s, and therefore

the duration of any conducting period is independent of previous conducting periods and dependent only upon the control voltage.

In the particular example given, the full response is obtained in the second half-cycle after the control voltage is applied.

Although the excitation circuit is isolated during the setting process, coupling occurs during sampling which causes a control current to flow. In the present ideal case this control current represents the only drain on the control source and in theory it may be made as small as we please by making the control circuit resistance sufficiently large. This will not affect the value of the load current during conduction which is independent of the control circuit since the core is saturated.

When the control circuit resistance is increased indefinitely, however, we are no longer justified in assuming ideal core characteristics, and at this stage it is important to realize that a finite control current is necessary to set the flux. This being the case, it is not possible to increase R_c above the limit imposed by this restriction. Otherwise the actual setting voltage at the coil terminals would be reduced to a small value and the sensitivity correspondingly reduced, due to the comparatively large amount of power drawn from the control source and dissipated in the control circuit resistance. Even more significant than the magnetizing current, however, are the losses due to eddy currents, which may be represented by the resistance shown dotted in *Figures* 9.1*b* and *d*. These losses also draw current which causes a fall in potential along R_c and therefore reduced sensitivity to a control voltage.

The essential function of the control voltage, namely setting the flux, is only performed during negative half-cycles of the supply voltage. It therefore follows that control could be effected by means of an alternating voltage synchronized with the supply and in phase with it. This is a convenient way of using this type of transductor when the signal source gives an a.c. output since it avoids rectification. Such outputs are obtained from magslips, synchros, a.c. transducers, etc.

The core characteristics and iron losses limit the power amplification of this circuit to 30–50. A balanced version [9(a)] of the circuit having more practical application is described in section 10.21.

9.4 The Ramey Circuit [9(b)]

It was seen in the previous section that the control voltage performs no useful function during the sampling and conducting

periods. Furthermore, in the ideal case, no control current is required during the setting periods (since the constant component corresponding to the width of the hysteresis loop may be supplied from a constant current bias supply). During sampling and conduction, however, even in the ideal case an induced current

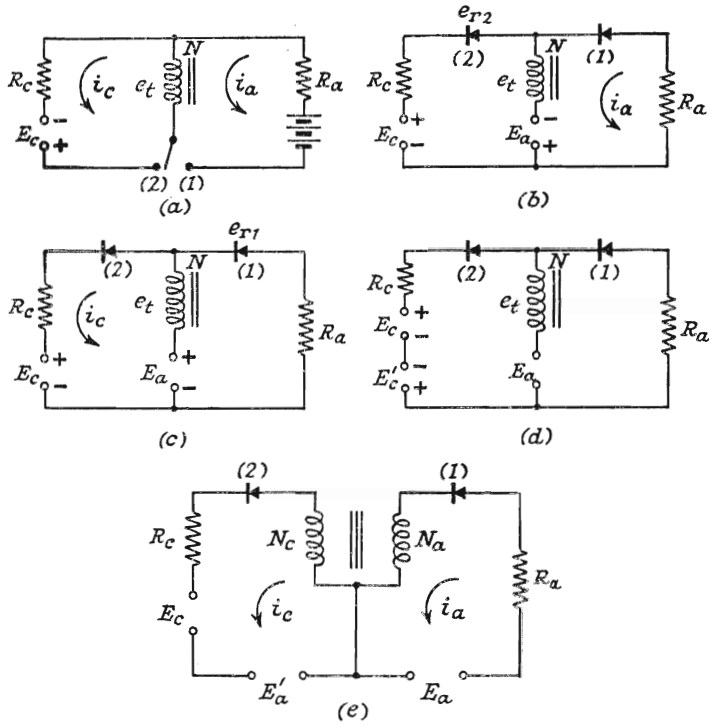


Figure 9.3. Half-wave transductors in which sampling and setting processes are completely isolated.

(*a*) Equivalent of Ramey circuit using mechanical chopper.
(*b*) Ramey circuit for half-wave operation (positive half-cycles).
(*c*) Ramey circuit for half-wave operation (negative half-cycles).
(*d*) Circuit showing use of voltage bias.
(*e*) Ramey circuit with unequal turns ratio.

flows in the control circuit and this was reduced by connecting a high resistance in series with the control source. Under ideal conditions this resistance does not dissipate power during the setting period since no magnetizing current is required.

In practice, of course, this is not the case and therefore on these grounds a low control circuit resistance is desirable to ensure that the whole control voltage is applied directly to the windings to

give the maximum possible power amplification. An ingenious arrangement which allows this without causing large induced control currents during the sampling period has been described by Ramey.

This is achieved in principle by isolating the supply voltage during setting periods as previously, but in addition isolating the control voltage during sampling and conduction. A simple circuit used by the author for investigations on the Ramey amplifier will now be considered.

Referring to *Figure* 9.3*a*, the control voltage E_c and the supply voltage E_a are both supplied from d.c. sources and are each in series with resistors R_a and R_c, representing the load and control circuit resistances respectively. The circuits are alternately connected in series with the coil by a continuously vibrating relay, which closes the load circuit when contact (1) is made and the control circuit when contact (2) is made. It is assumed that the changeover time is comparatively small, and that both contacts are made for equal periods. When the relay is at position (1) the supply voltage is applied to the load circuit and the core flux increases to the saturation value, after which the load current suddenly increases being limited only by the load resistance. When the contact is broken the load current drops to zero and the core remains at positive saturation. Immediately afterwards contact (2) is made and the flux decreases at a rate E_c/N during the setting period. The total flux change is $\frac{E_c}{N}\frac{T}{2}$, where T is the period of the relay, and therefore the amount of voltage-time stored in the core from the saturation datum is determined solely by the control voltage E_c since T is constant. At the end of the setting period, the contact (2) is broken and (1) is made. During the transit time of the relay the flux remains at the level set by the control voltage after which the core now absorbs positive voltage-time during the sampling period and eventually saturates once more. The rate of flux increase during sampling is E_a/N and this is constant, therefore the time taken to sample depends only upon the value to which the flux is set. No current is ever drawn from the control source excepting that required to magnetize the core and, in the ideal case of a **B–H** curve having vertical sides the control current will be constant, the current gain infinite and hence the power gain infinite. Furthermore, the response time is equivalent to one half-cycle and the mean value of the load

current is proportional to the control voltage. The usual form of curve obtained when the control voltage is plotted against the mean load current is shown in *Figure* 9.4*a*. When the control voltage is zero, the flux is never set from the saturation value and therefore the mean load current is at its maximum value. The current decreases as the control voltage is increased, and since in this case the turns ratio is unity the load current is zero when the control voltage has the same magnitude as the supply voltage.

In the more practical form of the circuit, the switching action

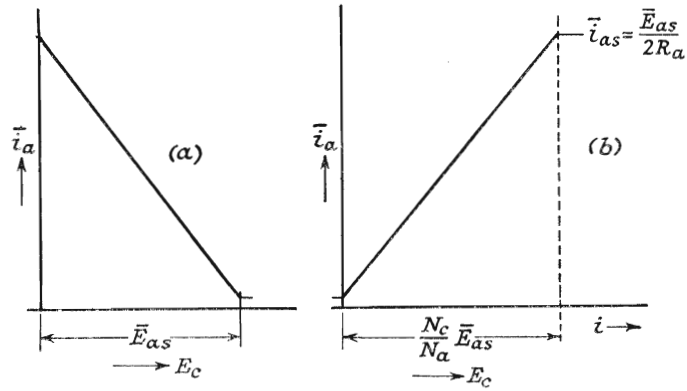


Figure 9.4. Control characteristics of half-wave transductors (*a*) using mechanical chopper ; (*b*) half-wave Ramey circuit.

is obtained by using a combination of rectifier elements and an alternating voltage (*Figure* 9.3*b*). Once again the square waveform will be considered for descriptive simplicity.

A square wave voltage of peak value $\hat{E}_a$ is supplied continuously to the coil as shown in *Figure* 9.3*b*, where the polarity of the supply is shown for a positive half-cycle of voltage. In these circumstances a current flows through the resistance R_a and rectifier (1). During this period, assuming a square loop core characteristic, rectifier (2) will be cut off and the left-hand circuit therefore open. The applied voltage will cause the core flux to increase to positive saturation after which the current i_a will increase, being limited only by the load and circuit resistance R_a. At the end of the positive half-cycle the voltage suddenly drops to zero and, since the core is incapable of energy storage, the current simultaneously becomes zero. During the negative half-cycle *Figure* 9.3*c* is applicable, rectifier (1) is cut off and,

provided $|\hat{E}_a| > E_c$, rectifier (2) conducts. The load circuit is now open and the voltage applied to the coil is negative and has a magnitude $|\hat{E}_a| - E_c$. This causes the flux to decrease at a constant rate until the end of the negative half-cycle.

The alternating voltage E_a should be given such a value that, for $E_c = 0$, the core flux varies exactly between positive and negative saturation, the former occurring by conduction through the load circuit and the latter by conduction through the control circuit. This state of affairs is shown by the waveforms in *Figure* 9.5 up to a time $T/2$. At this instant it is assumed that a constant control voltage is switched on. During flux setting periods (negative half-cycles) the control voltage reduces the total negative voltage-time applied to the core, as shown by the shaded area in *Figure* 9.5*c*, and therefore by the end of the setting process the flux falls short of negative saturation at ϕ_Q by an amount $\frac{E_c}{N}\frac{T}{2}$ (*Figure* 9.5*e*). It follows that on the next positive half-cycle, saturation must occur before the end of the half-period at t_q (*Figure* 9.5*c* and *e*) where t_q will be fixed by the fact that the positive and negative flux changes must be equal. Control of this circuit is therefore effected by varying the control voltage E_c and complete isolation is achieved during the setting and sampling processes. It should be noted that no induced currents can flow in the control circuit and hence the current in the control circuit is that required to magnetize the core and supply losses during the setting period. It should also be noted that the control voltage does not set the core flux positively, but rather prevents the alternating voltage E_a from setting to the full extent. For this reason, if a magnetizing current flows in the control circuit during setting, its direction is opposed to the control voltage.

It thus appears that power is absorbed by the control voltage source and the input power therefore negative. These considerations, however, are not significant, for if one considers an arrangement whereby the control voltage E_c is maintained at a constant value being considered part of the supply and the actual control is effected by a second steady voltage E'_c, as in *Figure* 9.3*d*, the control current then flows in the same direction as the control voltage E'_c.

The circuit in *Figure* 9.3*b* or *c* has a control characteristic of the form shown in *Figure* 9.4*b*, while the alternative arrangement with an extra control voltage E'_c has the same characteristic

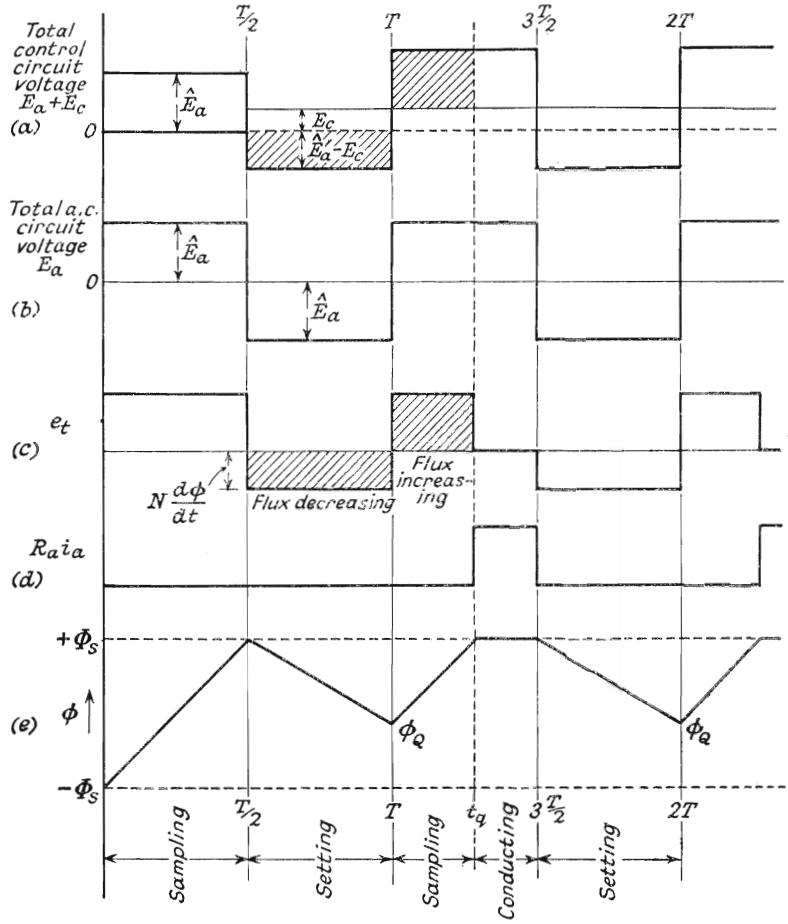


Figure 9.5. Voltage current and flux waveforms for half-wave Ramey circuit using square-wave excitation.

(*a*) Voltage applied to control circuit $E_a + E_c$.
(*b*) Voltage applied to load circuit E_a.
(*c*) Coil voltage e_t.
(*d*) Load voltage $R_a i_a \propto$ load current
(*e*) Core flux Φ.

as that obtained using the mechanical vibrator (*Figure* 9.4*a*). The important feature about this arrangement is the way in which the rectifiers cut off at the appropriate times to enable the sampling and setting processes to be isolated. This is dependent upon the fact that, due to the rectangular hysteresis loop of the core material, no current is maintained at the end of a half-cycle. It is

also dependent upon the existence of reverse voltages e_{r1} and e_{r2} of sufficient magnitude appearing across the rectifiers at the appropriate times. These reverse voltages will now be considered.

On positive half-cycles (*Figure* 9.3*b*) we have for the load circuit :

$$E_a - e_t = R_a i_a$$

and for the control circuit :

$$E_a + E_c - e_t = e_{r2}$$
(i_c assumed negligible)

Thus

$$e_{r2} = E_c + R_a i_a \qquad . \quad . \quad . \quad (9.6)$$

On negative half-cycles (*Figure* 9.3*c*) for the control circuit :

$$E_a - E_c - e_t = R_c i_c$$

and for the load circuit :

$$E_a - e_t = e_{r1}$$
(i_a assumed negligible)

Thus

$$e_{r1} = E_c + R_c i_c \qquad . \quad . \quad . \quad (9.7)$$

Since the currents flowing are very small during unsaturated periods, the reverse voltage is substantially equal to the control voltage (equations 9.6 and 9.7). During saturated periods the control rectifier experiences a reverse voltage equal to the control voltage plus the load voltage (equation 9.6).

This brings out the important point that the control voltage must be unidirectional. If, for example, the control voltage were alternating and anti-phased to the supply, *Figure* 9.3*c* would still be applicable to setting half-cycles and the load rectifier would be cut off according to equation 9.7. During sampling periods (*Figure* 9.3*b*), however, the control voltage would change sign and therefore the *reverse* voltage on rectifier (2) would be negative (equation 9.6) ; this rectifier would therefore conduct. Thus the control rectifier would conduct during setting *and* sampling and the circuit would be equivalent to the one using half-wave rectified excitation (section 9.3) with its inherent disadvantages.

Apart from this requirement no other restrictions are placed on the control voltage. To obtain linearity, however, the latter must be of the same form as the supply voltage would be if rectified. Thus if the alternating voltages are sinusoidal the control voltage should be a rectified sine wave, while if the alternating

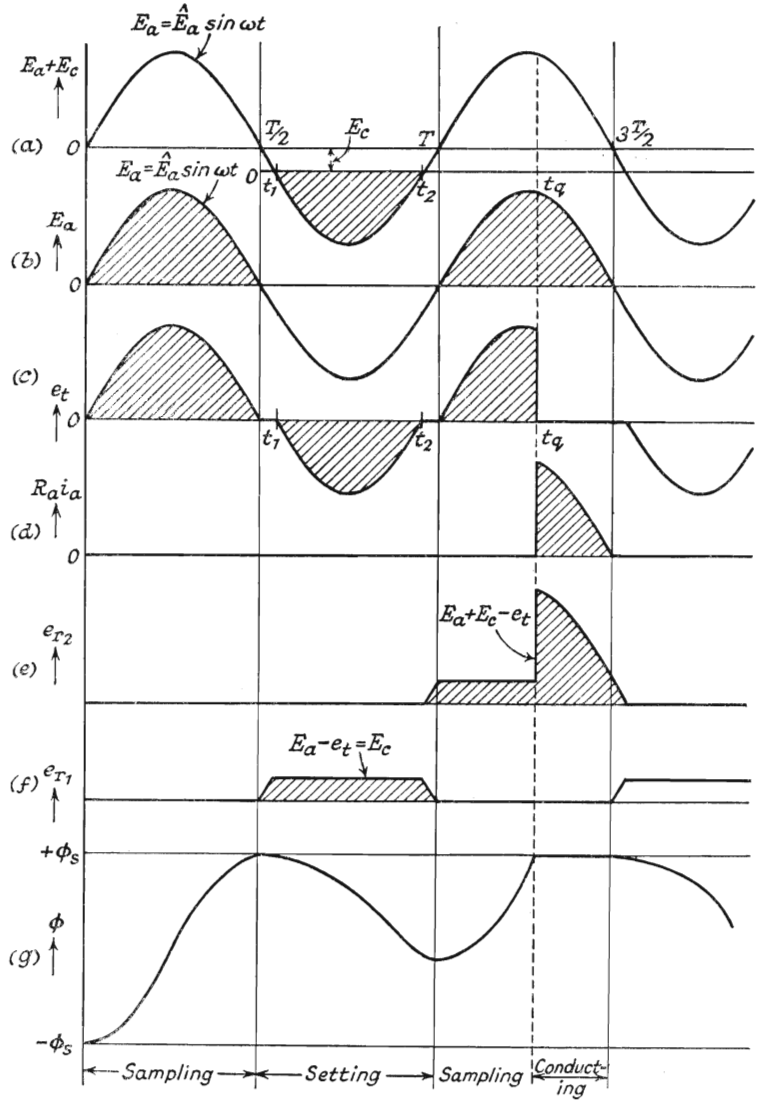


Figure 9.6. Voltage current and flux waveforms for half-wave Ramey circuit using sinusoidal excitation.

(*a*) Voltage applied to control circuit. (*b*) Voltage applied to load circuit. (*c*) Coil voltage. (*d*) Load voltage. (*e*) and (*f*) Rectifier voltages. (*g*) Core flux.

voltages are square waves, the control voltage should be a steady direct voltage. Otherwise, although the circuit operates, it will tend to be non-linear.

Figure 9.6 shows the waveforms applicable to the case where the alternating voltages are sinusoidal, the magnetizing currents are neglected and the control is effected by a steady direct voltage applied at time $T/2$. In *Figures* 9.6*a* and *c*, it is seen that flux setting occurs over the period t_1 to t_2. In *Figure* 9.6*c* the total flux set is proportional to the shaded area over this interval, which is not directly proportional to the control voltage (see *Figure* 9.6*a*). The flux sampled is equal and opposite to the flux set, and is given by the shaded area between T and t_q.

9.5 Voltage Amplification

These circuits have all been considered on the basis of a single coil replacing a mutual inductance. This has implied equal turns on the control and a.c. coils.

In the general case, where the control turns N_c and a.c. turns N_a are unequal, it is necessary to apply two alternating voltages E_a and E'_a as shown in *Figure* 9.3*e*. Thus referring to *Figure* 9.5*a*, *c* and *e* it is seen that the total *decrease* of flux during setting from $T/2$ to T is equal to

$$\frac{1}{N_c}\int_{T/2}^{T} e_t dt = \frac{1}{N_c}\int_{T/2}^{T}\Big[\,|\,\hat{E}'_a\,| - E_c\Big]dt.$$

The total *increase* of flux during sampling from T to t_q

$$= \frac{1}{N_a}\int_{T}^{t_q} e_t dt = \frac{1}{N_a}\int_{T}^{3T/2}\Big[\,|\,\hat{E}_a\,| - |\,i_a R_a\,|\,\Big]dt.$$

(This follows from *Figures* 9.5*b*, *c* and *d*.)

Since the total flux sampled must be equal to the total flux set. the above two equations may be combined and putting $\hat{E}'_a = \frac{N_c}{N_a}\hat{E}_a$

gives
$$\frac{1}{N_c}\int_{T/2}^{T} E_c dt = \frac{R_a}{N_a}\int_{T}^{3T/2} |\,i_a\,|\,dt \qquad . \; . \; . \quad (9.8)$$

The left-hand side of equation 9.8 may be integrated directly.

When considering the load current, however, it must be remembered that this only flows every other half-cycle and the right-hand integral is therefore the mean load current multiplied by the total period T, thus :

$$\frac{E_c}{N_c} \cdot \frac{T}{2} = \frac{R_a \bar{\imath}_a}{N_a} T \qquad . \quad . \quad . \quad (9.9)$$

giving

$$\bar{\imath}_a R_a = \frac{1}{2} \frac{E_c N_a}{N_c} \qquad . \quad . \quad . \quad (9.10)$$

Thus two facts emerge from this : firstly the alternating voltage E_a' in the control circuit must be adjusted to the value $E_a N_c/N_a$; the two supply voltages thus correspond to equal and opposite flux changes. Secondly, under the above conditions the voltage amplification is equal to one-half the turns ratio, i.e. $N_a/2N_c$.

The power amplification of the half-wave circuits is limited by the losses in the core, the current required to magnetize the core and the ability of the rectifiers to block at the appropriate times.

It is usually considerably less than that obtainable with the slow-response transductors and is of the order of 100 with existing core materials.

9.6 Response Time

Although the time taken to achieve full response is normally quoted as one cycle, it should be noted that the control voltage will not usually be applied exactly at the beginning of a setting period. Full response will in fact be obtained in the first conducting period after the control voltage has been applied for a whole setting period. Thus if the control voltage is applied in the first half-cycle just after the commencement of a setting period, then the full output will appear in the fourth half-cycle. In this respect the response is random within two cycles, depending upon the time when the input signal changes.

Full-Wave High-Speed Transductors

Up to the present, the high-speed circuits considered have been characterized by a half-wave mode of operation whereby the load circuit was ungated only once every cycle, thereby leading to an output having a very poor form factor, a response time which could be as high as two cycles and a maximum possible voltage gain equal to one-half the turns ratio. Full-wave circuits will now be described which enable a considerable all-round improvement to be realized.

Hitherto the *change* of control current required to produce a given flux change has been neglected in the high-speed circuits by assuming that the hysteresis loop has sides of infinite slope. It will now be assumed that they have a certain finite slope leading to a linear relationship between flux and control current over the operating range (*Figure* 9.7*c*).

9.7 The Full-Wave Ramey Circuit [9(b]

The diagram in *Figure* 9.7*a* shows the circuit of a full-wave transductor having half-cycle response in which the switching action is achieved by using two synchronized relays driven from some independent alternating supply. It will be assumed that the **B–H** curve for the core materials is of the form shown in

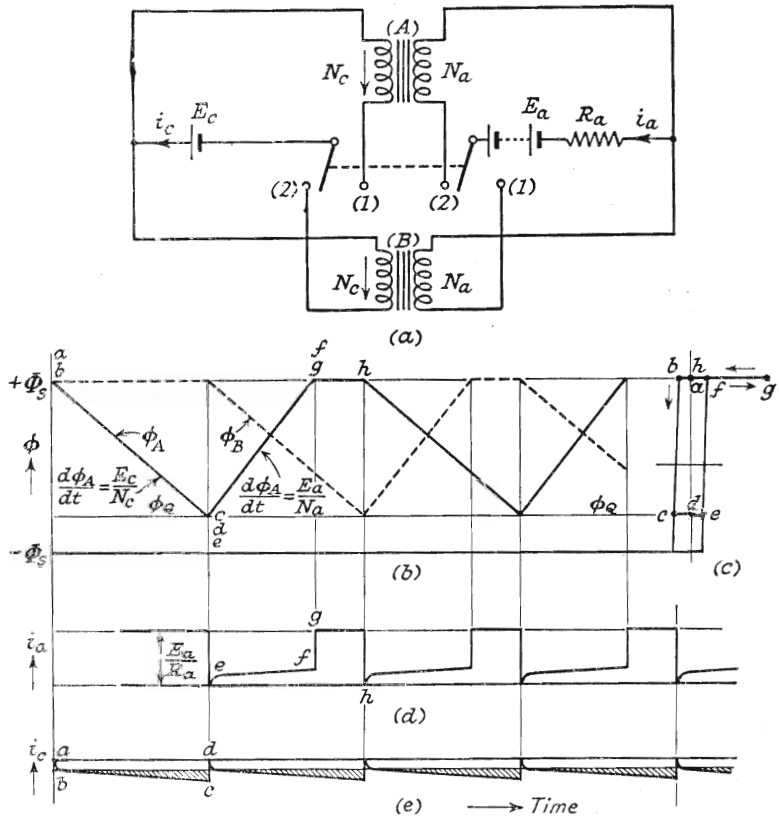


Figure 9.7. Full-wave high-speed circuit using relay switching.
(*a*) Circuit arrangement. (*b*) Core fluxes Φ_A and Φ_B. (*c*) Hysteresis loop for cores. (*d*) Load current i_a. (*e*) Control current i_c.

Figure 9.7*c*. As before, this type of magnetization loop is characterized by a remanent flux which is equal to the saturation flux and a saturation which is sharply defined. This implies that the core flux is able to remain at any value up to saturation in the absence of an m.m.f. The effect of departures from these conditions will be mentioned later.

In the present circuit, the control voltage is E_c and the supply voltage E_a (*Figure* 9.7*a*), both of which are unidirectional and smooth. The two relay armatures are continuously vibrating and are synchronized so that, for example, both contacts (2) are broken simultaneously after which both contacts (1) are made simultaneously, the time of dwell on contacts (1) being equal to the time of dwell on contacts (2). Consider the case of both cores initially at positive saturation denoted by the point (*a*) on the **B–H** curve in *Figure* 9.7*c*, the relay armatures first move to the contacts (1). The control voltage E_c is applied to the control winding on core A and the supply voltage E_a is applied to the load winding on core B. The control current instantaneously assumes the value corresponding to the point (*b*) on *Figures* 9.7*c* and *e* and thereafter increases linearly with time in the negative direction as the flux decreases at a rate nearly equal to E_c/N_c (*Figure* 9.7*b*). Meanwhile the supply voltage acts in such direction as would cause an increasing flux in core B ; since this core is already saturated, however, no flux change can occur and the whole voltage is applied to the external circuit resistance R_a throughout the first half-cycle. The relay armatures remain at the contacts (1) for one-half period of the relay by which time the flux in core A has been set to the value ϕ_Q point (*c*). When the contacts (1) are broken, the load current decreases instantaneously to zero and core B remains saturated. The control current also decreases instantaneously to zero (point (*d*)), the flux remaining at the set value ϕ_Q. Contact (2) is made immediately after contact (1) is broken and the supply voltage is applied to the load winding on core A. The load current instantaneously increases to the small value corresponding to point (*e*) after which the flux increases at a rate approximately equal to E_a/N_a during the sampling period until point (*f*) is reached. At this stage core A saturates and the supply voltage is ungated and transferred to the load (*g*). The duration of the conducting period depends upon the time taken to sample, which in turn depends upon the total flux set in the previous half-cycle. Furthermore, since for a given

control voltage the flux is always set at a constant rate from the saturation value, the final set value is independent of events occurring in previous half-cycles and therefore full response is attained in the first conducting period after the control voltage has been applied for a full half-cycle. The operation is almost identical with that described for the half-wave circuit and the sequence of events can be followed from *Figure* 9.7.

The more practical form taken by this circuit achieves the switching action by using rectifiers and alternating voltages. The

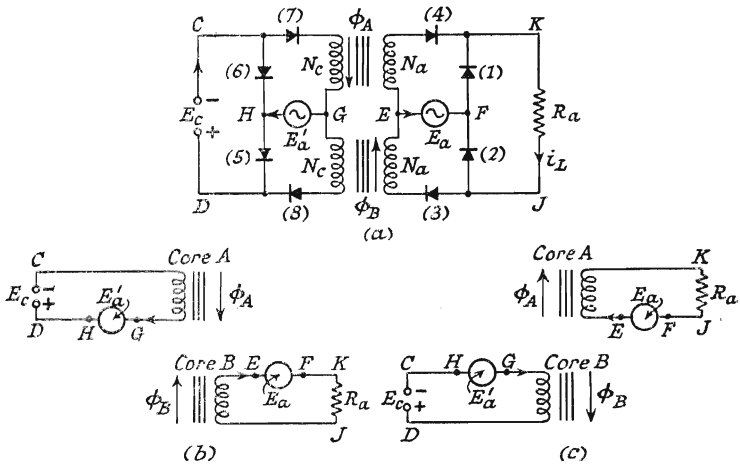


Figure 9.8. The Full-Wave Ramey circuit.
(*a*) Complete circuit.
(*b*) Equivalent circuit (positive half-cycles).
(*c*) Equivalent circuit (negative half-cycles).

arrangement is shown in *Figure* 9.8*a*, and it is seen that the load circuit is identical to that of a bridge-connected auto-excited transductor (*Figure* 5.1*c*). Thus the alternating supply voltage E_a is applied across the points EF and the total circuit resistance is lumped in R_a which is shown connected across the terminals JK. The control circuit now takes the same form as the load circuit, having an alternating voltage E'_a applied across HG and the control voltage E_c across CD. The directions of the arrows GH and EF represent a positive half-cycle of the voltages E_a and E'_a which are in phase and may be obtained from isolated windings on the same supply transformer.

The voltages E'_a and E_a should be adjusted to such a value that

each one produces the maximum flux change of $2\Phi_s$ when applied to its appropriate coil for one half-cycle. It therefore follows as before that $E'_a/N_c = E_a/N_a$.

If the control voltage E_c is equal to zero then, provided the rectifiers are ideal and the hysteresis loop for the cores is characterized by 100% remanence, the core fluxes will be decreased from $+\Phi_s$ down to $-\Phi_s$ due to the voltage E'_a and restored to positive saturation by the voltage E_a.

When the voltages E'_a and E_a are positive the rectifiers 1, 3, 5 and 7 are conducting, therefore the circuit may be simplified to that shown in *Figure* 9.8*b*. Similarly, when the alternating voltages become negative the rectifiers 2, 4, 6 and 8 are conducting so that *Figure* 9.8*c* applies.

In both cases it is seen that currents flowing through the load and the control source are unidirectional. When a control voltage is applied to the transductor it always acts in such a direction as to reduce the magnitude of the total voltage applied to the control coils for setting the flux. The magnitude of this voltage is $|E'_a| - E_c$ and, provided $|E'_a| > E_c$, its direction is such as to cause a decreasing flux. Thus it is seen that the rectifiers switch the circuit as though the coils were resistive (p. 235), thereby leading to the ideal mode of operation where a core is only associated with one circuit at a time. This purely resistive action is a direct result of using square loop cores since no flux change occurs when the magnitude of the m.m.f. decreases to zero either from positive saturation or from the negative set flux ϕ_Q (see section 9.9).

As in the half-wave cases the operation is similar when using sine waves instead of square waves, and the operation tends to be non-linear if a sine wave of supply is used with a steady direct control voltage.

9.8 Power Amplification

The amplification of the full-wave circuit may easily be calculated for the case where the core hysteresis loop has a finite slope in the unsaturated region. It will be assumed that this slope corresponds to inductances L_{cw} and L_{aw} for each control winding and a.c. winding respectively.

When considering the gain of the high-speed circuits it must be remembered that the control voltage is not being applied continuously but is being switched from one core to the other, commencing with the same initial conditions each time. There-

fore the mean control current is never able to build up to the value E_c/R_c but is determined largely by the effective *inductance* and iron losses associated with the control circuit.

If we consider the transductor circuit in *Figure* 9.7, the mean current amplification can be determined on the basis of zero iron losses. Referring to the waveforms in *Figure* 9.7*e*, the control current contains a constant component *ab* and a variable one. If the control voltage is increased the rate of increase of the variable component during the setting half-cycle becomes greater with the result that the mean value of the control current considered over one-half period increases. The constant component can be supplied from a d.c. bias supply of high impedance and therefore when considering the current amplification we are interested only in the mean value of the variable component corresponding to the shaded area in *Figure* 9.7*e*. The change of control current over one half-cycle, associated with the flux change from saturation down to the minimum value ϕ_Q, is equal to $\frac{E_c}{L_{cw}} \cdot \frac{T}{2}$ where L_{cw} applies to one control coil. Therefore the mean value of the variable component of control current over one half-cycle corresponding to the shaded area is given by

$$\bar{\imath}_c = \frac{E_c}{2L_{cw}} \cdot \frac{T}{2} \qquad \ldots \quad (9.11)$$

Also the voltage amplification is obtained by the same method as for the half-wave circuit except that the factor of one-half disappears due to the full-wave operation thus :

$$\frac{\bar{\imath}_a R_a}{E_c} = \frac{N_a}{N_c} \qquad \ldots \quad (9.12)$$

Combining this with equation 9.11 and putting $T/2 = 1/(2f)$ gives a current gain equal to g_i

where $$g_i = \frac{2}{\pi} \frac{\omega M_{ca}}{R_a} \qquad \ldots \quad (9.13)$$

Thus if the effect of losses is neglected the expression for the current amplification is the same as that obtained for the fully self-excited transductor (Chapter 4). This of course does not imply that the mean power amplification is the same since in the high-speed transductors the control voltage levels are considerably greater.

It has already been shown that the voltage amplification is

given by the turns ratio N_a/N_c (equation 9.12), and this result may be combined with the current amplification to give the mean power amplification thus :

$$G_p = \frac{\bar{i}_a^2 R_a}{E_c \bar{i}_c}$$

$$= \frac{\bar{i}_a}{\bar{i}_c} \cdot \frac{\bar{i}_a R_a}{E_c}$$

$$= g_i \frac{N_a}{N_c} \quad . \quad . \quad . \quad (9.14)$$

Therefore $$G_p = \frac{2}{\pi} \frac{\omega L_{aw}}{R_a} \quad . \quad . \quad . \quad (9.15)$$

The last two expressions will be recognized as giving the ampere-turns ratio, and it is interesting to note that in equation 8.4 this is equal to the power amplification of the low-speed transductor when the time constant is one half-cycle.

In the auto-excited transductors the control voltage is equal to $\bar{i}_c R_c$ which can be made small by reducing R_c to a minimum. In the present circuits, however, the control current is virtually being switched on every half-cycle from the same initial conditions each time and therefore its mean value is limited by the inductance (equation 9.11).

The control voltage necessary to maintain a given mean control current must therefore overcome the induced e.m.f. due to the flux changes as well as the resistance. The effect of the latter is generally small compared with the inductive effect and thus the control voltages associated with the high-speed transductor are considerably greater than those required for low-speed operation.

In view of this it is not strictly true to refer to the power amplification since the greater part of the control circuit impedance is now reactive and therefore the power amplification G_p is better defined as the ratio of the mean output volt-amperes to the mean control volt-amperes.

In practice iron losses may account for an appreciable increase of control current and these, coupled with the non-linearities in the unsaturated region of the hysteresis loop, often make it difficult to predict the exact magnitude of the control current under any given set of conditions.

In practice other effects also arise which cause the transductor characteristics to depart appreciably from the ideal case ; such effects will now be considered.

9.9 Remanent Flux Less than 100%

Hitherto it has been assumed that at the end of a conducting period the current in the load windings decreases to zero with the core still saturated, and in order to produce a decrease of flux it is first necessary to apply a negative magnetizing force of magnitude F_{ab} (*Figure* 9.9*a*). Similarly, when the flux level is at *c*, the m.m.f. must be increased by an amount F_{ce} to the positive value corresponding to F_{de} before the flux will increase. This pronounced hysteresis effect makes the core incapable of energy storage since energy is necessary to make the flux increase or decrease, whereas

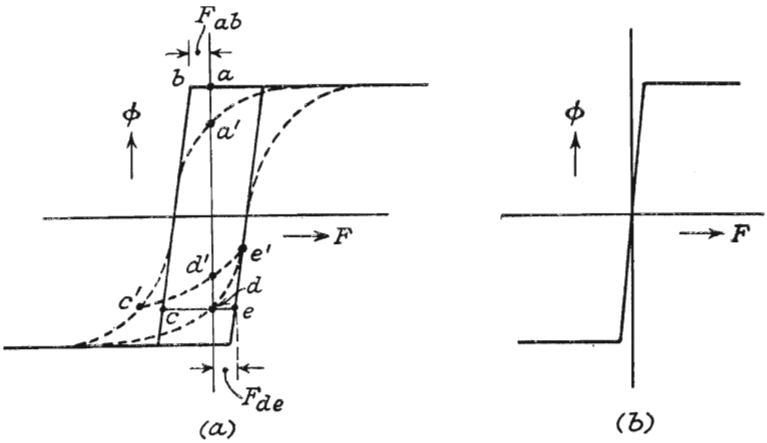


Figure 9.9. (*a*) Hesteresis loops showing effect of incomplete remanence. (*b*) Curve without hysteresis.

the alternative characteristic shown in *Figure* 9.9*b* exhibits no hysteresis and all the energy required to increase the flux is recovered when the flux returns to its original value.

There are three main core requirements for the type of transductor so far discussed in this chapter. Firstly 100% remanence, secondly minimum eddy current losses and thirdly high slope in the unsaturated regions. Unfortunately, in practice, the materials which best fulfil the two latter requirements do not exhibit 100% remanence. In such cases the remanent flux is appreciably less than the saturation flux so that the flux decreases to some point *a'*, *Figure* 9.9*a*, as the current tends to zero. Furthermore, this flux change produces an e.m.f. which tends to maintain the current in the load circuit after the supply voltage has changed sign. The

drooping flux characteristic combined with a slightly lagging current tends to reduce the voltage and power amplification. The effect of the lagging load current gives rise to a short overlap interval during which the load circuit rectifiers have not cut off when the corresponding control circuit rectifiers become conducting. Therefore, over this interval normal transformer coupling occurs between the control circuit and the load circuit, during which induced currents could flow in the control circuit and thereby reduce the effective power amplification.

A more serious effect, however, is the premature setting of the core flux to the value a' (*Figure* 9.9*a*).

This reduces the voltage amplification for large outputs but has less effect on lower parts of the voltage control characteristic. For the case of 100% remanent cores, if the control voltage E_c were equal to E'_a, then during a setting half-cycle the flux would remain at positive saturation. Therefore the supply voltage E_a would be available to drive current through the load throughout the following half-cycle and the mean load current would have its maximum possible value $\bar{E}_a/R_a$.

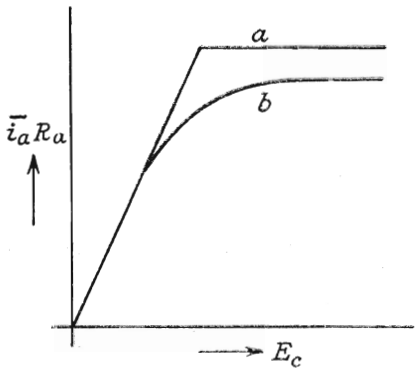


Figure 9.10. Curves showing effect of incomplete saturation on voltage control characteristic.

(*a*) Control characteristic with cores having 100% remanence.

(*b*) Control characteristic with cores having a remanent flux appreciably less than the saturation flux.

If, however, the cores are not 100% remanent then, although still no setting occurs due to the control voltages, premature reset to the point a' reduces the duration of the following conducting period which cannot start until the flux has been restored to saturation. The effect of premature reset arising out of incomplete remanence is therefore to reduce the maximum output which could ideally be obtained. This is shown in *Figure* 9.10 where curve (*a*) shows the voltage control characteristic obtained using a core material for which the remanence is 100%, while (*b*) shows the effect of using a core material having a remanent flux appreciably below the saturation value.

9.10 Effect of Rectifier Leakage [9(c)]

Rectifier leakage in the reverse direction has been shown to have an important bearing on the control characteristics of the high-speed circuits. It also places a lower permissible limit on the width of the core hysteresis loop, i.e. the coercive force.

In general, the poorer materials, e.g. those having a comparatively large coercive force, have better resetting characteristics when the rectifier leakage is appreciable.[9(c)] Since the control current increases with the width of the hysteresis loop, however, there is a drain of energy from the control source which plays no part in the actual setting of the flux. Thus, in order to obtain the best amplification characteristics by employing narrow-loop materials, the rectifiers must have a reverse leakage current less than that corresponding to the m.m.f. F_{ab}. The effect of rectifier leakage is shown by the curves in *Figure* 9.11*a* and *b*; both these curves apply to the same core material, but in curve (*a*) the rectifier leakage is negligible whereas in (*b*) the leakage is sufficient to cause spurious reset, leading in particular to a reduction of the maximum possible output and a loss of linearity for large outputs.

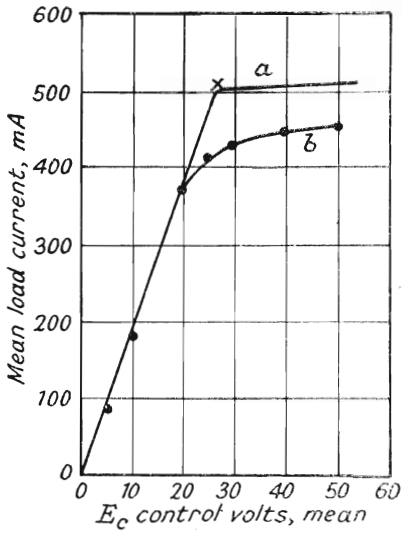


Figure 9.11. Control characteristic showing effect of rectifier leakage. (*After Johnson.*)
(*a*) Using a rectifier in which the leakage current was less than that necessary to produce a flux decrease.
(*b*) Using a rectifier where the leakage is sufficient to cause flux reset by the supply voltage.

9.11 Fast Response by Voltage Feedback

It is to be expected that negative voltage feedback applied to an auto-excited transductor has the effect of increasing the speed of response of the amplifier while at the same time reducing and linearizing the voltage amplification. The full implication of this and its bearing on the fast-response circuits was first appreciated

by Scorgie,[9(d)] who showed that the earlier circuits developed by Ramey represent a special case of a more general arrangement.

It has been shown in earlier chapters that the voltage amplification of an auto-excited transductor can be expressed in the general form

$$G_v\overline{Y}_v(D) = \left[\frac{g'_v}{1 + \tau_t D}\right]$$

where

$$g'_v = \frac{1}{\pi}\frac{\omega M_{ca}}{R_c}$$

and

$$\tau_t = L_{cw}/R_c.$$

If a portion H_v of the output voltage is fed back and injected into the control circuit in series with the control voltage then the overall voltage amplification becomes

$$\left(\frac{g'_v}{1 + g'_v H_v}\right)\left[\frac{1}{1 + \left(\frac{\tau_t}{1 + g'_v H_v}\right)D}\right].$$

If H_v is made equal to N_c/N_a, then by substituting for g'_v and τ_t it is easy to show that, provided $g'_v H_v$ is large compared with unity, the voltage amplification is equal to N_a/N_c and the time-constant equal to $\frac{1}{2f}$.

These results, however, cannot be considered to illustrate more than the general effects of voltage feedback since the time-constant has a special meaning which can no longer apply when dealing with durations as short as a half-cycle of the supply frequency. A better understanding of the circuit is obtained if we consider the actual sequence of events following the application of a control voltage to a transductor with voltage feedback.

The circuit in *Figure* 9.12 shows an auto-excited transductor using the bridge connection in which a fraction of the output voltage e_a is fed back to provide an additional source of control e in series opposition to the actual control voltage E_c.

If this circuit is biased off to give zero output for zero input and, furthermore, if under these conditions the a.c. fluxes are varying from positive to negative saturation, the operation may be considered in three stages. Firstly a control voltage is applied to the control circuit at a time $t = 0$. After a given time t, this produces

a mean flux proportional to the magnitude of the total voltage-time integral applied to the windings from the instant $t = 0$. Secondly, this flux gives an output voltage e_a, the mean value of which is proportional to the mean flux level. Thirdly, a fraction of the output voltage is fed back to oppose the control voltage.

If the mean value of the voltage fed back is equal to the control voltage the former will oppose the control voltage as soon as the output appears. Therefore the total voltage-time applied to the control circuit will be the difference between that due to the control voltage and that due to the voltage fed back.

Now since the control voltage is applied at $t = 0$, the output voltage will not appear until one half-cycle later and thus in all

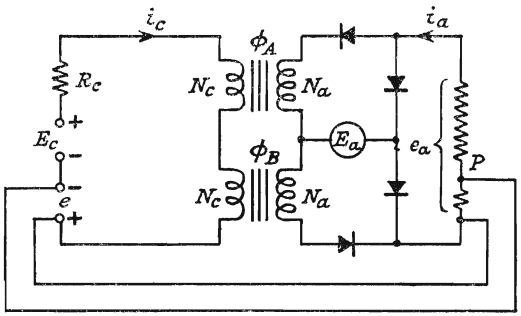


Figure 9.12. Auto-excited transductor with voltage feedback.

half-cycles after the first the applied voltage time must be zero no matter how long a time interval is considered, since the mean voltage fed back is equal to the control voltage. Therefore the control voltage is only acting unopposed for the first half-cycle during which time the applied voltage time is equal to E_c $(T/2)$. It follows that the mean flux can only increase during the first half-cycle and the final output will therefore be established after this time. This, therefore, is an effective method of limiting the integration time to one half-cycle as discussed in section 9.1.

It will be noticed that the above discussion has ignored the voltage drop in the control circuit due to the inevitable build-up of mean control current.

Provided the core permeability is large and the control circuit resistance is small, however, the control current cannot build up sufficiently in the first half-cycle to produce an appreciable voltage in opposition to the control voltage.

The foregoing discussion will now be considered on the basis of the circuit equations thus :

$$E_c - e = N_c\left[\frac{d\phi_A}{dt} + \frac{d\phi_B}{dt}\right] \text{ (ignoring } i_c R_c\text{)}.$$

Also the mean flux $\phi_0 = \dfrac{\phi_A + \phi_B}{2}$,

giving $$E_c - e = 2N_c \frac{d\phi_0}{dt}.$$

Integrating between the limits 0 and t,

$$\phi_0 = \frac{1}{2N_c}\left[\int_0^t E_c dt - \int_0^t e dt\right].$$

Now the control voltage-time integral may be considered to consist of two components, i.e. the integral over the first half-cycle added to the integral over the remaining half-cycles, the last one of which ends at time t.

Also the feedback voltage-time integral is zero in the first half-cycle. Thus from these considerations

$$\phi_0 = \frac{1}{2N_c}\left[\int_0^{T/2} E_c dt + \int_{T/2}^t E_c dt - \int_{T/2}^t e dt\right].$$

Since the a.c. fluxes for no signal vary over the whole unsaturated range of the core, we have

$$\phi_0 = \phi_c = \frac{\bar{e}_a}{4fN_a} \text{ (see section 3.13)}$$

where $\bar{e}_a$ is the output voltage. Thus putting $e = H_v \bar{e}_a$ gives

$$\bar{e}_a = \frac{2fN_a}{N_c}\left[\int_0^{T/2} E_c dt + \int_{T/2}^t E_c dt - \int_{T/2}^t H_v \bar{e}_a dt\right].$$

If H_v is fixed by the load tapping P so that the two last integrals cancel, then we have

$$E_c = H_v \bar{e}_a \qquad \ldots \quad (9.16)$$

In these circumstances integrating over the first half-cycle and putting $T/2 = 1/2f$ gives

$$\bar{e}_a = \frac{E_c N_a}{N_c} \qquad \ldots \quad (9.17)$$

and from 9.16 and 9.17 $\quad H_v = \dfrac{N_c}{N_a}$.

Thus this arrangement has characteristics which are very similar to those of the switched rectifier type of circuit devised by Ramey, the most important being a linear voltage amplification equal to the turns ratio and a response time of one half-cycle. It has also two distinct advantages, firstly it is not so dependent upon a square loop characteristic, and secondly it is more easily adjusted to the correct working condition.

Finally, it will be appreciated that the voltage fed back exercises the same type of constraint as the voltage drop in the control circuit of an auto-excited transductor due to the growth of control current. Both of these constraints reduce the voltage gain and speed up the response ; the disadvantage of the latter method, however, arises because it is not the output which opposes the input but, in effect, the control current. Therefore any time delays or non-linearities arising outside this local feedback loop will not be influenced by it. Two such characteristics are immediately evident :

(*a*) The time-delay introduced by the a.c. windings (section 8.5).
(*b*) Non-linearity between the control current and the flux, this being the most non-linear characteristic of the transductor.

When using the voltage feedback type of circuit both of these effects are included within the feedback loop and it is therefore probably true to say that feedback is the most effective and easiest way of producing a fast response.

9.12 Three-Phase Circuits

Three-phase transductor circuits have been examined and successfully applied to control applications [9(e)] (see section 14.8). These circuits may be synthesized from the half-wave circuit of *Figure* 9.3*b*.

Firstly the coil in this figure may be considered in two parts, the rectifiers also being duplicated to give the circuit of *Figure* 9.13*a*. If two more inductors are added, as in *Figure* 9.13*b*, full-wave operation is obtained when using a direct control voltage E_c and an alternating voltage E_a. Here each reactor is designed to sustain half the alternating supply voltage. Thus on positive half-cycles (as shown) coils (1) and (3) are having their flux sampled prior to conduction which takes place round the circuit $AJKLMH$. In coils (2) and (4) the flux is being set by the control voltage-time due to $|E_a|-E_c$ acting round $ABCDEFGH$. On negative half-cycles the two pairs exchange their operation.

A third pair of inductors may be added to the above arrangement which makes the circuit immediately applicable to three-phase operation, each of the three points A, H and N being connected to the three respective phases of the supply as shown in *Figure* 9.13*c*.

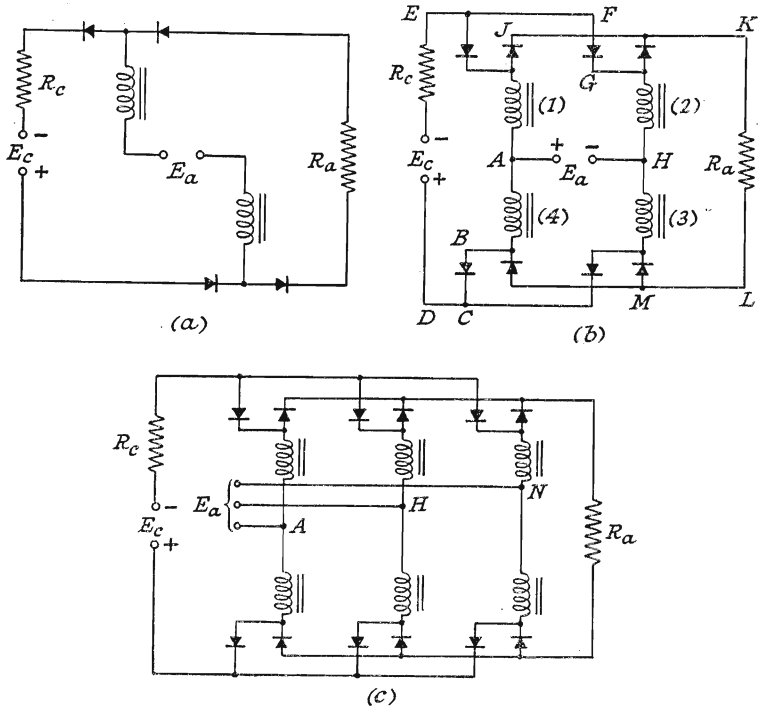


Figure 9.13. Synthesis of three-phase circuit.
(*a*) Half-wave single-phase circuit.
(*b*) Full-wave single-phase circuit.
(*c*) Three-phase circuit.

Experimental work [9(e)] on three-phase high-speed transductors of this type using H.C.R. cores excited at 400 c/s showed that the device tends to behave as a pure time-constant of $1/2f$ seconds but with a phase shift which is linear with signal frequencies due to a fixed lag of $1/2f$ seconds as described in section 8.6. Experimental frequency responses are plotted in *Figure* 9.14.

If galvanic isolation is required or a voltage gain greater than unity then, as previously, double-wound reactors may be used. The circuit for this is shown in *Figure* 9.15. As in the case of the

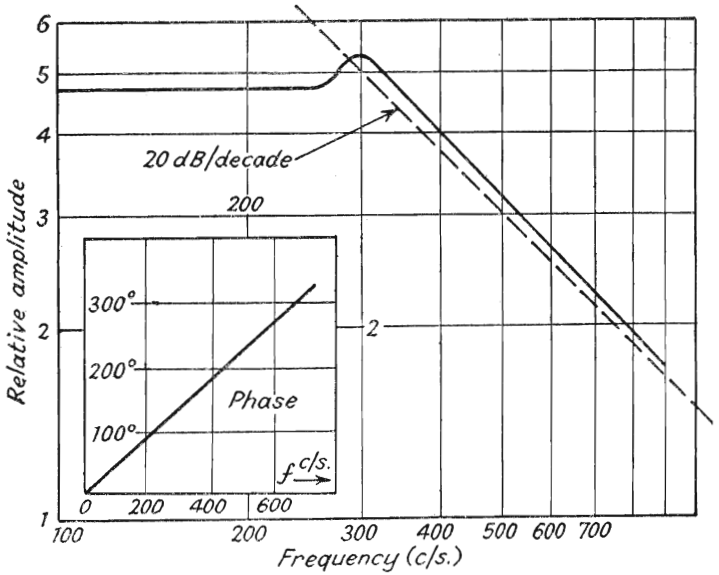


Figure 9.14. Frequency response of three-phase circuit. (*After Maine.*)

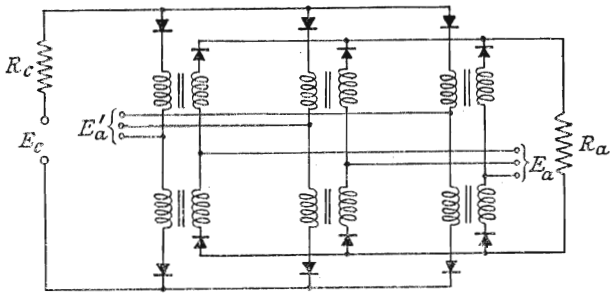


Figure 9.15. Three-phase transductor with isolation between control and load circuits.

single-phase circuits, two alternating voltages are required such that

$$E'_a = \frac{N_c}{N_a} E_a.$$

The power gain per stage which can be obtained from a high-speed three-phase amplifier is of the order of 1,000 and hence multi-stage operation is often desirable.

Fast response may also be obtained with three-phase circuits

by the use of negative feedback as described in section 9.11. This has the advantages discussed for the single-phase amplifier in that no control rectifiers are required, making leakage problems easier. Also, as previously, the amplifier characteristic is less dependent on complete remanence in the core characteristic (*Figure* 9.16).

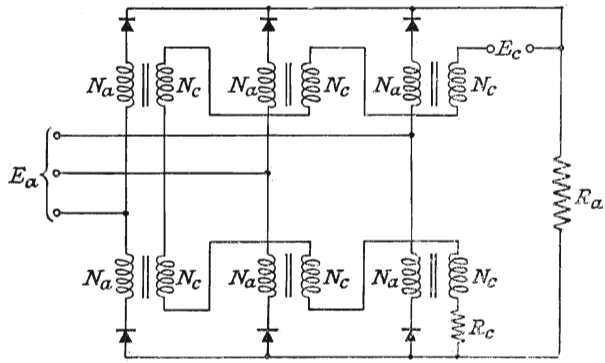


Figure 9.16. Three-phase circuit giving fast response by voltage feedback.

9.13 Comparison of Transductors having Single-Cycle and Multi-Cycle Setting

Let us now compare the fundamental operation of the slow-setting full-wave transductors with that of the fast-setting circuits. After a control voltage is applied the initial flux set in the first half-cycle is approximately the same for both types of circuit. In the former type of circuit, however, the flux set in each half-cycle is integrated over several cycles due to the fact that the core proceeding towards saturation always inherits the total flux change from previous setting periods. The ultimate output is determined by the final mean flux level which is limited by the mean control current ; this cannot increase beyond the value E_c/R_c. The total flux which is set may therefore be large although the time taken to achieve this is extended over many cycles.

In the fast-setting transductor, after the application of a control voltage, the flux is always set to the same value from saturation and there is no inheritance of voltage-time from previous setting periods. It follows, therefore, that no integration of the flux takes place over more than one half-cycle, thereby confining the response time to this period.

There are, therefore, two alternative ways of using a transductor ; in the former the power output for a given voltage input is limited by the circuit parameters (e.g. the control circuit resistance) while the response time is allowed to take any value consistent with the circuit amplification. In the latter the time-constant is limited to one half-cycle while the power output depends upon the total flux which can be set in this time.

On the basis of a hysteresis loop of infinite slope, the power gain of the fast-setting transductors is infinite, and therefore it might appear that the device has a power gain comparable with the slow transductor having 100% self-excitation. This, however, is not the case since it does not take into account the finite core permeability which associates a change of m.m.f. with a flux change. This may be seen qualitatively from elementary energy considerations. Whenever a flux change occurs in a magnetic core an energy interchange takes place (see section 1.6). In the high-speed transductor the flux is set in one half-cycle, therefore the total energy interchange W between the source and transductor occurs in this time. When the energy level is set to this value, sampling takes place and over the next setting half-cycle the core flux is reset. Therefore the rate at which energy is transferred between the source and the control circuit is $2W/T$ units per second. In the slow transductor the same energy interchange W is necessary to set the flux in order to produce the same output, but this occurs over n half-cycles and therefore the mean energy interchange per second will be $2W/nT$. The quantity $2W/T$ is the rate of energy interchange or the power required to achieve setting in the fast transductor, and this is n times the power used in the slow transductor, and therefore the amplification is only $1/n$ times that of the slow transductor. Losses complicate the problem still further and tend to reduce the power amplification of the high-speed transductor. It should also be noted that the voltage applied to the control circuit of the high-speed transductor is considerably greater than the control voltage for the low-speed transductor. This follows again from the fact that if the control flux is to change by a given amount in one half-cycle, it must be supplied from a much larger e.m.f. source than if a larger number of cycles were available in which to set the flux.

The advantages and disadvantages of the two modes of operation can only be discussed with reference to a particular application. But in cases where power gain may be sacrificed for

improved response, the high-speed circuits may be preferable. On the other hand, in many cases it would be advantageous to sacrifice speed of response and thereby enable the control current to build up over a large number of cycles to a higher value for a given control voltage.

It is of interest to compare the amplifying process of a transductor with that of an electronic valve. In the transductor the input is being sampled every half-cycle, the output then responding to the results of the sampling process. Thus the input may be constant over a considerable period of time ; nevertheless it is periodically sampled, although the information resulting from this sampling may be " stale." In the electronic valve, on the other hand, the grid charges up to a certain potential and the input information is sampled continuously, i.e. at infinite frequency. The time-constant of the transductor is therefore limited by the supply frequency whereas that of the valve is limited only by the rate at which energy can be stored in the grid capacitance.

CHAPTER 10

BALANCED MAGNETIC AMPLIFIER CIRCUITS

10.1 Factors Governing Choice of Circuit

In all the transductor circuits so far described the output cannot be reduced to zero, and no matter how the control system is biased the output will never be less than a minimum value equal to $\bar{\imath}_{a_0}$ (section 2.4). In certain circumstances this standing current does not matter. If, for example, the load is an electromagnetic relay or a heating element a small load current is unlikely to produce any appreciable effect.

In other cases where, for example, the load is a second-stage of magnetic amplification or a motor field, it may be possible to compensate for this standing current by actually biasing the load device itself. In the latter examples the loads are electromagnetic and the outputs are m.m.f.s which magnetize a core.

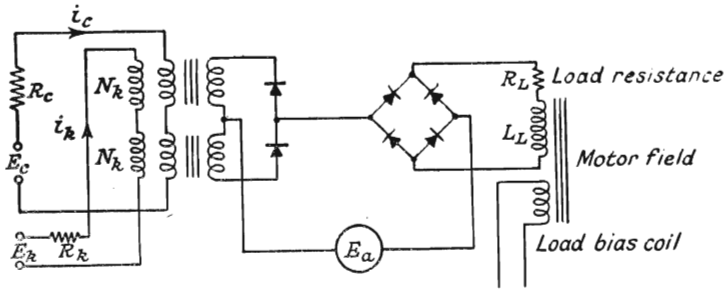


Figure 10.1. Single auto-excited transductor giving push-pull output by using bias in the load.

It is sometimes required that the direction of the output will reverse for a change of signal direction. This may be achieved by the provision of suitable biasing at the input of the amplifier as well as in the load. A simple example is shown in *Figure* 10.1,

where the load is the d.c. field of a motor and the amplifier is a single parallel auto-excited transductor. Bias windings N_k are provided on the transductor, and the bias supply is adjusted so that for zero signal the output m.m.f. stands at half its maximum value. A bias field is also provided on the motor to balance out this standing m.m.f., thereby ensuring that the motor field m.m.f. will be proportional to the control voltage and of the same sign. This technique is particularly applicable when the load is another transductor, in which case bias windings may readily be added to the second stage.

Although this arrangement as an amplifier is probably the most economical in terms of cost and size, it suffers from two disadvantages. Firstly it is often impracticable to incorporate bias in the load, and secondly the zero stability of such an arrangement is comparatively poor. It will be shown in Chapter 11 that better zero stability could be obtained with a single transductor output stage by using an even harmonic pre-amplifier. Otherwise the zero level associated with a single transductor is seldom better than about 10^{-7} watt, referred to the signal.

If the load cannot be designed or modified to suit this type of operation, however, it is necessary to use some form of balanced circuit to obtain an output whose sign changes with that of the input. A number of push-pull circuits of this type are available, and they generally employ a balanced pair of transductors operating in such a way that the difference between the two individual outputs operates on the load. Some of these circuits give a d.c. output and others give an a.c. output whose phase reverses with the sign of the input signal. The latter type of circuit may be used for feeding directly into an a.c. load such as one of the phases of a two-phase servo motor.

A number of magnetic amplifier circuits have been published in a book by Geyger.[10(a)] There are, however, a comparatively small number of basic circuits which are quite adequate for the majority of applications ; these are discussed in this chapter and the next. The circuits described in this chapter are primarily single-stage amplifiers employing orthodox transductors, while Chapter 11 mainly deals with low-level and multi-stage amplifiers. Push-pull amplifiers employing half-wave transductors are characterized by a high speed of response, and almost invariably require two or more stages of amplification. Such an amplifier is described in sections 10.21 and 10.22.

Push-pull amplifiers consist of two transductors arranged in a circuit to give a balanced output. This may be achieved in a number of ways but in all cases it is first necessary to make each transductor sensitive to signal polarity. Auto-excited or self-excited transductors are, in any case, polarized ; unself-excited transductors may be made sensitive to signal polarity by various methods which are outlined in sections 10.16 to 10.18. Auto-excited transductors are undoubtedly the most useful, however, and the alternative systems are therefore of comparatively minor importance.

10.2 Types of Push-pull Circuit giving a D.C. Output

A very useful paper on push-pull magnetic amplifiers has been published by Milnes and Law[5b]; the problem has also been studied graphically by Butcher and Willheim.[10(b)] Many aspects of push-pull transductor operation still remain obscure, however, particularly in the case of magnetic amplifiers working into highly inductive loads where, in certain circumstances, cross-coupling occurs between the two transductors of a push-pull pair.

When a d.c. load is used, the final output is obtained by subtracting the outputs from each transductor.

The following methods are available for combining the two outputs :

(*a*) Voltage mixing ;
(*b*) Current mixing ;
(*c*) Ampere-turn mixing.

In addition, any given circuit may be operated in either of two ways. Firstly that shown in *Figure* 10.2*a* ; two transductor characteristics, A and B, are each biased to the mid-point O so that a positive control current causes the output from A to increase while the output from B diminishes. If the control current is reversed, then B increases and A diminishes. For any particular control current the output is proportional to a line such as LM, which becomes zero when the control current is zero and negative for negative control currents. Secondly, *Figure* 10.2*b* shows the transductors each biased down to the bottom bend of their characteristics. The same general principles apply excepting that in this case only one of the transductors is operating at a time, the other giving almost no change of output. Thus positive outputs are almost entirely due to A while

negative outputs are supplied by B. For any given mixing circuit, either of these bias conditions may be used, depending upon the type of characteristic required.

The various transductor arrangements will each be studied on the basis of simple equivalent circuits of the type described in section 5.14. When doing this it must be remembered that the voltage generator representation for a transductor primarily

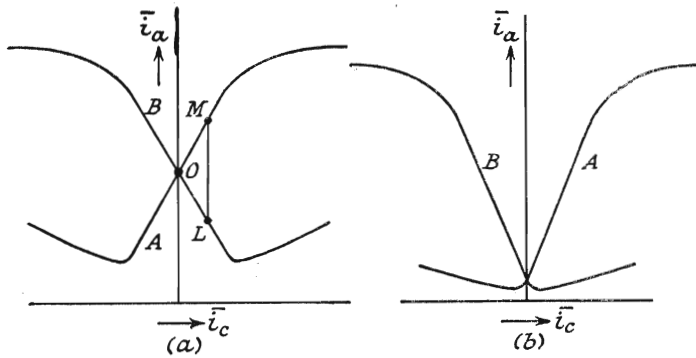


Figure 10.2. Two methods of biasing transductors for push-pull operation.
(*a*) To mid-point (quiescent firing angle = 90°).
(*b*) To bottom bend (quiescent firing angle = 180°).

specifies the slope of the characteristic, while the standing current corresponding to the point O can be made to assume any value by applying the appropriate bias.

The analysis given in Chapters 4 and 5 showed that for auto-excited transductors the standing output is equal to half the maximum output. This result was based on idealized cores without losses ; in practice, however, the characteristics are often shifted in such a direction that the standing output is nearer the maximum value. For this reason it is nearly always necessary to employ a bias winding if only for trimming purposes.

D.C. Voltage Mixing Circuit (Series Summing)

In the following section the same assumptions as those made in Chapters 4 and 5 will be applicable ; thus the core will have a high permeability in the unsaturated region, losses will be neglected and the control current will be assumed to be smooth ($\lambda = 1$).

10.3 Equivalent Circuit

When the load is not centre-tapped the voltage mixing circuit is probably one of the most universally applied ; it will therefore be considered in some detail, not only from the equivalent circuit standpoint but also from first principles.

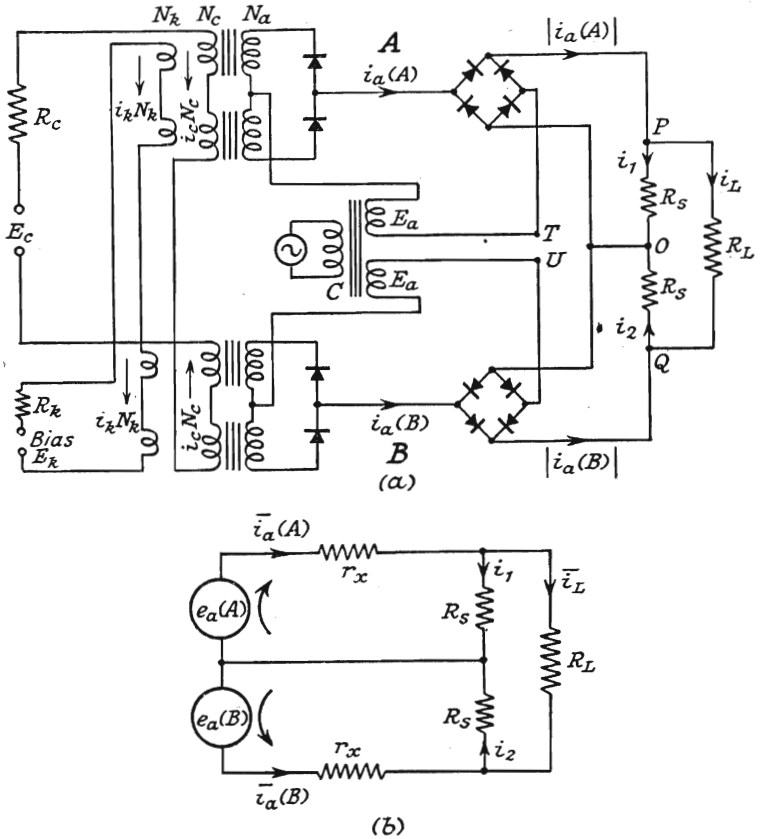


Figure 10.3. Push-pull transductors with voltage mixing (series summing).
(*a*) Complete circuit.
(*b*) Equivalent circuit where $r_x = r_{aw} + r_{ar} + 2r_{Lr}$.

The diagram in *Figure* 10.3*a* is the complete circuit for a push-pull magnetic amplifier employing voltage mixing. Two auto-excited transductors A and B are each fed from isolated windings on a transformer C ; this is necessary to avoid short circuiting the rectifier elements due to a common connection of two points such as T and U. The two d.c. outputs feed into equal

ballast resistors R_s, one end of each being common to both circuits at O. The load R_L is then placed across the outer ends of the resistors R_s and the rectifier polarities are so arranged that when the two outputs are equal in magnitude the total voltage applied to the load is zero.

The input to both transductors is biased to the appropriate point on each characteristic so that the total m.m.f. controlling a core of transductor A is equal to

$$i_k N_k + i_c N_c$$

and that controlling transductor B is

$$i_k N_k - i_c N_c.$$

Thus if the control current is increased from zero, the control ampere-turns on A increase while those on B decrease, whereas if the control current is decreased the effect is reversed. The outputs follow the same trend and therefore in the former case P becomes positive with respect to Q while in the latter the relative polarities of P and Q reverse.

If we consider the total a.c. circuit resistance of each transductor, it can be seen that besides the impedance across PQ, each transductor has an internal resistance due to a winding and auto-excitation rectifier ($r_{aw} + r_{ar}$). In addition the currents $i_a(A)$ and $i_a(B)$ must each flow through two elements of a bridge rectifier. Thus if r_{Lr} is the forward resistance of one element of the bridge the total additional circuit resistance is equal to $r_{aw} + r_{ar} + 2r_{Lr}$. Since the points P and Q will both always be positive with respect to O, the currents i_1 and i_2 will always flow in the directions shown by the arrows. Therefore there is never any possibility that all four elements in either rectifier will become conducting and so short circuit a ballast resistor. For this reason the equivalent circuit may be represented as in *Figure* 10.3*b* in which the only effect of the bridge rectifiers is to introduce an additional series resistance equal to $2r_{Lr}$ in each circuit. The amplification and maximum output of this circuit will now be considered by using the simple equivalent network shown in *Figure* 10.3*b*.

10.4 Amplification and Output of Voltage Mixing Circuit

There are two possible operating conditions for this circuit, firstly that in which both transductors are biased to the mid-point of their characteristic as in *Figure* 10.2*a*, and secondly when they

are both biased to the bottom bend as in *Figure* 10.2*b*. In the former case the conducting period for zero control current occurs at a time $T/4$ (90 electrical degrees) for both transductors. While in the latter case conduction just commences in either one or the other as the control current is increased from zero, we will first consider the case of a 90° firing angle.

In this case both transductors are active at all times and the following simple relationships may be written down for the equivalent circuit.

Putting $r_x = r_{aw} + r_{ar} + 2r_{Lr}$

we have from *Figure* 10.3*b*

$$\bar{e}_a(A) - \bar{e}_a(B) = [\bar{i}_a(A) - \bar{i}_a(B)]\, r_x + \bar{i}_L R_L \quad . \quad . \quad (10.1)$$

Also $$R_s(i_1 - i_2) = R_L \bar{i}_L \quad . \quad . \quad . \quad (10.2)$$

$$\bar{i}_a(A) = i_1 + \bar{i}_L \quad . \quad . \quad . \quad (10.3)$$

and $$\bar{i}_a(B) = i_2 - \bar{i}_L \quad . \quad . \quad . \quad (10.4)$$

Subtracting (10.4) from (10.3), eliminating $(i_1 - i_2)$ then eliminating $[\bar{i}_a(A) - \bar{i}_a(B)]$ from (10.1) gives

$$\bar{e}_a(A) - \bar{e}_a(B) = \left[r_x \left(\frac{R_L}{R_s} + 2 \right) + R_L \right] i_L \quad . \quad . \quad . \quad (10.5)$$

From the results in sections 5.3 and 5.9 we also have

$$\bar{e}_a(A) = \frac{1}{\pi}\, \omega M_{ca} \left[\bar{i}_k \frac{N_k}{N_c} + \bar{i}_c \right]$$

and $$\bar{e}_a(B) = \frac{1}{\pi}\, \omega M_{ca} \left[\bar{i}_k \frac{N_k}{N_c} - \bar{i}_c \right].$$

Combining this with 10.5 gives :

$$\bar{i}_L = \frac{2}{\pi} \cdot \frac{\omega M_{ca}}{R_L + \left(\frac{R_L}{R_s} + 2 \right) r_x} \cdot \bar{i}_c \quad . \quad . \quad . \quad (10.6)$$

This shows that if the internal resistance is zero then the current amplification is twice that obtained with a single auto-excited transductor from which we obtained

$$\bar{i}_L = \frac{1}{\pi} \frac{\omega M_{ca}}{R_L}\, \bar{i}_c \text{ (for zero internal resistance).}$$

The power amplification will also be twice that of a single transductor since, although the square of the current gain has increased

by a factor of four, the power input to the control circuit is doubled due to the presence of two control windings in series.

In the push-pull circuit the effect of the internal resistance is much more pronounced since it is multiplied by the factor $\left(\frac{R_L}{R_s} + 2\right)$. It is therefore important, when dealing with transductors for push-pull operation, to have an internal resistance which is as small as possible. This may easily be seen from the following simple numerical example.

Considering first a transductor for which the internal impedance is equal to $0{\cdot}2 \times$ the load resistance, it will be shown in the following section that a reasonable value for the summing resistance R_s will be such that

$$R_s = R_L.$$

The current amplification for the push-pull unit is therefore, according to equation 10.6,

$$G_i = \frac{1}{\pi} \frac{\omega M_{ca}}{0{\cdot}8\, R_L}$$

and for a single transductor of this internal impedance we have :

$$G_i = \frac{1}{\pi} \frac{\omega M_{ca}}{1{\cdot}2\, R_L}.$$

Thus, remembering that for the push-pull unit the control winding impedance is doubled, we have

$$\frac{\text{power amplification of push-pull amplifier}}{\text{power amplification of unbalanced transductor}} = 1.12.$$

If, on the other hand, the internal impedance were equal to the load, the above ratio becomes $0{\cdot}5$. The power gain of the single transductor has decreased by a factor of 2.77 but that of the push-pull pair has decreased by a factor 6.25. Thereby showing that the effect of increasing the internal resistance has been much more serious in the latter case.

10.5 Power Output and Efficiency

Provided the internal resistance is small, it has been shown that there is little difference between the power amplification of the single transductor and that of the push-pull unit. When considering the power actually wasted in the two circuits, however, it immediately becomes evident that in this respect the push-pull circuit is far less efficient than the single unit.

When the control signal is a maximum, one transductor of the pair will deliver full output and the other will be cut off. If we consider *Figure* 10.3*a* under these conditions, transductor B is cut off and A will be feeding a total load consisting of its own internal resistance in series with the parallel combination of R_s and $R_L + R_s$. This corresponds to a total impedance given by

$$R_E = r_x + \frac{R_s(R_L + R_s)}{R_L + 2R_s} \quad . \quad . \quad . \quad (10.7)$$

and the total power delivered is $\dfrac{\bar{E}_a^2}{R_E}$.

Only a fraction of this power is actually dissipated in the load, however, and this fraction defines the efficiency η of the circuit.

The total transductor current at maximum output is $\bar{\imath}_{as}$ and is equal to $\bar{E}_a/R_E$, and by considering the fraction of this in the load we have

$$\bar{\imath}_L\,(\max) = \frac{\bar{E}_a}{R_s + R_L + r_x\left(\dfrac{R_L}{R_s} + 2\right)} \quad . \quad . \quad . \quad (10.8)$$

Thus, having fixed on the load resistance R_L, a practicable value (as low as possible) is chosen for the internal resistance r_x. Then, using equation 10.8, it may easily be shown that for maximum current in the given load, the summing resistor should be given by

$$R_s = \sqrt{R_L r_x} \quad . \quad . \quad . \quad (10.9)$$

The full-line curves in *Figure* 10.4 show the maximum load current $\bar{\imath}_L$ expressed as a fraction of $\bar{E}_a/R_L$ and plotted against R_s/R_L for various values of internal resistance. It is seen from these curves that maximum current flows when $\dfrac{R_s}{R_L} = \sqrt{\dfrac{r_x}{R_L}}$, which is in accordance with equation 10.9. It may also be shown that the efficiency is given by

$$\eta = \frac{R_L R_s}{\left[\left(R_L + R_s\right) + \left(\dfrac{R_L}{R_s} + 2\right)r_x\right]\left(R_L + 2R_s\right)} \quad . \quad (10.10)$$

These expressions enable us to obtain the efficiency for various values of the circuit constants.

Firstly, in the ideal case of zero internal impedance, by putting $r_x = 0$ in equation 10.10 and equating $\frac{d\eta}{dR_s}$ to zero, the efficiency is a maximum when $R_s = R_L/\sqrt{2}$; this corresponds to an efficiency of 17.2%. Such a low efficiency is of course due to the presence of the summing resistors which are necessary in order to provide an artificial centre tap for the load.

In practice, however, the internal impedance r_x must be taken into account, and the dotted curves in *Figure* 10.4 show the

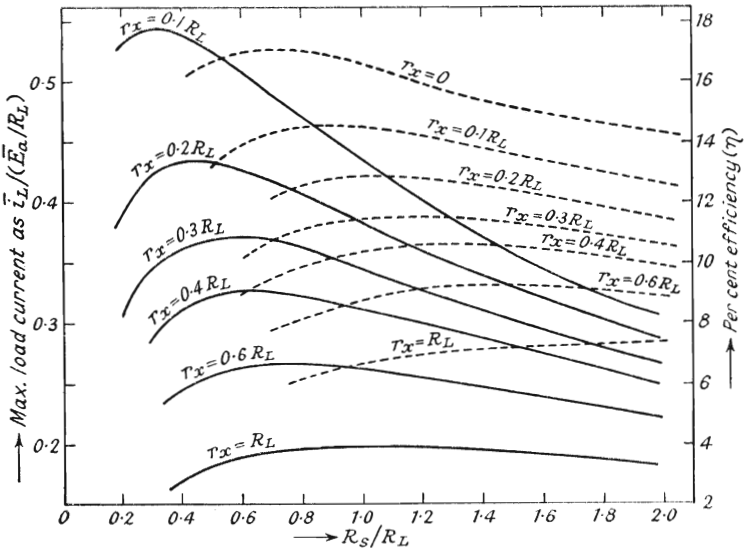


Figure 10.4. Curves showing effect of R_s/R_L on push-pull transductor performance.

(*a*) (dotted) R_s/R_L plotted against efficiency.
(*b*) (full line) R_s/R_L against fractional load current.

efficiency plotted as a function of R_s/R_L for different values of the internal resistance. It is seen that the maximum efficiency and maximum output do not occur together. It should also be noted, however, that the maximum output (equation 10.8) is derived on the assumption that one transductor is completely quiescent. This is not exactly true at the limit of linearity and the effect is to make the maximum output occur at a slightly higher value of summing resistance than that given by equation 10.9. In practice each summing resistance is usually

made equal to the load resistance ; this, in most cases, gives a fair compromise.

10.6 Basic Operation of Voltage Mixing Circuit (Quiescent Firing Angle Equal to 90°)

Although the more important relationships for the voltage mixing circuit have already been determined from the simple equivalent circuit, the basic operation of the pair of transductors acting together will now be considered for this particular circuit. When dealing with the other types of circuit, the equivalent circuit derivation will be used since the internal operation in all cases follows a similar pattern to that about to be described.

Considering *Figure* 10.3*a*, if the control voltage is zero, both transductors are subjected to exactly the same control m.m.f. per core, $i_k N_k$, and therefore both transductors commence conduction at the same instant during a cycle. If the bias is adjusted so that this instant corresponds to the quarter period time $T/4$, and if, furthermore, both transductors are identical in all respects, then over the period from 0 to $T/4$ all four cores will be unsaturated and neither supply voltage will be applied to the load circuit. When both transductors commence conduction, both supply voltages drive current through the summing resistors in opposite directions, the potentials at P and Q are still equal and the load current is still therefore zero. If a signal is now applied to the input, the magnetizing m.m.f. for A is increased by $i_c N_c$ while the m.m.f. for B is decreased by $i_c N_c$. It therefore follows that conduction commences in A earlier than in B.

During the period after A has fired but before B fires, two factors must be considered. Firstly, a voltage is applied to the total circuit resistance R_E as a result of which a load current flows ; if the instantaneous voltage is E_a, then as in equation 10.8 we have

$$i_L = E_a/R'$$

where

$$R' = R_s + R_L + r_x\left(\frac{R_L}{R_s} + 2\right) \quad . \quad . \quad . \quad (10.11)$$

Secondly, a voltage equal to $i_L R_s$ is developed across QO which opposes the supply voltage applied to the second transductor. Therefore the total voltage acting in the a.c. circuit of B is equal to

$$E_a - R_s i_L \quad . \quad . \quad . \quad (10.12)$$

it thus follows that flux changes in B after A has fired are governed

by this resultant voltage and not the supply voltage acting alone.

After transductor (A) has fired at a time $t_q(A)$, one of the cores in (B) will still be proceeding to saturation ; if this core saturates at $t_q(B)$, the total flux change $\Delta\phi$ occurring between $t_q(A)$ and $t_q(B)$ is obtained by considering the total voltage-time integral applied to the windings of (B) over this interval. Since the control current is smoothed the total voltage-time is that applied to the a.c. windings between $t_q(A)$ and $t_q(B)$. Thus from (10.11) and (10.12) we have

$$\Delta\phi = \frac{(1 - R_s/R')}{N_a} \int_{t_q(A)}^{t_q(B)} E_a dt \qquad . \quad . \quad . \quad (10.13)$$

Also, since a voltage is only applied to the load over the period from $t_q(A)$ to $t_q(B)$, we have from equation 10.11

$$\bar{\imath}_L = \frac{2}{T} \int_{t_q(A)}^{t_q(B)} \frac{E_a\, dt}{R'} \qquad . \quad . \quad . \quad (10.14)$$

Combining 10.13 and 10.14 gives

$$\bar{\imath}_L = \frac{2N_a\Delta\phi}{T} \cdot \frac{1}{R' - R_s} \qquad . \quad . \quad . \quad (10.15)$$

When the flux in a core of transductor (A) reaches saturation, that in the corresponding core of (B) is behind saturation by an amount $\Delta\phi$. It follows that while both transductors were unsaturated, i.e. before $t_q(A)$, the difference in their respective flux levels was $\Delta\phi$ and that this was due to the difference $2N_c\bar{\imath}_c$ between the m.m.f. controlling (A) and that controlling (B). Since a linear magnetization curve of the form shown in *Figure* 3.1c is assumed, we have

$$\Delta\phi = \frac{2N_c\bar{\imath}_c}{S}$$

where S is the reluctance of one of the cores. Substituting this result in equation (10.15) and evaluating the term involving R' from (10.11) we have

$$\bar{\imath}_L = \frac{2}{\pi} \frac{\omega M_{ca}}{\left[R_L + \left(\frac{R_L}{R_s} + 2\right) r_x\right]} \bar{\imath}_c$$

which is the same result as that obtained by the equivalent circuit.

The most important factor which emerges from this detailed

treatment is the fact that the supply voltage E_a to each transductor may be rated somewhat over the value normally used in the case of a single-sided device.

This is because the effective voltage in the a.c. circuit is reduced due to the current flowing in the summing resistor R_s. Thus when the amplifier is delivering full output transductor (A) may be conducting throughout a half-cycle while (B) never conducts ; the mean voltage actually applied to the a.c. windings of (B) will therefore be

$$\bar{E}_a\,(1 - R_s/R').$$

This voltage must be equal to the value $\bar{E}_{as}$, which corresponds to saturation, so that

$$\bar{E}_a = \frac{\bar{E}_{as}}{(1 - R_s/R')}.$$

Thus

$$\bar{E}_a = \bar{E}_{as}\left[1 + \frac{R_s}{R_L + r_x\left(\frac{R_L}{R_s} + 2\right)}\right] \quad . \quad . \quad . \quad (10.16)$$

The permissible supply voltage may readily be determined for any known circuit parameters ; thus in the example in section 10.4 for which

$$r_x = 0{\cdot}2\ R_L \text{ and } R_s = R_L$$

we have from equation 10.16

$$\bar{E}_a = 1{\cdot}62\ \bar{E}_{as}.$$

10.7 Quiescent Firing Angle Equal to 180°

When the quiescent firing angle of the two transductors is biased back to 180° each transductor operates independently of the other and the control characteristic of each one is of the form shown by the curves in *Figure* 10.2*b*.

In this case it is seen that for positive control currents the amplification will be given very nearly by the slope of A, while the slope of B is usually sufficiently small to have only a negligible effect. The amplification of a push-pull unit biased in this way may therefore be obtained by considering that one of the transductors only applies a voltage to the external circuit resistance denoted by R_E which consists of r_x in series with the parallel combination of R_s and $R_L + R_s$. The ratio of the mean alternating current $\bar{\imath}_a$ to the control current $\bar{\imath}_c$ is therefore the same as that

applying to a single unit having a total circuit resistance R_E, thus

$$\bar{\imath}_a = \frac{1}{\pi} \frac{\omega M_{ca}}{R_E} \bar{\imath}_c.$$

Evaluating $\bar{\imath}_L$ in terms of $\bar{\imath}_a$, R_s and R_L, and substituting in the above result, gives

$$\bar{\imath}_L = \frac{1}{\pi} \frac{\omega M_{ca}}{R_s + R_L + r_x\left(\frac{R_L}{R_s} + 2\right)} \bar{\imath}_c \quad . \quad . \quad . \quad (10.17)$$

This is seen to differ from the previous result (equation 10.6) obtained for a 90° quiescent firing angle in two respects, firstly the constant factor $\frac{2}{\pi}$ is now replaced by $\frac{1}{\pi}$, and, secondly, the effective circuit impedance appearing in the denominator is increased by an amount R_s.

The curves in *Figure* 10.5 compare the various theoretical

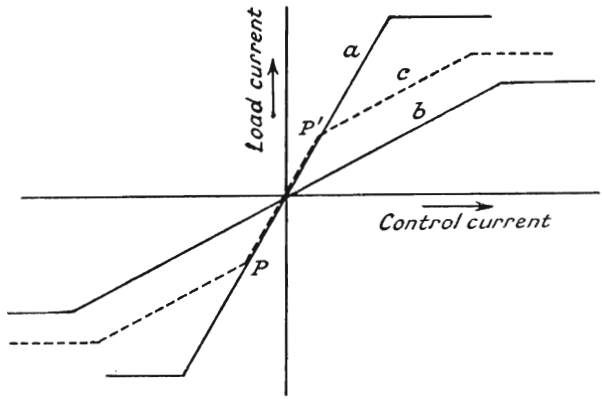


Figure 10.5. Control characteristics for voltage mixing push-pull transductors. (*After Milnes.*)

(*a*) 90° quiescent firing angle.
(*b*) 180° quiescent firing angle.
(*c*) Intermediate firing angle.

characteristics obtained for the different bias conditions. Curve (*a*) shows the final characteristic obtained for a 90° quiescent firing angle. Here we have a large slope and a large value of the maximum output due to the fact that the supply voltage can be appreciably uprated. In curve (*b*), since only one transductor is operating at a time, no uprating of the supply voltage is permissible

and the maximum output is therefore only about two-thirds of that given by (*a*) ; the amplification is also considerably reduced, as explained previously. Curve (*c*) shows a combination of these two effects. For small control currents the curve follows (*a*) while both transductors are active. At the points P and P' one transductor of the pair cuts off and the characteristic is then parallel to (*b*). Such a relationship is non-linear and therefore not particularly useful.

When the quiescent firing angle is equal to 180°, the efficiency is the same as that obtained for the conditions of a 90° quiescent firing angle and therefore, if the amplifier is liable to be in continuous use, the latter arrangement is generally preferable. In cases where the amplifier must remain switched on but is only called upon to operate at infrequent intervals, the former mode of operation is preferable since it has the advantage of a low quiescent current.

PUSH-PULL CIRCUITS WITH M.M.F. MIXING (MAGNETIC SUMMING)

Of the various types of d.c. push-pull amplifier, the ones employing magnetic summing circuits are the most efficient. They can, however, only be used when the load device is centre-tapped and since such loads are usually electromagnetic devices of some kind, this method of obtaining the resultant of two separate outputs has been termed magnetic summing.[5(b)]

10.8 Amplification of Magnetic Summing Circuit

Figure 10.6*a* shows the complete circuit of a pair of auto-excited transductors arranged to give a d.c. push-pull output into a centre-tapped load. In this case the effective output is the m.m.f. produced in the load coil which has N_L turns on each side of the centre-tap. If the self and mutual inductance of the two halves of the load are appreciable then the operation is complicated by cross-coupling between the individual transductors. This may be reduced by methods which have been discussed in a later section. Meanwhile it will be assumed that the inductive effects are not appreciable and the equivalent circuit may be represented as shown in *Figure* 10.6*b*. The following simple relationships are immediately seen to apply thus :

$$\bar{e}_a(A) = \bar{\imath}_a(A)\left[r_x + \frac{R_L}{2}\right] \quad . \quad . \quad . \quad (10.18)$$

$$\bar{e}_a(B) = \bar{\imath}_a(B)\left[r_x + \frac{R_L}{2}\right] \quad . \quad . \quad . \quad (10.19)$$

Effective load m.m.f. $= [\bar{\imath}_a(A) - \bar{\imath}_a(B)]\,N_L$

so that the effective load current $\bar{\imath}_L$ is equal to $\bar{\imath}_a(A) - \bar{\imath}_a(B)$. Combining 10.18 and 10.19 and substituting as previously gives

$$\bar{\imath}_L = \frac{2}{\pi}\,\frac{\omega M_{ca}}{\left(r_x + \frac{R_L}{2}\right)}\,\bar{\imath}_c \quad . \quad . \quad . \quad (10.20)$$

This arrangement is seen to have twice the effective current amplification of a single auto-excited transductor operating into a load of $R_L/2$.

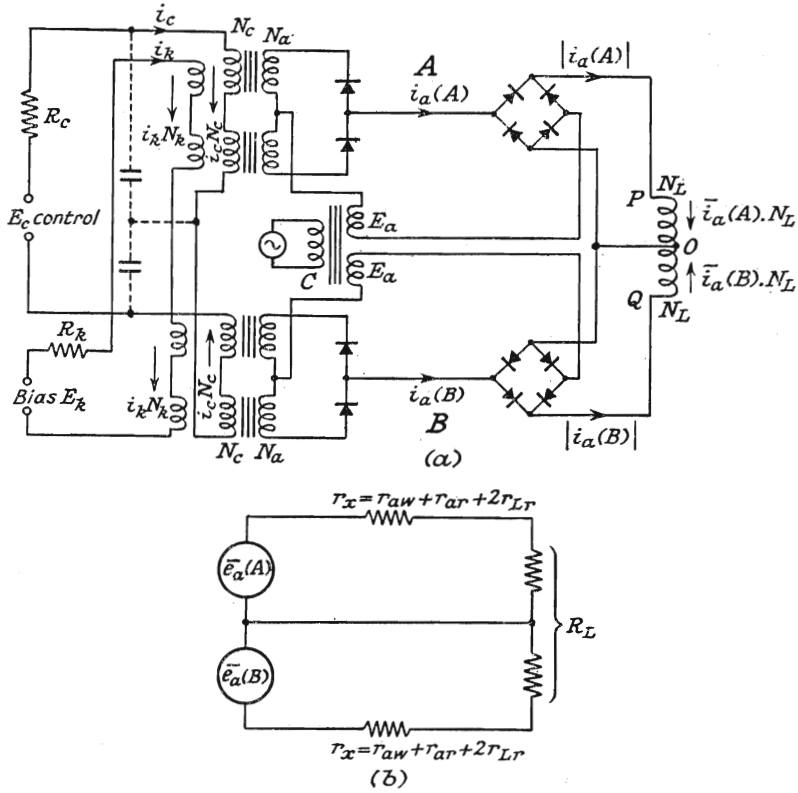


Figure 10.6. Push-pull circuit using m.m.f. mixing.
(*a*) Complete circuit.
(*b*) Equivalent circuit.

The greater efficiency of this circuit compared with the voltage summing circuit is to be expected since in the present case no summing resistors are necessary and each transductor may be treated as a single unit. Magnetic summing is therefore utilized whenever possible, since efficiencies as high as 90% can be obtained.

Finally, it should be noted that when the twin load coils are completely isolated from one another there is no need for the isolating transformer C, and the two circuits can be coupled in parallel to the same supply voltage E_a.

10.9 Push-pull Circuits Using Current Summing

Another circuit which is sometimes used for combining two independent outputs is the current summing network. This

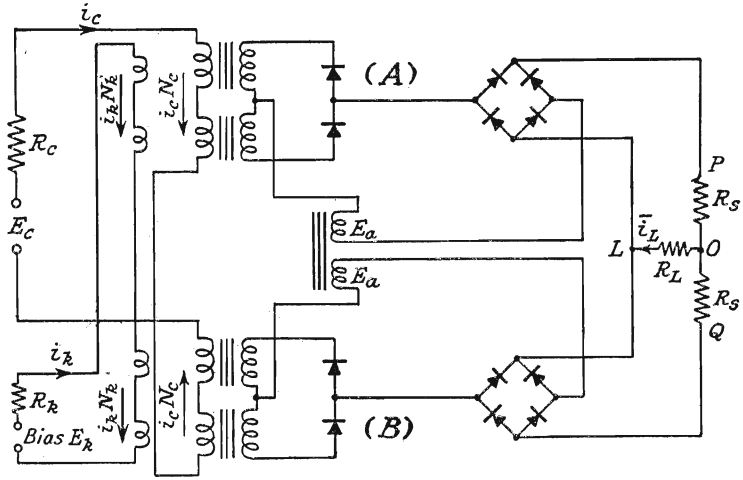


Figure 10.7. Push-pull transductor using current summing circuit.

arrangement is shown in *Figure* 10.7. It is even more inefficient than the voltage summing circuit and is not often used ; it does, however, occasionally find application in cases where the load has a very low impedance.

10.10 Push-pull Circuits with (Group 2) Bridge Transductors

The push-pull circuits described so far have all involved the auto-excited parallel transductor. They apply equally well, however, to the bridge connection and, by way of example, the voltage summing push-pull circuit is shown in *Figure* 10.8. Here

we have a very similar arrangement to that shown previously ; indeed, the only basic difference lies in the effective internal resistance r_x. In the present case the transductor current $i_a(A)$, for example, flows through one a.c. winding with its associated auto-excitation rectifier having a total resistance $r_{aw} + r_{ar}$ and in

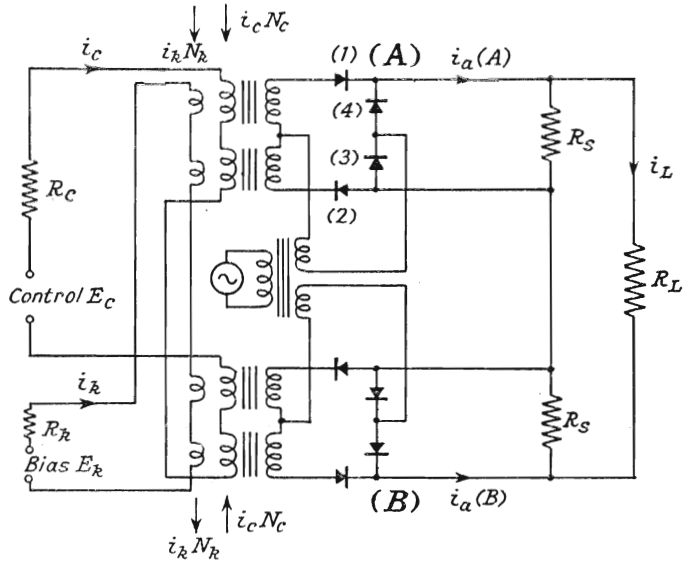


Figure 10.8. Push-pull voltage mixing circuit using bridge connected (Group 2) transductors.

addition the current path includes only one of the load rectifiers (3) and (4). Thus the total internal resistance r_x is given by

$$r_x = r_{aw} + r_{ar} + r_{Lr} \quad . \quad . \quad . \quad (10.21)$$

Provided the appropriate value of the internal resistance is inserted in the relevant equations, an analysis of this push-pull circuit using the bridge transductors yields the same results as those obtained using the auto-excited parallel transductors. The same observations apply to the magnetic summing circuit and the choice of parallel or bridge connection depends upon the same factors as those applying when considering the single unit ; this was discussed in Chapter 8 and was shown to depend primarily on the required dynamic performance.

10.11 Push-pull Circuits Using Self-Excited Transductors

The equivalence between the self-excited and auto-excited

transductors was discussed in Chapter 5, and this result enables one to write down the expressions for the push-pull circuits using self-excited transductors. These circuits, however, are seldom used for reasons already given. In cases where self-excited circuits are employed it is often found that better results are obtained if

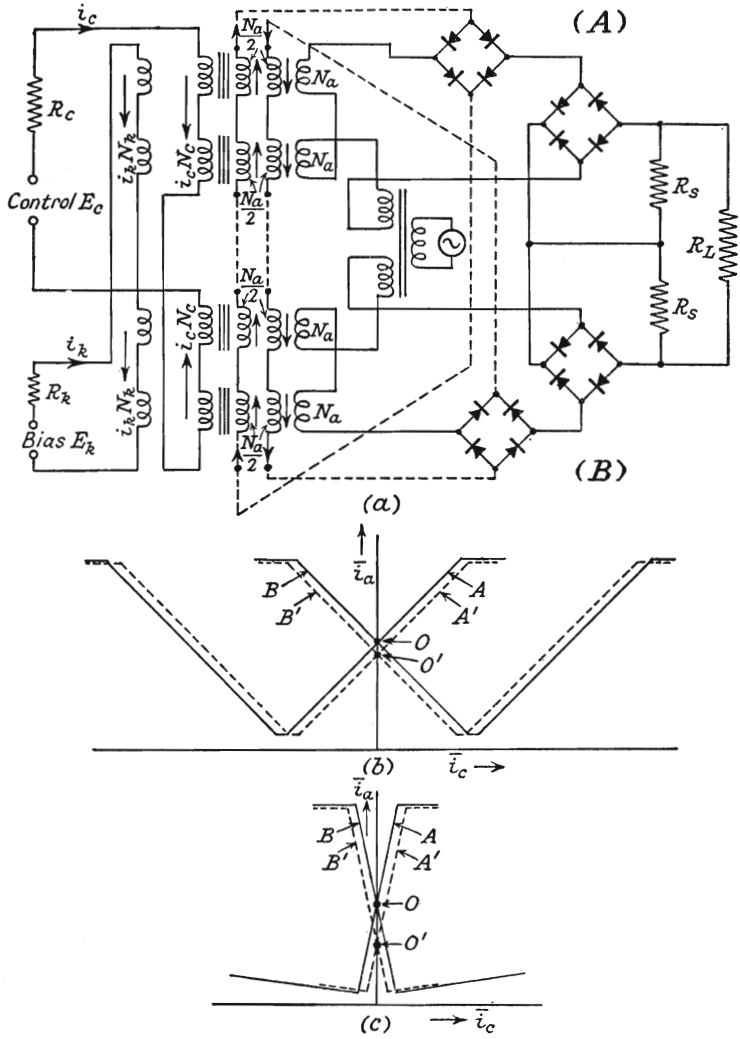


Figure 10.9 (*a*). Push-pull voltage mixing circuit using crossed self-excitation. (*b*). Effect of drift on push-pull amplifier with cross-excitation. (*c*) Effect of drift on push-pull circuit with normal self-excitation.

cross-feedback is used between the two transductors of a push-pull pair.

A circuit diagram of this arrangement is shown in *Figure* 10.9*a* for the condition of complete self-excitation. Here the self-exciting circuits are shown dotted and it is seen that each transductor has two such windings, each one having $\frac{1}{2}N_a$ turns per core. One is fed from its own a.c. circuit and the other fed from the adjacent circuit. It follows that when both transductors are balanced the total self-exciting ampere-turns on each is zero.

Initially the transductors are biased so that when there is no self-excitation each gives one-half its maximum output for no signal. This state of affairs is shown in *Figure* 10.9*b*, where the characteristics A and B are those of unself-excited transductors.

Thus it is seen that the self-excitation only becomes effective when the two transductors become out of balance, e.g. due to a control signal. If normal self-excitation were used, the balance point might tend to shift due to drift to a lower part of the control characteristic as shown by the dotted lines in *Figure* 10.9*c*, and although this would maintain zero output provided both units shifted similarly, the operating point on each characteristic would no longer be symmetrical with respect to the individual characteristics. With the present circuit this effect is considerably reduced since the balance is independent of the self-excitation. The circuit is also found to give somewhat improved zero stability and more effective matching of the pair.

Effect of Inductive Loads and Smoothing Capacitors on Push-pull Circuits

The effects obtained when the load is appreciably inductive are somewhat complicated ; they have, however, been investigated experimentally by Milnes and Law,[5(b)] and the conclusions are that in general improved performance is obtained by using capacitive smoothing.

10.12 Voltage Summing Circuit

The effect of a smoothing capacitor across the d.c. load of a push-pull amplifier using voltage summing is to increase the amplification and to extend the linear range of the device ; its function is thus very similar to that of a single unit with capacitor smoothing. When the conducting period is short, a capacitor across

the load will store energy, and tend to give rise to pulse lengthening as described for the single unit in Chapter 6. (*Figure* 10.10 shows the final control characteristics obtained with a push-pull amplifier using voltage summing.) Curve (*a*) applies to the case of a pure resistive load of 120 ohms. Curve (*b*) shows the

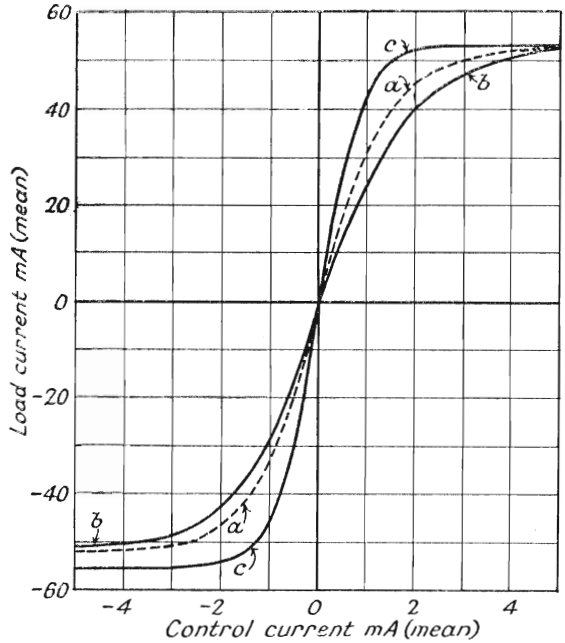


Figure 10.10. Curves showing effect of capacitive smoothing on inductive load when using push-pull circuit with voltage summing. (*After Milnes.*)
(*a*) Resistive load (120 Ω).
(*b*) Inductive load $R_L = 120\ \Omega\quad L_L = 0{\cdot}6$ H.
(*c*) As (*b*) but with 8μF capacitor across load.

effect of adding an inductance of 0·6 henries while maintaining the same total resistance. Finally, curve (*c*) illustrates the improvement brought about by a smoothing capacitance of 8μF in parallel with the inductive load. In the latter case it is seen that the linear range is extended and the amplification is increased. The improvement is similar to that shown for a single unit in *Figure* 6.12 and the value of load capacitance to produce a reasonable result is given approximately by the relation

$$R_L C_L > \frac{1}{2f} \qquad . \quad . \quad . \quad (10.22)$$

10.13 Magnetic Summing

When the magnetic summing circuit is used, a load with appreciable inductance can give rise to cross-coupling effects due to the mutual inductance between the two load coils. This effect appears as a reduced sensitivity of the characteristic near the origin as shown in *Figure* 10.11*a*. If separate capacitors are connected

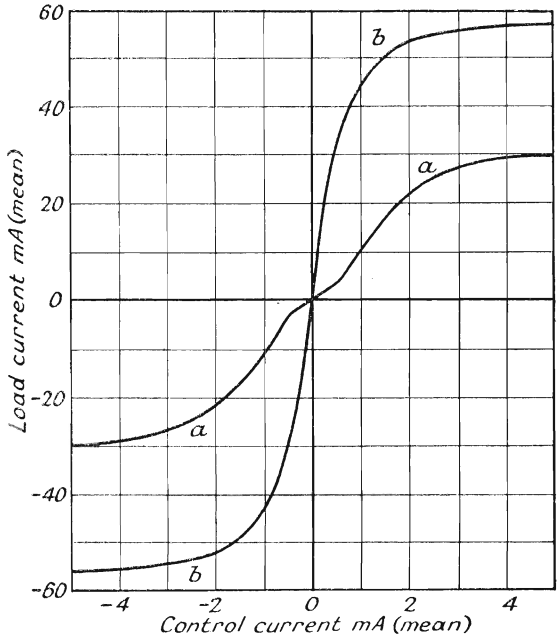


Figure 10.11(*a*). Effect of cross coupling in push-pull circuit with magnetic summing (240 Ω + 0·6 H).
(*b*) Improvement due to parallel capacitor $C_L = 4\mu$F across each coil.
(*After Milnes.*)

across the control windings (shown dotted in *Figure* 10.6), the effect is somewhat reduced, and by placing a capacitor across each individual load coil, the effect can usually be eliminated. The characteristic in this latter case is shown in *Figure* 10.11*b*. The appropriate value of load capacitance is again given by equation 10.22.

Push-Pull Circuits with an A.C. Output

A pair of transductors operating in push-pull are often used as an output stage for servo amplifiers. In many cases the output

stage delivers a.c. power to a two-phase induction motor whose direction of rotation reverses when the input to one phase changes polarity relative to the other. Such a motor has two windings, one is fed from an a.c. supply of constant amplitude and the other is supplied with the output from the magnetic amplifier. The latter

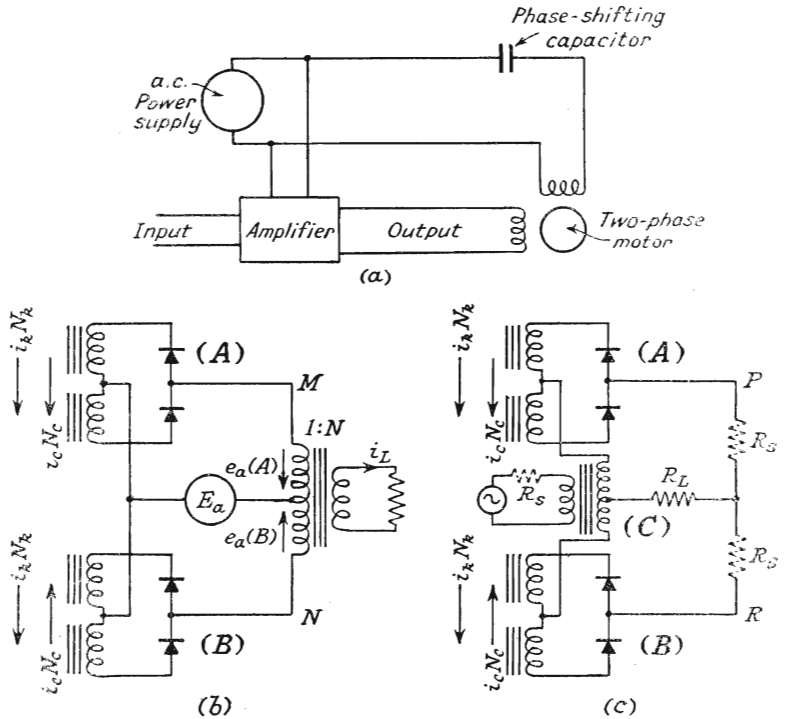


Figure 10.12(*a*). Variable speed two-phase motor.
(*b*) and (*c*). Two alternative types of push-pull circuit giving an a.c. output.

supply is also alternating and is capable of a phase reversal of 180° when the input signal changes sign. The supplies to each winding of the motor must be displaced in phase by 90°. This is achieved by means of a capacitor in series with the fixed phase as shown in *Figure* 10.12*a*. The amplifier output must therefore be alternating and capable of a phase reversal of 180° when the input signal changes sign.

10.14 A.C. Output with Auto-Excited Transductors

There are two auto-excited circuits which may be used to

give an a.c. output with phase reversal ; these are shown in *Figures* 10.12*b* and *c*, both of which can be regarded as systems which are balanced when the control current is zero. The control current causes one transductor to fire before the other and thereby introduces unbalance in the course of which an output is delivered to the load.

Referring to *Figure* 10.12*b*, when both transductors are non-conducting up to a time $t_q(A)$ the whole supply voltage E_a appears across the transductors and consequently the output is zero. If the transductor internal resistance is ignored then the output delivered to the load is given by

$$e_L = N[e_a(A) - e_a(B)].$$

where N is the turns ratio between the primary and secondary windings of the transformer.

The voltages $e_a(A)$ and $e_a(B)$ are either zero or have the same instantaneous polarity as the supply and are in fact represented by the ungated portions of the supply voltage. It follows that if the polarity relationship between the output and the supply is regarded as positive when (A) fires first then it will be negative when (B) fires first. When one transductor has fired and the other is still unfired an output equal in magnitude and sign to that of the fired transductor appears across the load transformer, but after both transductors have fired $e_a(A) = e_a(B)$ and hence the resultant output is again zero.

During the period when transductor (A) is conducting, a voltage is developed across the load which in turn is induced back into the circuit of transductor (B) ; its direction is such as to increase the total voltage applied to the a.c. windings of (B). This leads to the requirement that the supply voltage must be slightly derated to a value which is generally about 75% of the normal design value $\bar{E}_{as}$ for a single transductor. The same phenomenon is seen to arise in the case of *Figure* 10.12*c* ; here the voltage developed across the load during the period of conduction for (A) is applied in series with the supply voltage of transductor (B).

Returning to *Figure* 10.12*b*, during the interval $t_q(A)$ to $t_q(B)$ we have

$$N i_L = i_a$$

and hence

$$N \bar{i}_L = 2f \int_{t_q(A)}^{t_q(B)} i_a dt$$

and since over this period $i_a = \dfrac{E_a}{r_x + R_L/N^2}$

we have $$N\bar{i}_L = 2f \int_{t_q(A)}^{t_q(B)} \left[\frac{E_a dt}{r_x + R_L/N^2}\right] \quad . \quad . \quad . \quad (10.23)$$

also the total voltage applied to transductor (B) after (A) has fired is equal to $E_a + \dfrac{R_L i_L}{N}$, substituting for i_L gives:

$$\text{voltage applied to } (B) = E_a \left(1 + \frac{R_L/N^2}{r_x + R_L/N^2}\right) \quad . \quad . \quad . \quad (10.24)$$

The difference in flux level $\Delta\phi$ between the two transductors arises as before due to the control current and we have

$$\Delta\phi = \frac{2N_c\bar{i}_c}{S} = \left[1 + \frac{R_L/N^2}{r_x + R_L/N^2}\right] \int_{t_q(A)}^{t_q(B)} \frac{E_a}{N_a}\, dt.$$

Combining with 10.23 and collecting terms :

$$\bar{i}_L = \frac{2}{\pi} \frac{\omega M_{ca}}{r_x + 2R_L/N^2} \bar{i}_c/N \quad . \quad . \quad . \quad (10.25)$$

It is worth noting that apart from the factor N the current amplification is almost the same as that found for a single transductor instead of twice the value ; this can be seen if the numerator and denominator of equation 10.25 are both divided by 2. The reason for this is that after (A) has fired the voltage applied to (B) is almost doubled (equation 10.24), so that (B) fires at almost the same time as it would have done if there were no control signal. Thus on applying a control signal, the firing time of (A) is advanced, but that of (B) is not appreciably retarded. Equation (10.24) also shows that the supply voltage requires to be derated to such a value that

$$\bar{E}_a = \bar{E}_{as}\left[\frac{r_x + R_L/N^2}{r_x + 2R_L/N^2}\right] \quad . \quad . \quad . \quad (10.26)$$

In these expressions the internal resistance includes one a.c. winding and one rectifier unit and therefore

$$r_x = r_{aw} + r_{ar}.$$

In both the circuits of *Figure* 10.12*b* and *c* it is seen that when both transductors have fired after $t_q(B)$, the current circulating in both transductors is limited only by their internal

resistances ; therefore, in order to avoid undue power wastage, limiting resistors may be included in series with each transductor ; these are shown in *Figure* 10.12*c*, and in 10.12*b* would be placed in series with each primary winding on the transformer. Since this is, in effect, an increase of the internal resistance, the various expressions already derived for the circuit of *Figure* 10.12*b* apply provided we put

$$r_x = r_{aw} + r_{ar} + R_s.$$

The need to include R_s in the internal resistance is removed if it is placed as a limiting resistor in series with the supply and shown dotted in *Figure* 10.12*c*. In these circumstances it does not reduce the gain of the circuit.[10(c)]

From the functional point of view there is very little to choose between the two alternative circuits in *Figure* 10.12*b* and *c*, but the former has one practical advantage when considering standardization. Since the a.c. load is preceded by a transformer, a standard amplifier can be used for most applications, each one having its own matching transformer.

10.15 A.C. Output Stages Using Simple Series Transductor

Sometimes the pre-amplifier for a servo is electronic and the output stage is magnetic ; in these circumstances the control

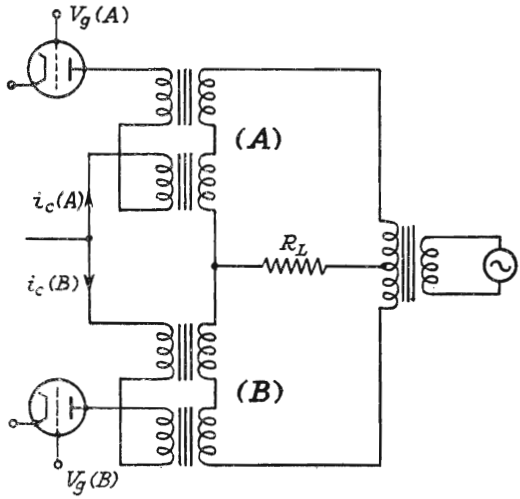


Figure 10.13. A.C. output stage using a pair of series transductors in push-pull operation.

windings can be driven by two pentode valves in push-pull ; this will ensure a short time-constant due to the high impedance control circuit. In these circumstances the power gain required from the magnetic amplifier is often very small and a pair of simple transductors without self-excitation is adequate. Several balanced circuits of this type are available, and an example of one is shown in *Figure* 10.13.

In this case the bias is provided by the standing currents in the two valves. When push-pull inputs are applied to the pentodes, the control current from one anode increases while that from the other anode is reduced. This results in an output whose magntiude and phase depend upon the magnitude and relative sign of the differential input.

Methods of Making Transductor Polarity Sensitive for Push-pull Operation

It will have become evident from the foregoing remarks that one of the main requirements for a transductor used in push-pull circuits of the type so far discussed is that it shall be sensitive to signal polarity. Thus, as we have seen, the application of a bias m.m.f. to the transductor or the use of self- or auto-excitation gives this effect. The latter arrangement is the one most usually employed, and other methods are seldom used. Some of the alternatives have points of interest, however, and will therefore be mentioned briefly.

10.16 Use of Permanent Magnets for Making Transductors Polarity Sensitive

A permanent magnet may be used with a particular type of core construction in order to make the transductor sensitive to signal polarity.[2(c)] A diagram of the arrangement is shown in *Figure* 10.14*a*. This system requires the centre limb type of core construction in which the a.c. windings are wound on the two outer limbs A and C, while the control winding is wound on the centre limb (section 2.13). Fluxes produced by positive current in the control winding are shown dotted ; these fluxes thread the outer limbs, producing magnetization of the core to give normal transductor action.

The magnet is arranged to produce a flux bias in the centre limb, and if the dimensions are appropriately arranged, this bias can reduce the flux in the centre limb to correspond to the negative

knee of the saturation curve. Apart from this, the magnet alone has little effect on the alternating current components in the transductor a.c. circuit. Thus if a positive control voltage is applied the resulting m.m.f. produces normal transductor action in the positive direction. If a negative control current flows, however, since the centre limb is almost saturated in the negative direction, the additional control m.m.f. saturates this limb. Under these circumstances the centre limb virtually becomes an air gap and the control m.m.f. is therefore incapable of producing an appreciable flux in the outer limbs no matter how large. The

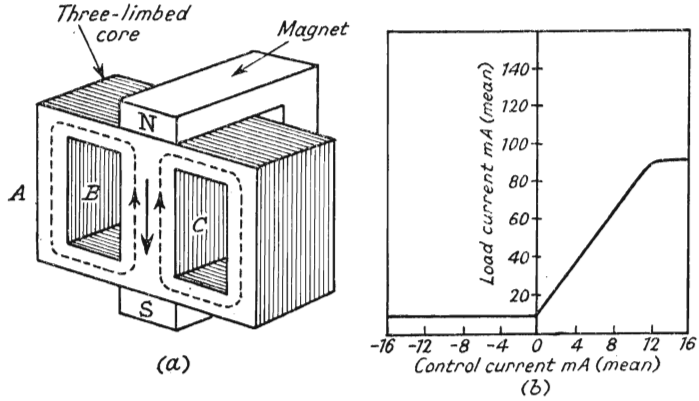


Figure 10.14. Use of a magnet to make transductor polarity sensitive.
(*a*) Assembly of magnet and core.
(*b*) Control characteristic obtained with a permanent magnet flux bias on the centre limb.

effect of this is therefore to make the amplifier effective only for positive signals.

A transductor characteristic using this polarizing arrangement is shown in *Figure* 10.14*b*, where it is seen that while normal operation is obtained for positive signals, the transductor is insensitive to negative signals.

If push-pull action is required, it is easily arranged by using a pair of bar magnets in place of the U magnet ; the remote ends of the bars are then arranged to make magnetic contact with a similar transductor unit which will then be polarized in the reverse direction.

The disadvantage of this system is that it necessitates the use of the centre limb construction which, in general, has poorer zero-stability than the alternative arrangements. Its advantage lies

in the fact that the effect is achieved without using up winding space on the cores, also that it is cheap and relatively easy to manufacture.

The single unit is particularly applicable to relay safety circuits where it might be necessary to operate the transductor for positive signals only.

10.17 Switched Control Circuit Using Rectifiers

If the signal level is sufficiently high, rectifiers may be arranged in the control circuit in such a way that positive control currents are routed through one control winding and negative currents are routed through the adjacent winding. The arrangement is shown in *Figure* 10.15 ; if the control voltage is positive, current is routed through control winding A, and if negative through B. Independent gain adjustment can be made by means of the separate resistors R_c. Sometimes, particularly when the control circuit impedance is low, the rectifiers modify the transductor action due to the rectification of even harmonic components flowing in the control circuit. This gives rise to a form of auto-excitation with a consequent increase of gain and loss of linearity. If the effect is not required it can be reduced by placing capacitors in parallel with each set of control windings, thereby providing an alternative circuit for the even harmonics.

Like the permanent magnet system (section 10.16), this arrangement has the advantage of a low output current in the quiescent transductor. It is only suitable at signal power levels which are sufficiently high to overcome contact potential effects in the rectifiers.

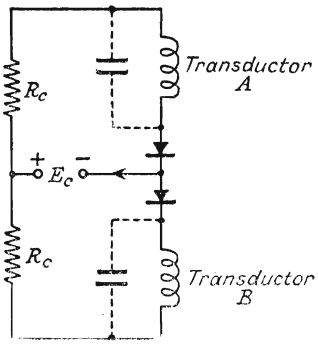


Figure 10.15. Use of rectifiers to make transductors polarity sensitive.

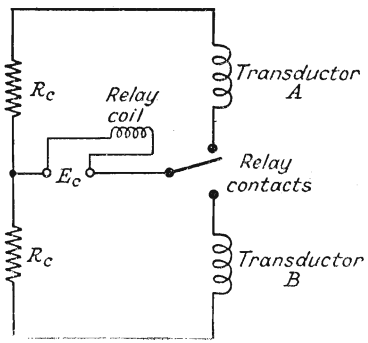


Figure 10.16. Use of relay, to make transductors polarity sensitive.

10.18 Switched Control Circuit Using Polarized Relays

The rectifiers used for routing the control current in the previous example can be replaced by a two-way polarized relay energized by the control current as shown in *Figure* 10.16. For positive control currents the relay makes contact at B, and for negative control currents contact is made at A. This arrangement does not cause interference with the transductor gain as in the previous case, but requires a signal power level sufficient to operate the relay. Other disadvantages arise due to the introduction of a mechanically moving component subject to hysteresis and also the time-constant of the relay, which must be added to that of the transductor.

Transient Response of Push-pull Circuits

10.19 Time-Constant when Load is Resistive

When using auto-excited transductors in push-pull, the transient response is independent of the load or the load circuit configuration provided that it is purely resistive. In these circumstances transients are governed by the simple flux time-constant as already found for the single unit in sections 8.3 to 8.5.

Thus if the quiescent firing angle is 90° (section 10.6) and the effect of circulating currents in the a.c. windings is ignored this time-constant is equal to $2L_{cw}/R_c$ where L_{cw} is the inductance of one control coil as previously and R_c is the total control circuit resistance. Since the control circuit resistance for a push-pull pair may be expected to be twice that of a single unit, the overall time-constant will be the same as that of a single unit.

When the quiescent firing angle is 180° (section 10.7), one of the transductors is virtually out of action since it is operating over the low gain region of its control characteristic. The effective control winding inductance applicable to this unit is therefore very much lower than that of the operating unit and hence its contribution to the control circuit time-constant is comparatively small. The total effective time-constant will therefore be approximately L_{cw}/R_c, i.e. half that of the previous case in which both transductors are fully operative since the same value of control circuit resistance R_c applies to both.

10.20 Inductive Load

When cross-coupling occurs with an inductive load, as in the case of the magnetic summing circuit, the dynamic performance

is liable to be complicated. In the case of the voltage summing circuit, however, the transient performance may be approximated to two simple time-constants, one due to the transductor flux changes and the other due to the load. In addition there will be a constant delay of one half-cycle as found for the single unit.

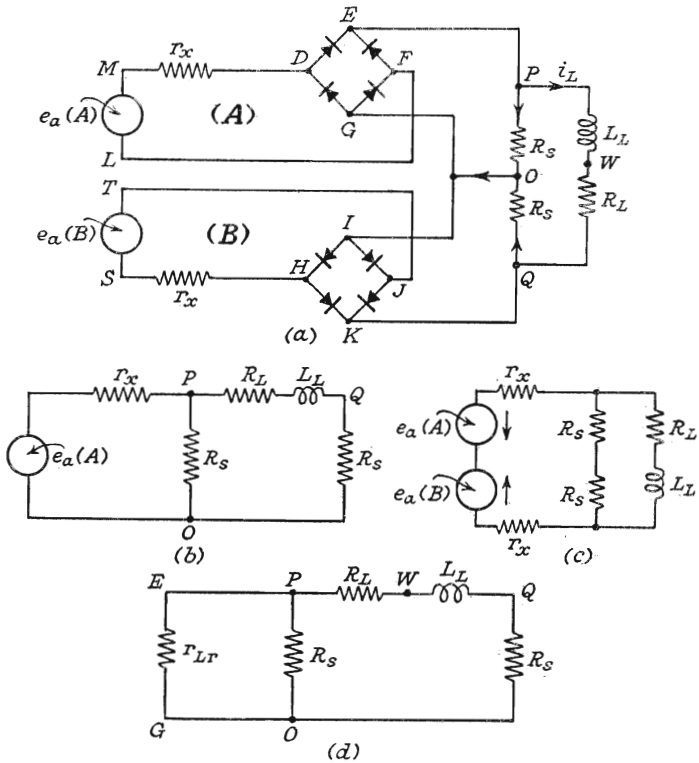


Figure 10.17. Circuits showing effect of inductive load for various modes of transductor operation.

(*a*) Push-pull circuit (throughout cycle).
(*b*) Equivalent circuit when (*A*) is conducting.
(*c*) Equivalent circuit when both transductors are conducting.
(*d*) Equivalent circuit when neither transductor is conducting.

In order to obtain an approximate value for the load time-constant, let us consider *Figure* 10.17*a* under such conditions that transductor (*A*) fires first, thereby tending to make P positive with respect to Q. This output from (*A*) will tend to drive a current through the inductance; the time-constant associated with

these changes will be that of the circuit shown in *Figure* 10.17*b* and will be equal to

$$\frac{L_L\left(1+\frac{r_x}{R_s}\right)}{R_L+R_s+r_x\left(\frac{R_L}{R_s}+2\right)} \quad . \quad . \quad . \quad (10.27)$$

where $r_x = r_{aw} + r_{ar} + 2r_{Lr}$.

When transductor (B) fires, since the load is inductive, the load current cannot instantly become zero, but since the circuit is now symmetrical, and neither of the rectifiers are relaxing, the discharge path consists of the two resistors R_s, each in parallel with the internal resistance of the corresponding transductor unit as shown in *Figure* 10.17*c* ; in this case the time-constant may be represented by the expression

$$\frac{L_L\left(1+\frac{r_x}{R_s}\right)}{R_L+r_x\left(\frac{R_L}{R_s}+2\right)} \quad . \quad . \quad . \quad (10.28)$$

Finally, the third state corresponds to that in which both transductors are non-conducting. If current is still being maintained in the inductance, this current flows in the path *PWQOP* (*Figure* 10.17*a*). This makes Q positive with respect to O and therefore blocks the load rectifier B, which is unable to conduct. Relaxation can occur in load rectifier A, however, and this has the effect of placing the effective resistance r_{Lr} from G to E in parallel with R_s, giving the additional path *PWQOGEP*. Thus in this case the circuit is as shown in *Figure* 10.17*d*, the time-constant of which is

$$\frac{L_L\left(1+\frac{r_{Lr}}{R_s}\right)}{R_L+R_s+r_{Lr}\left(\frac{R_L}{R_s}+2\right)} \quad . \quad . \quad . \quad (10.29)$$

It was shown in Chapter 8 that when considering a single unit the mean time-constant of an inductive load was a variable quantity depending upon the duration of the conducting period and that the time-constant actually had two values, depending upon whether the transductor was non-conducting or conducting. In the present instance we have three possible values for the

time-constant, depending upon whether one unit, both units or neither unit is conducting.

In order to get some idea of the relative magnitudes of these three time-constants we will consider the voltage summing circuit operating under the conditions for which

$$r_x = 0{\cdot}2\ R_L$$
$$R_s = R_L$$

as in section 10.4. If it is also assumed that the resistance r_{Lr} of one element of the load rectifier is equal to one-quarter of the total internal resistance r_x, then the following results are obtained :

(*a*) *One transductor conducting*, time-constant $= 0{\cdot}46\ \dfrac{L_L}{R_L}$

(*b*) *Both transductors conducting*, time-constant $= 0{\cdot}75\ \dfrac{L_L}{R_L}$

(*c*) *Neither transductor conducting*, time-constant $= 0{\cdot}49\ \dfrac{L_L}{R_L}$

This shows that under normal operating conditions for a push-pull pair with voltage summing the effective load time-constant τ_L may be taken as something rather less than the simple time-constant under d.c. conditions.

10.21 Half-Wave Push-pull Amplifier for High-Speed Operation [9(*a*), 10(*d*)]

It was shown in section 9.3 that when using a single-core transductor with half-wave rectified excitation, although the power gain of such an arrangement was greatly reduced, the response time was also reduced to the minimum value dictated by the supply frequency. Such an amplifier has been extended to operate in a push-pull arrangement by using two half-wave units in a bridge circuit with a common load.

The circuit is shown in *Figure* 10.18 and it consists of four equal coils *A*, *B*, *C* and *D*, linked by two cores. It should be noted that coils *A* and *B* correspond to a pair of a.c. windings on one common core (*ab*) while coils *C* and *D* correspond to the other unit having a separate core (*cd*). Thus the a.c. circuit for one half-wave unit is from the supply through coil *A*, the load and coil *B* ; the other circuit comprises coil *C*, the load and coil *D*. Each of these half-wave units is directly comparable with the basic circuit of *Figure* 9.1*a*.

If neither core (*ab*) or (*cd*) is saturated, all four coils form a balanced bridge and no current flows in the load. If either one

of the pair saturates, however, the bridge is unbalanced and the direction of the load current depends upon which core is saturated. If both cores are saturated all four coils are of a low impedance and the bridge is again balanced. The supply voltage is connected to the remote ends of the coils as shown and in series with a resistance r_{ag} sufficiently large to avoid overheating during the period when all four cores are saturated. Bias windings G, H link the cores (ab) and (cd) respectively ; the control windings I, J also link these cores.

Ignoring the bias and control coils, it is seen that on the first positive half-cycle the supply voltage will drive both cores into

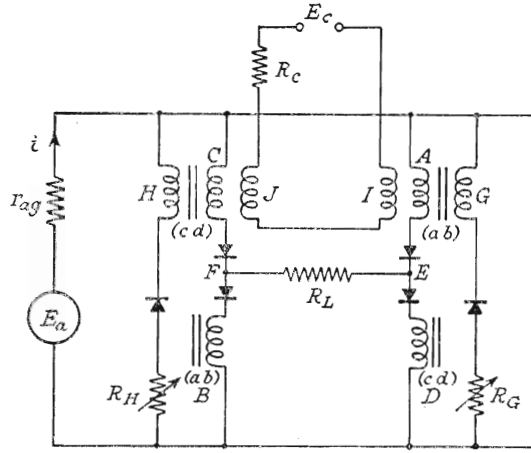


Figure 10.18. Half-wave push-pull amplifier for fast operation.

saturation, and on all succeeding half-cycles the cores will remain permanently saturated provided they are 100% remanent. Thus during each positive half-cycle no load current flows but equal half-wave rectified currents flow down each limb, AD and CB. Each current will be equal to

$$\frac{\frac{1}{2}E_a}{r_{ag}+r_{aw}+r_{ar}}$$

where r_{aw} is the resistance of one coil and r_{ar} that of a rectifier element.

Since the current is a half-wave rectified sinusoid its r.m.s. value (section 2.11) will be

$$\frac{1{\cdot}11}{2\sqrt{2}}\,\frac{\bar{E}_a}{r_{ag}+r_{aw}+r_{ar}}.$$

Therefore the series resistor r_{ag} must be sufficiently large to prevent the windings from overheating under these conditions. During negative half-cycles the supply voltage is not applied to the windings and therefore the cores remain saturated. The bias windings G and H, however, can be made to cause a predetermined decrease of flux during negative half-cycles by adjustment of the resistors R_G and R_H, which should be very large.

Thus in the no-signal condition the cores saturate in each conducting half-cycle at a time depending upon the amount of flux set by the bias windings in the previous half-cycle. If a control voltage is applied, it will oppose the flux setting due to the bias from coil G and assist the flux setting due to coil H. Thus the resultant flux set in the core (ab), for example, will be less than that set in core (cd) by an amount depending upon the magnitude of the control voltage. Therefore in the following conducting half-cycle core (ab) will saturate before core (cd), and over the period when the bridge is unbalanced a current will flow in the load. The mean value of this current will depend upon the duration of unbalance which in turn depends upon the flux level in core (cd) when (ab) saturates.

Thus by varying the control voltage the mean load current will also be subjected to a corresponding variation.

The following important points should be noted :—

(a) Provided the supply voltage is sufficient to vary the flux from negative to positive saturation in one half-cycle, both cores will be saturated by the end of every positive half-cycle of the supply, therefore the initial condition must be the same at the beginning of every flux setting half-cycle. This ensures a fast response.

(b) The only relevant action due to the control voltage occurs during setting periods, therefore the control can be effected by an alternating voltage equal in phase and frequency to the supply voltage. The amplitude of this voltage then determines the flux change in the core during flux setting periods.

(c) If the control voltage reverses in phase by 180° the difference between the flux set in the cores changes sign so that core (cd) saturates first and therefore the load current reverses. The control windings are therefore suitable to operate from a synchro which is supplied from the voltage source E_a.

(d) During the period when current flows in the load the control

windings are coupled to the a.c. windings, and induced currents can flow in the control circuit. These currents may be limited by the resistance R_c, but the latter will raise the control power and therefore reduce the gain (section 9.3).

In addition, the setting action of the control voltage is limited to one half-cycle, and this also will limit the gain as in other high-speed circuits.

The total effect of these factors is therefore to give a low gain with a fast response, and it is therefore important to be able to cascade these units together in order to obtain a reasonable overall amplification. In practice it is generally possible to achieve a power gain of between 30 and 40 per stage, thus giving a gain of over 1,000 in a two-stage unit, for a response time of two to three cycles.

10.22 Setting up Cascaded Half-Wave Amplifiers

When setting up these amplifiers there are certain requirements which must be satisfied. Firstly an unstable triggering action can

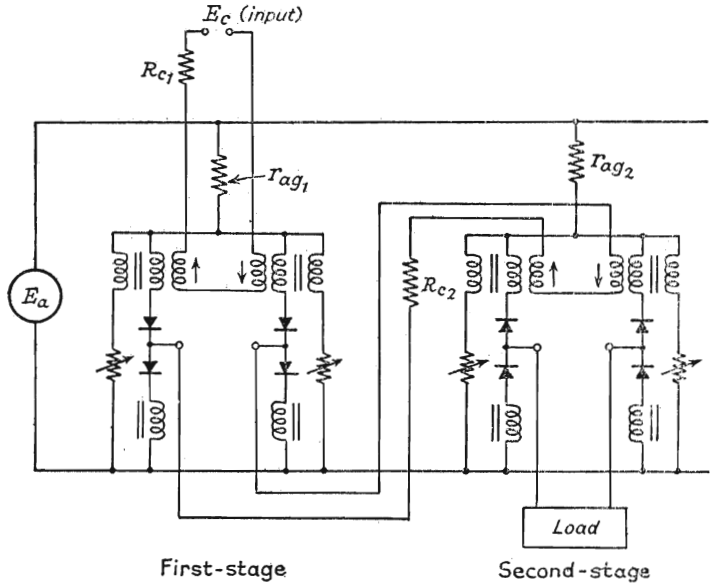


Figure 10.19. Two-stage half-wave amplifier for push-pull operation.

occur if the source resistance r_{ag} is too large, and the critical condition to give instability occurs when the source resistance is equal to the load.

Secondly, when amplifiers are cascaded, as in *Figure* 10.19, it is clear that the output from the first stage must occur during the flux setting period for the second stage, and it therefore follows that the output appears from the second stage when the flux is being set in the first. During conduction in the second stage a voltage is induced in the control windings of this stage, and if this voltage is sufficiently large it could cause one of the rectifier pairs in the first stage to become conducting. The control windings of the first stage would therefore be coupled to the a.c. windings and the flux setting in the first stage would be disturbed. If there were sufficiently few turns on the control windings of the second stage such an induced voltage would not be large enough to give rise to this effect.

Thus each of these amplifiers, when cascaded and operating under the appropriate conditions, should be arranged to have an appreciable voltage gain.

Finally, the bias windings should consist of a few turns of wire in series with a large resistance, and since these windings are similar to high impedance control windings, the bias rectifiers can be disposed of in practice without any deterioration of performance.

There are three main advantages of this circuit. Firstly the cascading which is necessary due to the low gain is achieved with comparatively simple circuitry. Secondly, winding and overall construction is relatively easy due to the fact that there are only two cores each comprising three coils. Thirdly, a.c. and d.c. signals may be superimposed at the input either by simple adding or, as is more usual, by means of an extra control winding. This latter point is very important when using a d.c. tachogenerator to supply velocity feedback from an a.c. motor. A d.c. rate signal can be fed back and mixed with the a.c. control.

10.23 Summary

The two most useful circuits using transductors in push-pull to give a d.c. output are the voltage summing circuit and the magnetic summing circuit. The latter is considerably more efficient than the former but can only be used when twin load coils are available or when the load is centre tapped.

The magnetic summing circuit is liable to have reduced sensitivity near the origin due to mutual coupling between the load coils ; this may be improved by connecting parallel capacitors

across each of the two control windings and also across each of the load windings.

The voltage summing circuit has a low efficiency, approximately 11%, but an amplification comparable with that of the magnetic summing circuit. The optimum circuit proportions are not unduly critical and the summing resistors are usually about equal to the load R_L, while the internal resistance r_a should not, if possible, be greater than one-fifth the load resistance. These proportions strike a reasonable compromise between maximum power output and maximum efficiency. With this circuit the maximum supply voltage can be somewhat higher than that normally used for single transductors. The factor by which the supply voltage may be increased depends upon the quiescent firing angle and the circuit proportions. It varies from unity when this angle is 180° to about 1·6 when the angle is 90°.

Transductors with an a.c. output are usually suitable as output stages for driving two-phase servo-motors. A push-pull pair delivering an a.c. output has a sensitivity of the same order as that of a single unit. The supply voltage normally requires to be somewhat less than that used for single transductors and is usually about 0·6 $\bar{E}_{as}$.

The time-constant of a push-pull transductor is the same as that of a single unit and the additional time-constant of an inductive load fed from the voltage summing circuit is rather less than the load time-constant

$$\frac{L_L}{R_L}.$$

Half-wave amplifiers can be used in a bridge circuit to give a push-pull output, and a number of these amplifiers can readily be cascaded to give a useful gain for a fast response. This type of amplifier is very suitable to amplify the output from a.c. position transmitters to a level suitable for operating a two-phase induction motor for position servos.

Table of Equations derived in Chapter 10

Voltage Mixing Circuit.

(*a*) 90° Quiescent Firing Angle

(i) $$G_i = \frac{2}{\pi} \frac{\omega M_{ca}}{R_L + \left[\frac{R_L}{R_s} + 2\right] r_a}.$$

(ii) Optimum Value of $R_s = \sqrt{R_L r_x}$ (for maximum current).

(iii) Voltage Uprating Factor

$$K = 1 + \frac{R_s}{\left[R_L + \left(\frac{R_L}{R_s} + 2\right) r_x\right]}.$$

(*b*) 180° Quiescent Firing Angle

(i) $G_i = \frac{1}{\pi} \frac{\omega M_{ca}}{R_L + R_s + \left[\frac{R_L}{R_s} + 2\right] r_x}.$

(ii) Voltage Factor $K = 1$.

M.M.F. Mixing Circuit (Magnetic Summing).

(i) $G_i = \frac{2}{\pi} \frac{\omega M_{ca}}{(r_x + R_L/2)}.$

Transformer A.C. Output Circuit.

(i) $G_i = \frac{2}{\pi} \left[\frac{\omega M_{ca}}{r_x + 2\, R_L/N^2}\right] \frac{1}{N}.$

(ii) Voltage Factor $K = \frac{r_x + R_L/N^2}{r_x + 2R_L/N^2}.$

CHAPTER 11

LOW LEVEL AMPLIFICATION AND MULTI-STAGE AMPLIFIERS

General Problems of Amplification at Low Power Levels

When determining whether a magnetic amplifier is suitable for a given application there are a number of considerations which have to be taken into account. Some factors by themselves place a definite limit on the extent to which the magnetic amplifier may be applied. For example, the response time of a magnetic amplifier cannot be less than one half-cycle of the supply frequency, and if the frequency is fixed then this limit is immediately imposed quite independently of other considerations. In other cases, however, it is often possible to make a compromise between two conflicting characteristics. Thus if a highly linear amplifier is required, it is generally possible to achieve this by negative feedback at the expense of gain.

In addition to the dynamic operation of magnetic amplifiers discussed in Chapter 8, their zero stability over long periods is also a fundamental limitation, and a considerable amount of work has been devoted to this subject. This work has been directed towards minimizing the effects of drift and developing circuits having characteristics which are inherently more consistent over long periods of time.

11.1 Zero Drift and Gain Drift

In order to understand the basic features relating to drift, let us first consider an hypothetical amplifier, the characteristic of which is AB shown in *Figure* 11.1*a*, and imagine that the output has been set to a definite value $\bar{\imath}_L$ for a fixed control current $\bar{\imath}_c$.

After a certain length of time has elapsed it might be observed that the output has drifted by an amount $\delta\bar{\imath}_L$, although the control current has been maintained constant. This change can occur in two ways, as shown in *Figure* 11.1*a* and 11.1*b*. In the first case

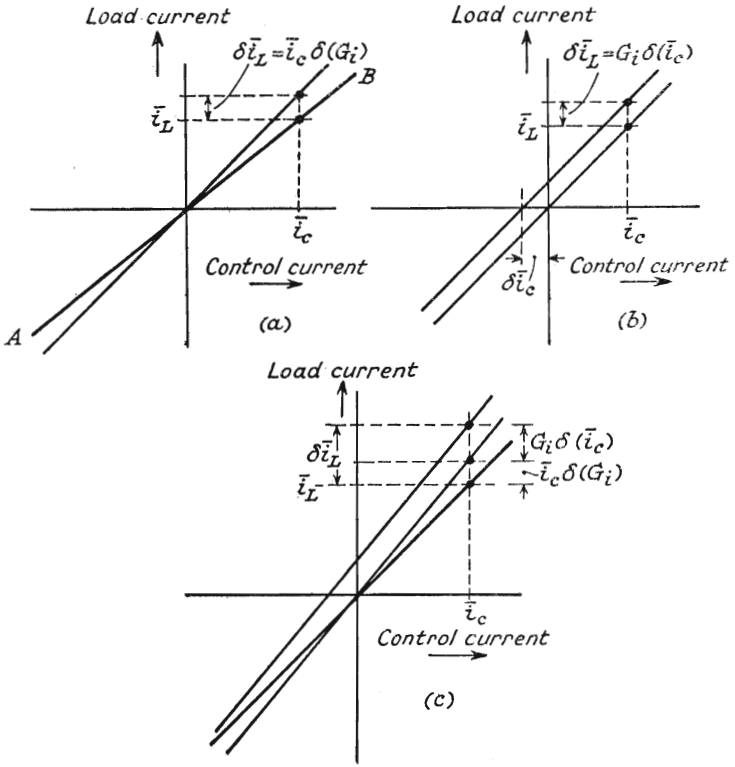


Figure 11.1. Diagrams illustrating effects of drift.
(*a*) Gain drift.
(*b*) Zero drift.
(*c*) Combined effect of zero drift and gain drift.

the change is brought about purely by a change in the slope of the characteristic and may be expressed in a general form, thus :

$$\delta\bar{\imath}_L = \bar{\imath}_c \delta G_i \qquad . \quad . \quad . \quad (11.1)$$

where $\delta\bar{\imath}_L$ is the change of load current resulting from a change δG_i in the current amplification. If the control current is reduced to zero, the load current also becomes zero since the drift term is proportional to the control current, as may be seen from equation 11.1 and the geometry of the diagram.

In the second case, shown in *Figure* 11.1*b*, the change occurs due to a shift in the zero, the gain having remained constant. It is easily seen from the diagram that if the control current is reduced to zero, the load current will not be zero, as it would if no drift had occurred, but will have a magnitude equal to that of the originally observed drift, namely $\delta\bar{\imath}_L$, which may be generally expressed in the form

$$\delta\bar{\imath}_L = G_i\delta\bar{\imath}_c \qquad . \quad . \quad . \quad (11.2)$$

The output may in fact be restored to zero by applying a negative control current equal to $\delta\bar{\imath}_c$.

The two types of drift generally occur together as shown in *Figure* 11.1*c*, and the change in output may be expressed as a function of both variables thus :

$$\delta\bar{\imath}_L = \bar{\imath}_c\delta G_i + G_i\delta\bar{\imath}_c \qquad . \quad . \quad . \quad (11.3)$$

The effect of zero drift is therefore precisely the same as that of a signal, the only difference being that in the latter case the change of output is intentional while in the former case it is spurious. Thus if the amplifier is arranged to give zero output, any zero drift which subsequently occurs is defined as the input which has to be applied in order to restore the output to zero.

The two important differences between zero drift and gain drift may be stated as follows :

(*a*) Zero drift gives rise to an error which is independent of and indistinguishable from an input signal while gain drift produces an error which is proportional to the input signal. It therefore follows that the magnitude of the zero drift in an amplifier determines the minimum signal which an amplifier can be expected to resolve. The occurrence of gain drift, on the other hand, places no such limit since the error is only a fraction of the signal no matter how small the signal is made.

(*b*) Gain drift can be corrected by means of negative feedback, whereas nothing can be done in this respect about zero drift. This may readily be seen as follows, for suppose we have any amplifier of gain G to which an input V_i is applied giving an output V_0 then

$$V_0 = GV_i.$$

If the gain changes by an amount δG then the change of input

necessary to ensure no change of output may be expressed as a fraction thus :

$$\frac{\delta V_i}{V_i} = -\frac{\delta G}{G}.$$

If, however, a proportion H of the output is fed back then we have

$$V_0 = \frac{G}{1 + GH} V_i$$

and the equivalent change of input expressed as a fraction is

$$\frac{\delta V_i}{V_i} = -\frac{\delta G}{G}\left[\frac{1}{1 + GH}\right].$$

Thus when feedback is used the percentage change is reduced by the factor $\frac{1}{1 + GH}$, which becomes smaller as the proportion of negative feedback is increased.

Considering the effect of zero drift, however, if we suppose that in the above case the gain remains constant but the output changes due to a drift which when referred to the input is equal to v, we have

$$V_0 = (V_i + v)G.$$

If a fraction H of the output is fed back then

$$V_0 = (V_i - HV_0 + v)G,$$

giving

$$V_0 = \left[\frac{G}{1 + GH}\right](V_i + v).$$

Thus, whether negative feedback is applied or not, the zero drift is still of the same magnitude in relation to the input signal.

The above considerations show that zero drift is far more serious than gain drift since it places a lower limit on the power level which the amplifier will handle and its effects cannot be reduced by negative feedback.

Zero drift is of course the same as noise and should, therefore, be quoted in relation to the bandwidth ; in the present context, however, the serious effects due to zero drift occur at a very low frequency which is generally much less than 1 cycle per minute and hence, when dealing with d.c. amplification, as is usually the case, it is not practicable to sacrifice bandwidth in order to improve the signal to noise ratio.

An amplifier may be arranged with a meter at the output and

the whole unit regarded as a measuring instrument where the quantity to be measured is the input. Any drift on the output will appear on the measuring instrument and may be regarded as an error in measurement of the input. The maximum variation of gain will then be expressed as the percentage error of the instrument and the maximum possible zero drift will be an absolute error expressed in definite units, e.g. volts, watts, etc. Alternatively, the zero drift is sometimes expressed more artificially as a percentage of the maximum output.

In a magnetic amplifier the zero drift is basically expressible as a certain control m.m.f., or more generally as a magnetizing field **H**. It is more usual, however, to consider the drift as an input power level ; thus if the basic drift of a given circuit using specified rectifiers and core materials is quoted as equivalent to a mean control field $\bar{\mathbf{H}}_c$, we have

$$\delta\bar{\mathbf{H}}_c = \frac{N_c}{l}\,\delta\bar{\imath}_c$$

where l is the mean length of the magnetic path in the core.

In addition, the control winding resistance is given by

$$2r_{cw} = \frac{8}{\pi}\frac{\rho b_c N_c^2}{A_c} \text{ (section 2.8).}$$

By combining these two relations the drift in terms of input power may be expressed as

$$\begin{aligned} 2(\delta\bar{\imath}_c)^2 r_{cw} &= \frac{8\rho b_c l^2}{\pi A_c}(\delta\bar{\mathbf{H}}_c)^2 \\ &= \text{linear dimension} \times (\delta\bar{\mathbf{H}}_c)^2 \end{aligned}$$

and therefore if the circuit, core materials and rectifiers are specified the absolute drift in terms of power is proportional to a linear dimension of the core. Among the smaller types of instrument transductor the linear dimensions do not vary by more than two or three to one, and the zero drift in terms of power levels is therefore often quoted as a constant although, strictly speaking, it does vary with the dimensions and, as would be expected, becomes greater in high power units.

11.2 Causes of Zero Drift in Magnetic Amplifiers

In the case of a single transductor, when no input is applied, there will be some standing output, the magnitude of this output

being dependent upon the bias conditions and the degree of self-excitation. The standing output is liable to drift for a number of reasons :

(*a*) Variation of supply voltage or frequency.
(*b*) Variation of the rectifier reverse characteristics.
(*c*) Variation of bias m.m.f.
(*d*) Variation of the core characteristics.

Other effects such as remanence in the cores due to a large signal having been applied, thermal e.m.f.s at junctions of dissimilar metals, etc., also cause the standing output to change. These latter effects are generally small compared with those listed above, however, and in fact most of the drift in a single transductor unit is due to variation of the supplies or changing rectifier characteristics with temperature. Although variations of frequency, rectifier characteristics and core also give rise to a drift of gain, in the present discussion the input is considered to be zero and therefore any drift occurring due to these factors can only be nullified by applying a correcting control signal. This correcting signal specifies the zero drift, and in the case of a single transductor it will be between 10^{-5} and 10^{-7} watt, depending on the extent to which the power supplies are stabilized, and the materials used for the core.

No improvement is obtained by balancing the standing output against a fixed supply unless the balancing is achieved by means of a supply which tends to vary with temperature and voltage in the same way as the transductor output ; in this case it is usually possible to achieve some degree of compensation. Probably the most effective way of achieving this is to use two transductors in push-pull operation as described in Chapter 10. If this is done the standing output is reduced to zero. It must be remembered, however, that it is given by the difference between two zero errors which are both dependent upon rectifier characteristics and therefore, unless the rectifiers drift by similar amounts, the difference between the two individual outputs will vary.

Supply voltage variations tend to affect both transductors similarly, and so also do bias variations. The resulting drift from these causes is therefore considerably reduced due to the compensating effect of one on the other. It is worth noting, however, that a variation of gain of either transductor can cause a zero drift, since variation of the gain will vary the effect of the bias and hence the balance point.

The push-pull arrangement is much more satisfactory than the single transductor and is generally expected to have a zero drift of between 10^{-8} and 10^{-9} watt for $\pm$ 10% variations of supply voltage and frequency up to supply frequencies of the order of 1 Kc/sec. No definite information is available at higher supply frequencies, but in view of deteriorating rectifier performance and increased iron losses, leading to increased standing currents, the zero drift generally increases appreciably with the supply frequency.

Many of the causes of zero drift in transductors are not yet fully understood, but the foregoing remarks will have shown that a significant part is played by the standing output which exists for zero input ; this output may be regarded as a zero error and it arises because even when the cores are unpolarized, the transductor has a finite impedance due to the unsaturated reactance X_{aw} of the a.c. windings.

In a push-pull amplifier, two transductor outputs are subtracted after each zero error has been subjected to rectification. The stability of the zero will depend upon the magnitude of the error, which will in turn depend upon the amount of drift occurring in the rectifiers.

If there were no zero error then zero drift from this cause would be absent and changing rectifier characteristics would merely give rise to drift of the gain. It therefore follows that better zero stability is achieved with core materials for which the unsaturated reactance is large and iron losses are small since both these effects govern the magnitude of the standing current.

11.3 Use of Even Harmonics to Obtain Improved Zero Stability

A large number of applications which are ideally suited to magnetic amplifiers require a zero stability which is one or two orders better than 10^{-9} watt. In such applications it is usual to employ a pre-amplifier which has a better zero stability than is obtainable with the orthodox circuits. This enables one to amplify the signal to a value well above the drift level of 10^{-9} watt before applying it to the input of a push-pull magnetic amplifier circuit. The problem is therefore to use a circuit having a good zero stability and sufficient amplification to improve the signal/drift ratio by an appreciable amount. Transductor circuits to do this are available and operate by using the even harmonic components in the core m.m.f.s.

It has already been shown in Chapter 3 that the output voltage e_a from a transductor contains odd harmonics only and, furthermore, that in practice this output is always finite (although small) even if the cores are perfectly matched and not polarized with a d.c. signal. It was also shown in Chapters 2 and 3 that the waveform of the control current for an unself-excited transductor contains even harmonics in addition to the direct component. By utilizing these even harmonics it is possible to use the transductor at lower power levels either as a magnetic amplifier or as a magnetic modulator to be followed by an electronic amplifying system. The former are discussed in this chapter and the latter in Chapter 12. Before discussing the various possible arrangements, however, the basic properties of such a system will be discussed.

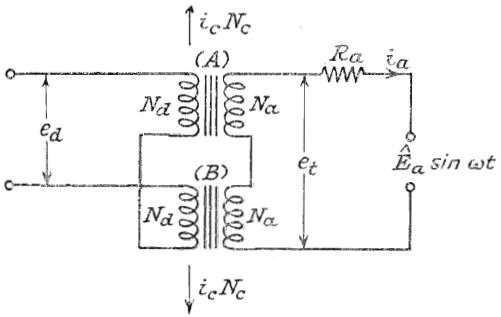


Figure 11.2. Basic magnetic modulator circuit.

In the transductor circuit of *Figure* 11.2, the a.c. supply voltage is assumed to be sinusoidal and we shall consider the voltages e_t and e_d which appear across each pair of windings when the cores A and B are polarized with signal ampere-turns $i_c N_c$ and $-i_c N_c$. The windings on the left will be termed the pick-off windings. These are connected in opposition relative to the a.c. windings and are in fact magnetically coupled to the control windings which are not shown. The latter, however, are assumed to be in series with a high impedance so that the coupling between these coils and the pick-off coils does not affect the alternating voltage e_d. The only function of the control windings is therefore to supply both cores with a steady direct m.m.f. depending upon the magnitude of the direct control current i_c. If the control current varies then the mean value of the voltages e_t and e_d considered over one half-cycle will vary correspondingly due to saturation of the cores.

Thus the system may be regarded as a magnetic modulator whereby the alternating voltages e_t and e_d are modulated by the input i_c. If the voltages e_d and e_t are considered at any instant t' and then again one half-cycle later at $t' + T/2$, it is shown in Appendix 11 that the following relations apply :

$$e_t(t') = -e_t(t' + T/2) \quad . \quad . \quad . \quad (11.4)$$

and

$$e_d(t') = e_d(t' + T/2) \quad . \quad . \quad . \quad (11.5)$$

where $e(t)$ is the instantaneous value of e at time t.

It is also shown that if the control current is zero then

$$e_t = 2N_a \frac{d}{dt}\left[\psi(N_a i_a)\right] \quad . \quad . \quad . \quad (11.6)$$

and

$$e_d = 0 \quad . \quad . \quad . \quad (11.7)$$

It is assumed that the supply is sinusoidal, that the cores and the coils in one pair are identical and that the magnetization curve is expressed in the form $\phi = \psi(F)$.

Now, since t' can be given any value, the relations defined by equations 11.4 and 11.5 apply at all times, and they state that the voltage e_t contains only odd harmonics of the supply frequency while e_d contains only even harmonics.

The circuit may be used as an odd harmonic device as in the case of the transductors so far described, in which case the output is governed by the voltage e_t, or as an even-harmonic device in which the output is given by e_d.

If the two alternative arrangements are compared, the following important differences may be noted.

(1) The effect of zero control signal may be seen from equations 11.6 and 11.7, which show that although the cores are perfectly balanced the odd harmonic output is finite and dependent upon the core properties. The even harmonic output, on the other hand, is zero for zero control signal, and hence, provided the cores are matched, the output is independent of the core properties.

(2) The effect of reversing the sign of the control current is also shown in Appendix 11. It is seen that the odd harmonic voltage e_t is the same whether the control current is positive or negative, whereas the even harmonic output e_d undergoes a phase reversal when the input changes sign.

Thus the even harmonic output exhibits no zero error and is sign conscious regarding the input while, as we have already seen,

the odd harmonic output exhibits neither of these characteristics, both of which can only be produced after rectification by means of one of the summing circuits described in Chapter 10.

In the ideal case, therefore, even harmonics can only be present when the core is asymmetrically excited, and asymmetry can be produced only by d.c. polarization. Thus, since ideally there is no zero error, there can be no zero drift. If temperature variations affect the components this can only cause gain drift which will give an error proportional to the input. Consequently the even harmonic output device will be responsive to any signal however small.

In practice the ideals are not fully realized, mainly due to imperfect matching; but even harmonic transductors are considerably less liable to zero drift than those discussed so far, and there are a number of ways of using the basic circuit, each one designed to have particular features.

In the present context the use of the circuit as a magnetic amplifier is very important and such an amplifier employing even harmonics will now be described in detail.

The Even-Harmonic Transductor [11a, b]

11.4 General Description

In the even-harmonic transductor the load R_L (*Figure* 11.3) is not connected in series with the a.c. coils (*a*), but with pick-off

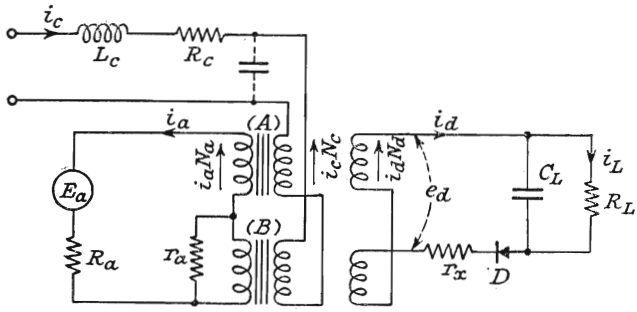


Figure 11.3. Even-harmonic transductor.

windings (*d*) which are geometrically disposed on the cores in the same way as the control windings (*c*). If the cores (*A*) and (*B*) are exactly matched, then when the control current i_c is zero no current flows in the pick-off coils no matter how large the supply voltage

is made, since any e.m.f. induced in the coil on core (A) is exactly opposed by an equal e.m.f. in the corresponding coil on core (B). When the cores are polarized with a d.c. signal, however, asymmetry is introduced, and it will be shown how this gives rise to an output current in the load R_L.

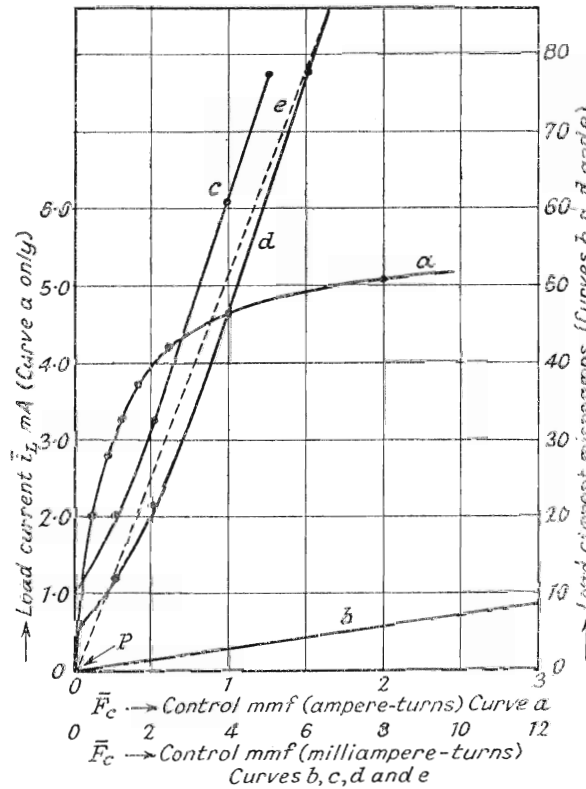


Figure 11.4. Control characteristic of even-harmonic transductor.
(*a*) Typical curve of direct load current plotted against control m.m.f.
(*b*) Observed values of curve (*a*) near the origin with zero load capacitance.
(*c*) Observed values of curve (*a*) near the origin with zero load resistance.
(*d*) Normal characteristic [curve (*a*) within the rectangle P].
(*e*) Calculated characteristic corresponding to curve (*d*).

An inductance L_c is placed in series with the control windings to prevent even-harmonic currents flowing in these windings and thus to confine them to the pick-off circuit. At the same time the d.c. resistance of the control windings is maintained at a low value. Balance of the system is achieved by adjustment of the

cores and by varying the value of r_a, which is a high resistance (usually of the order of 0·1 megohm) placed across one or other of the a.c. coils.

The total a.c. circuit resistance R_a must be sufficient to safeguard the windings from excessive currents when the cores are saturated. A rectifier D is placed in series with the load, and a capacitor C_L in parallel with the load.

If the mean value of the direct load current is plotted against the control m.m.f. a curve of the form shown in *Figure* 11.4*a* is obtained, and the power gain which can be obtained from such a unit is of the order of 1,000.

The device is generally employed as a highly sensitive first stage of amplification, and interest is usually centred on the characteristics within a small rectangle P at the origin in *Figure* 11.4. The characteristic within such a rectangle is replotted as curve (*d*) to expanded ordinate and abscissa scales. The other curves are referred to in section 11.6.

It will be seen immediately that the control ratio is very much larger than that which has hitherto been applicable to transductors. At the same time the standing current for zero signal is two or three orders smaller than the usual value.

In order to examine this circuit, the following assumptions will be made. Firstly that the magnetization characteristic of the cores is given by three straight lines (see *Figure* 3.1*c*). It will also be assumed that the cores are exactly balanced, that the rectifier reverse impedance is infinite, and that a.c. components are completely eliminated from the control windings so that these windings have no effect on the performance beyond polarizing the cores with a constant and unidirectional magneto-motive force $\bar{i}_c N_c$.

With careful balancing it is possible in practice to reduce the a.c. components in the control and pick-off windings to a very low value in the absence of a d.c. signal. The magnitude of the residual current corresponds to an m.m.f. in the order of 10^{-2} ampere-turns/metre, which is three or four orders lower than that normally associated with transductors. Correct balance of the cores is one of the underlying requirements for successful operation of this type of transductor.

11.5 Mode of Operation

In the even-harmonic transductor, both cores are driven well

into saturation by making the supply voltage large. This means that when the control m.m.f. is zero both cores are unsaturated over the first part of the voltage cycle ; over the rest of the cycle both become saturated. If the cores are polarized, however, there is a short interval during each half-cycle when one is saturated and the other unsaturated ; this will be made clearer by tracing the events over one whole cycle of the supply voltage. Referring to *Figure* 11.3, over the first part of the cycle when both cores are unsaturated we have :

Total m.m.f. in core $A = F_a + F_d + F_c = F_a + F_c$ ampere-turns since $F_d = 0$.

Total m.m.f. in core $B = F_a - F_d - F_c = F_a - F_c$ ampere-turns since $F_d = 0$.

If the m.m.f.s are increasing, core A will saturate before core B, the latter having $2F_c$ ampere-turns less than core A and therefore at the instant $t_q(A)$ (*Figure* 11.5*a*) when core A becomes saturated the flux in core B is short of saturation by $2F_c/S$ where S is the core reluctance. During the interval when core A alone is saturated, a current i_d flows in the pick-off coils and supplies a reverse m.m.f. $F_d = i_d N_d$; this current is permitted to flow by the rectifier D. Now core B will not saturate until the flux has changed by an amount $2F_c/S$, which will be equal to the voltage-time integral referred to the pick-off windings and considered over the conducting period of i_d.

During the conducting period the capacitor C_L is charged, but when core B becomes saturated at $t_q(B)$ the capacitor discharges slowly through R_L since the rectifier does not allow reverse currents. When desaturation occurs a small negative voltage pulse is induced across the pick-off coils, but this does not cause any current i_d since the rectifier prevents current in this direction. Therefore nothing further occurs until the following half-cycle, when the m.m.f.s for cores A and B are given by $(-F_a + F_c)$ and $(-F_a - F_c)$ respectively. This time core B becomes saturated before core A at $t_q(A) + T/2$ and an e.m.f. is induced in the pick-off coil on core A. This e.m.f. is in the same direction as that obtained previously, since the cycle is of reversed sign and the winding direction is also reversed. Therefore a current pulse again flows and the load capacitor is recharged.

Thus energy is transferred from the power supply to the load circuit for a short period in each half-cycle, and, as in the case of the auto-excited transductor, the time over which this takes place

is governed by the magnitude of the d.c. signal. Over the period when only one core is saturated the equivalent circuit is as shown in *Figure* 11.5*b*. If the capacitor C_L were absent the energy transferred would be very small due to the poor matching between the high impedance load and the comparatively low impedance

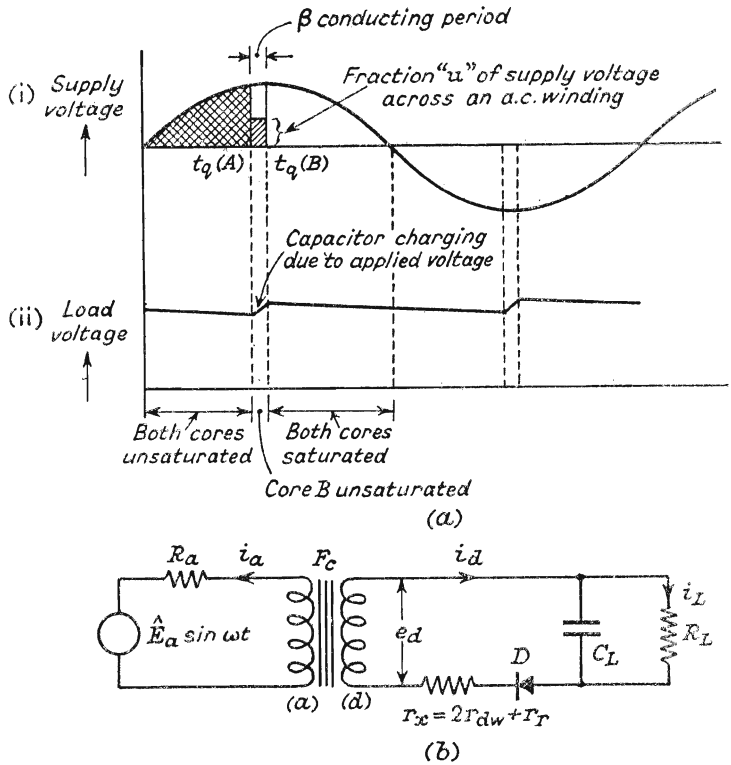


Figure 11.5(*a*). (i) Diagram showing waveform of voltages applied to windings of even-harmonic transductor.
(ii) Load voltage.
(*b*). Circuit of even-harmonic transductor applicable to conducting period $\beta = t_q(B) - t_q(A)$.

supply. In the present case, however, energy is stored in the capacitor during the conducting period, and if the capacitance is sufficiently large considerably more energy can be stored than could be dissipated in the load over the same period. The energy stored in this way is discharged into the load during the quiescent periods. Thus the mechanism is that of pulse lengthening, and brings about the greatest improvement when the conducting period from $t_q(A)$ to $t_q(B)$ is short, that is when the input signal is

small. Since our present interest is confined to a low-level amplifier having the maximum possible gain, the smoothing capacitor plays a very important part in making the even-harmonic transductor a useful amplifying device.

A paper on this circuit has been published and an analysis of the circuit is given in Appendix 12. It is shown that, provided the load capacitor C_L is greater than a certain limiting value, the current amplification is given by

$$\frac{\bar{\imath}_L}{\bar{\imath}_c} = \frac{2}{\pi} r_x \frac{\omega M_{cd}}{\left[1 + (2\beta/T)\frac{R_L}{r_x}\right]} \quad . \quad . \quad . \quad (11.8)$$

where M_{cd} is the mutual inductance between the control windings and the pick-off windings, and r_x is the output impedance, comprising the resistance of the pick-off coils and the forward resistance of the rectifier D, thus

$$r_x = 2r_{dw} + r_r.$$

This expression for the current gain (equation 11.8) will be denoted by G_i (max) since it is the maximum possible gain which would be obtained if the series inductance L_c were indefinitely large, thereby completely eliminating even harmonics from the control circuit, otherwise the actual gain G_i is some fraction of this, depending upon the magnitude of L_c (see section 11.13). It should be noted that if the signal polarity is reversed, the pulses of e.m.f. in the pick-off circuit during conduction are of reversed polarity. Flow of current i_d is therefore blocked by the rectifier (*Figure* 11.5*b*) and the capacitor will not charge. This circuit therefore has a very low gain for negative signals.

The expression for the current amplification is very similar to that found for the auto-excited transductor with capacitor smoothing. In the present case, however, the load is associated with the pick-off circuit, and the constants of this circuit, i.e. M_{cd}, r_{dw}, etc., replace the corresponding constants for the a.c. circuit with the auto-excited or self-excited connections (see equation 6.14).

It is seen from equation (11.8) that if the input level is low, giving only a small conducting period β, then the current amplification is given by

$$G_i \text{ (max)} = \frac{2}{\pi}\frac{\omega M_{cd}}{r_x} \quad . \quad . \quad . \quad (11.9)$$

In the present case our interest is likely to be confined to low

input levels and very seldom, if ever, will the output approach more than a small fraction of its maximum possible value. It follows, therefore, that equation 11.9 gives a realistic estimation of the current gain under normal operating conditions.

PRACTICAL FACTORS INFLUENCING CURRENT AMPLIFICATION

In order to explain many of the results obtained with the even-harmonic transductor, the current amplification expression given in equation (11.8) will be written in the form derived in Appendix 12 thus :

$$G_i\,(\max) = \frac{2}{\pi}\frac{\omega M_{cd}}{r_x}\left[\frac{1}{1 + \dfrac{\bar{\imath}_c}{2\pi\bar{\imath}_{cs}u\sqrt{K-1}}\left(\dfrac{R_L}{r_x}\right)}\right] \quad . \quad (11.10)$$

where $\bar{\imath}_{cs}$ is the control current necessary to saturate a core ; the factor K defines the mean supply voltage $\bar{E}_a$ such that

$$K = \frac{\bar{E}_a}{\bar{E}_{as}} > 1 \quad . \quad . \quad . \quad (11.11)$$

and u is a factor which tends to unity as the a.c. circuit resistance R_a tends to zero (equation 7, Appendix 12).

11.6 The Load Capacitance

The behaviour of the even-harmonic transductor is very dependent upon the load capacitance, but the performance varies with C_L only up to a limit, after which no further improvement is obtained by further increase of the value of C_L. This limiting value corresponds to infinite capacitance for which case equation 11.10 is applicable.

If the capacitor were absent the current amplification would be given by

$$G_i\,(\max) = \frac{2}{\pi}\frac{\omega M_{cd}}{(R_L + r_x)} \quad . \quad . \quad . \quad (11.12)$$

(see equation 3, Appendix 12)

and therefore by comparison with equation 11.9, for small input signals the effect of the capacitor is to increase the amplification by a factor $\dfrac{R_L + r_x}{r_x}$, which in practice may be of the order of 15.

It is also interesting to note from equation 11.12 that if the load resistance is zero the current gain is comparable with that obtained for small signals when using a capacitor and a normal

load resistance (equation 11.9). If the load resistance is zero, however, although the current gain is high there is no useful power gain.

A typical experimental characteristic for the case when $C_L = 0$ is shown in curve (*b*) of *Figure* 11.4, and a similar characteristic when R_L also is zero is shown in curve (*c*). The latter should be compared with curve (*d*), which is the characteristic over the working range using a normal value of load resistance and the limiting value of capacitance. These curves are taken for a mumetal-cored transductor,* and curve (*e*) is derived by calculation from equation 11.9, taking a value for μ of $2\pi \times 10^{-2}$ M.K.S. units (5×10^4 C.G.S. units).

The limiting value of load capacitance must be such that no appreciable discharge takes place during the quiescent intervals which are of duration $(T/2 - \beta)$, and since the conducting period β is usually small, the time-constant of the load circuit must be something greater than one half-period, i.e.

$$R_L C_L > \frac{T}{2}$$

or

$$R_L C_L > \frac{1}{2f}.$$

In the practical example chosen and illustrated in *Figure* 11.4 the supply frequency is 50 cycles per second and, for a load of 2,000 ohms, the limiting capacitance is 4μF.

Although the capacitor C_L gives greatly increased amplification it also causes a certain amount of non-linearity. This is shown in equation (11.10), where the gain is seen to be dependent upon the control current. As the control current increases the gain is reduced ; this is shown experimentally in *Figure* 11.6. Curve (*a*) shows the gain as a fraction of its maximum value plotted against the d.c. m.m.f. when no capacitor is used, while curve (*b*) shows the effect of a smoothing capacitor on the linearity. The slight non-linearity arising in curve (*a*) is probably due to the permeability of the cores not being constant but decreasing slightly with control current. It should be noted, however, that although the gain is reduced by a factor of 2 over the range shown, this range is considerably larger than the normal working range which is defined more typically in *Figure* 11.4.

* The experimental conditions under which the curves were obtained corresponded to the case of infinite control circuit inductance obtained by using an inductor many times larger than would normally be acceptable.

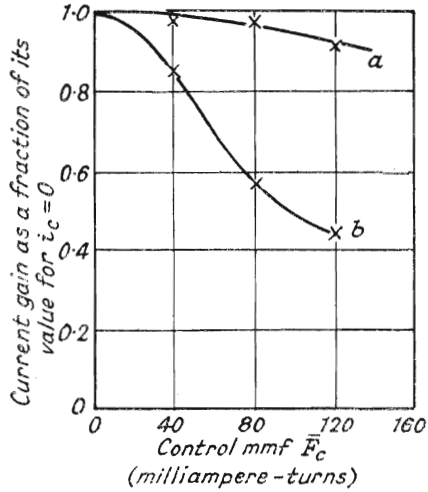


Figure 11.6. Reduction of current gain with increasing input.
(*a*) $C_L = 0$ $R_L = 2200$ ohms.
(*b*) $C = 4\mu F$ $R_L = 2200$ ohms.

11.7 Resistance of the Pick-Off Windings

For small input signals the current amplification is inversely proportional to the resistance of the pick-off windings, including the rectifier forward resistance. In order to obtain a high gain, therefore, these windings should be designed to have a minimum impedance for a given number of turns ; this implies that the pick-off windings should occupy a large proportion, usually at least one-half, of the total winding space available. The depen-

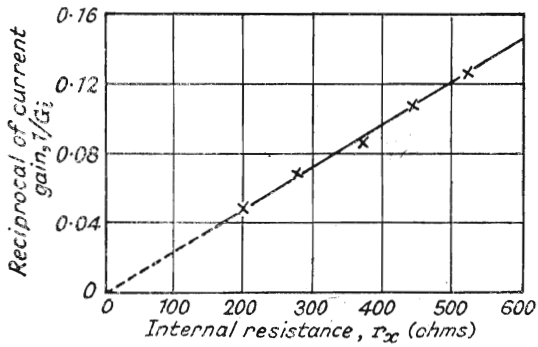


Figure 11.7. Curve showing the relationship between current gain and interna resistance r_x for an even-harmonic transductor.

dence of gain on the internal resistance is shown in *Figure* 11.7 where the internal resistance r_x is plotted against $1/G_i$.

11.8 Effect of Load Resistance R_L

According to the previous remarks, the current amplification is independent of the load resistance provided the input signal is very small. As the load resistance increases, however, the

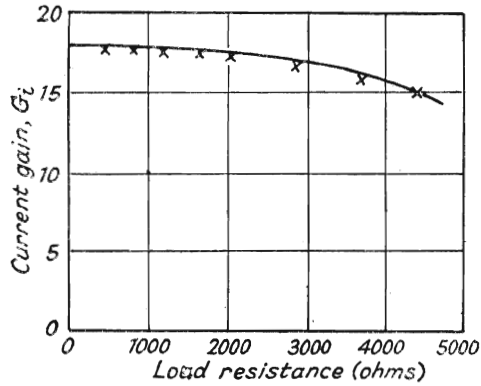


Figure 11.8. Current gain plotted against load resistance for even-harmonic transductor.

characteristic becomes less linear for a given control current, as is shown by equation 11.10. Thus if the ratio $\frac{R_L}{r_x}$ is increased by a factor of 10, the control current must be reduced by the same factor in order to restore the gain to its original value.

The linearity is determined by the factor

$$\frac{\bar{\imath}_c}{2\pi\bar{\imath}_{cs}u\sqrt{K-1}}\left(\frac{R_L}{r_x}\right)$$

which should be much less than unity, and clearly too large a load resistance does not help to give this condition.

In practice, the ratio $\frac{R_L}{r_x}$ is usually chosen to be about 15 ; this is found to give acceptable linearity and a reasonable power amplification. A typical curve showing the relationship between load resistance and current amplification is shown in *Figure* 11.8 ; this curve shows that from $R_L = 0$ to $R_L = 3{,}000$ ohms the change of gain is only 6%.

11.9 Effect of Mutual Inductance and Supply Frequency Variations

It is clear from the discussion in section 11.5 that for a high current gain the mutual inductance M_{cd} must be large. For a given number of turns, this is governed primarily by the core permeability, and hence a high permeability material is desirable. Other factors enter into this problem, however, which are not accounted for in the analysis, and one of the most important of these is iron losses. The effects of the latter increase with frequency, and both the balance of the cores and the gain deteriorates.

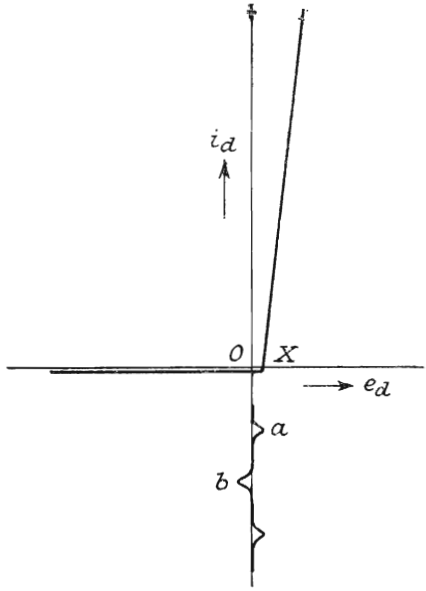


Figure 11.9. Curve showing effect of contact potential on rectifying in action even-harmonic transductor.

Thus although the gain is proportional to the frequency (equation 11.9), these secondary considerations generally limit the maximum practicable supply frequency to about 1 Kc. for mumetal-cored transductors of this type.

11.10 The Rectifier and Core Balance

In the analysis it was assumed that the rectifier characteristic was discontinuous at the origin. In practice, however, a high resistance also tends to be maintained for very small positive voltages and a better representation is given in *Figure* 11.9, where the voltage e_d appearing across the pick-off windings is plotted

against the resulting current i_a which flows through the rectifier. It is seen that in the example given the voltage pulses e_a due to a signal $\bar{\imath}_c$ must increase to a value OX before the rectifier conducts and allows any charge to flow into the capacitor. This means that for very small input signals the device will not operate and it becomes necessary to supply some form of bias to the pick-off circuit in order to enable the rectifier to operate for the smallest values of input. This immediately imposes a lower practical limit on the zero stability since it becomes dependent upon the bias and drift of the rectifier characteristics near the origin.

The positive cut-off voltage OX is normally termed the contact potential, and in practice it is necessary to unbalance the cores sufficiently to overcome this contact potential and thereby introduce the appropriate bias for the rectifiers. The voltage appearing across the pick-off windings as a result of this unbalance contains the fundamental and odd harmonic components of the supply frequency and the unbalance is produced by means of the resistor r_a shown connected across one of the a.c. windings in *Figure* 11.3.

The odd-harmonic bias voltage is shown in *Figure* 11.9 ; it normally occurs as alternate pulses which appear at the instant when the cores saturate. In practice, the effect of a positive signal is then to increase the positive pulses and to decrease the negative pulses, while a negative signal reduces the positive pulses and increases the negative ones. Increased negative pulses do not charge the capacitor C_L, however, due to the high rectifier resistance and therefore, as shown previously, the even-harmonic transductor will not respond to negative signals.

Thus it is seen that unless the rectifier exhibits no contact potential effect there is no virtue in having perfectly matched cores and that improvements to core materials are only of value provided the rectifier is capable of operating at a lower level than that imposed by existing core materials. It should be noted, however, that interest is now centred on zero errors considerably smaller than those normally encountered with auto-excited transductors, and therefore, although this type of transductor exhibits a standing current, its magnitude is two or three orders lower than that of a conventional transductor, as is clearly shown by *Figure* 11.4.

Apart from these effects, variation of the rectifier reverse resistance affects the gain due to the fact that some of the charge in the capacitor C_L leaks away through the reverse resistance during quiescent periods. In a typical example employing

selenium rectifiers it was found that by placing a resistor in parallel with the rectifier and equal to its reverse resistance, the current gain was reduced by a factor of approximately 0·7.

Electronic diodes give unsatisfactory performance in this circuit due to the cathode current which flows for zero anode voltage. This current flows in the pick-off windings and produces a large amount of polarization giving rise to a large output for zero control current and poor zero stability.

11.11 Mean Supply Voltage

The magnitude of the supply voltage $\bar{E}_a$ determines the stability and linearity of the characteristics. In the analysis of the circuit the gain was found to depend upon the non-linear term

$$\frac{\bar{\imath}_c}{2\pi\bar{\imath}_{cs}u\sqrt{K-1}}\left(\frac{R_L}{r_x}\right)$$

which occurred in the denominator of equation 11.10. Provided this term is small, it may vary without appreciably affecting

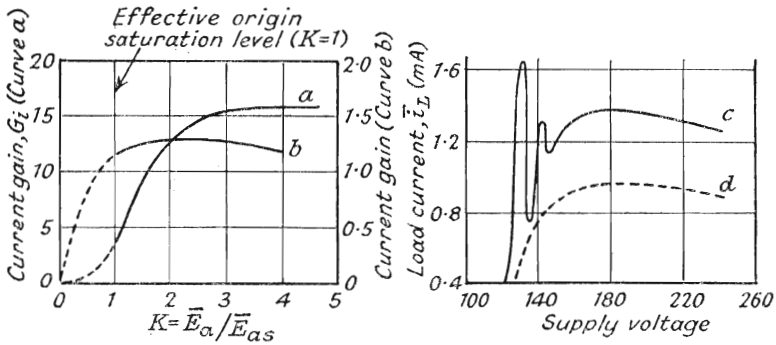


Figure 11.10. Curves showing the effect of variation of supply voltage on the gain of the even-harmonic transductor.

(*a*) Variation of gain when using a load capacitance. (Left-hand ordinate scale.)
(*b*) Variation of gain when $C_L = 0$. (Right-hand ordinate scale.)
(*c*) Curve showing variation of output gain when using a tuned control circuit.
(*d*) Curve obtained without tuning and corresponding to curve (*a*).

(*After Hemmingway and Atkinson.*)

the linearity. It has already been shown how the load resistance and control current influence the linearity, and it may be seen that since

$$K = \frac{\bar{E}_a}{\bar{E}_{as}}$$

the supply voltage also will govern the magnitude of the non-linear term.

The factor K must be greater than unity, otherwise the cores will never be saturated and the transductor will not operate. Also, if K is only slightly greater than unity the non-linear term will be large and the gain will be very dependent upon supply voltage. It is thus necessary to drive the cores well into saturation in order to make K large and the gain independent of supply voltage variations.

Figure 11.10*a* shows how the gain depends upon the supply voltage when a load capacitor is used, but in curve (*b*) it is seen that when this capacitor is zero the gain is virtually independent of supply voltage for values of $k > 1$. In the former case the factor K requires to be about 4 to make the gain reasonably stable.

11.12 Tuning the Control Windings

The current amplification can be increased by a factor of 1·5 to 2 by tuning the control windings with a small parallel capacitor (shown dotted in *Figure* 11.3). The necessary value of capacitor is critical and has a curious effect on the supply voltage regulation

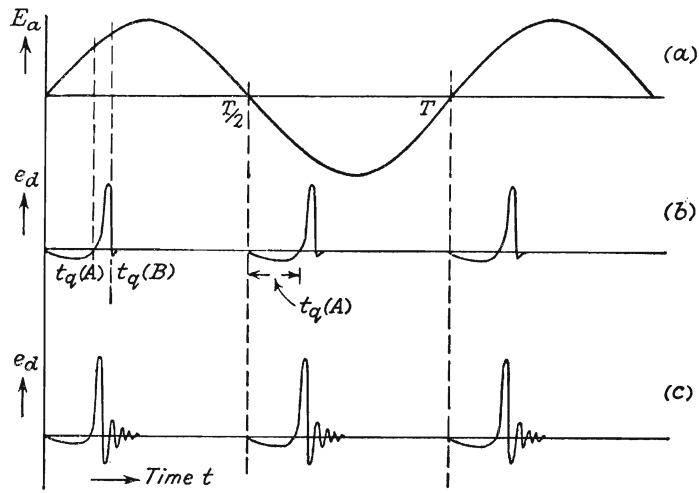


Figure 11.11. Waveforms illustrating the output pulses e_d in relation to the supply voltage.

(*a*) Supply voltage $\hat{E}_a \sin\omega t$.
(*b*) Voltage pulses e_d without tuning of control windings.
(*c*) Voltage pulses e_d with tuned control windings.

curve. The effect has not been completely explained, but a typical curve obtained is shown in *Figure* 11.10*c* ; curve (*d*) is for comparison and is for similar conditions to those of curve (*a*). It is seen that over the working range the gain is increased considerably and that for lower values of supply voltage large oscillations of sensitivity occur. For low values of supply voltage these oscillations are large and hence under these conditions stable operation is not obtained due to the dependence of the gain on the supply voltage.

The effect of this tuning also appears in the waveform of the voltage pulses e_d applied to the load circuit. In the absence of tuning, the output pulses are of the form shown in *Figure* 11.11*b*, but if the control windings are tuned a damped oscillation occurs immediately after the output pulse as shown in *Figure* 11.11*c*. The value of the tuning capacitor should be such that its natural period of oscillation with the inductance of the control windings is less than the time $t_q(A)$. For small signals this corresponds to the duration of the unsaturated period, which will be one-sixth of the total period if the supply voltage factor $K = 4$.

Power Amplification and Time-Constant of Even-Harmonic Transductors

As in the case of the conventional transductor circuits, the useful power amplification is defined as

$$(\text{mean current gain})^2 \cdot \frac{\text{load resistance}}{\text{input resistance}}.$$

As before, the power gain and the time-constant are closely related, as will be shown in the following sections.

11.13 Series Inductance in the Control Circuit

If the control circuit impedance were very low, then a large proportion of the even-harmonic currents would be dissipated in the control circuit with the result that the overall amplification would be considerably reduced. If, on the other hand, the control circuit resistance is increased, much of the energy in the control source is uselessly dissipated in this series resistance. The inclusion of an inductor L_c in series with the control circuit prevents the high-frequency components from flowing in the control circuit, thereby confining them to the pick-off circuit. The impedance of the control circuit to low-frequency signals, on the other hand, is still maintained at a low value. As the signal frequency increases

the inductance presents an increasing impedance to the control voltage, thereby reducing the control current. The required value of the series inductance L_c is therefore dependent upon the overall gain/bandwidth requirements of the amplifier.

The factors which influence the design of the inductor are complicated by the following considerations :

(*a*) The high-frequency components in the control circuit can cause appreciable iron losses in the inductor.
(*b*) The self-capacitance of the inductor windings will introduce appreciable errors due to these high-frequency components.
(*c*) The signal current in the d.c. windings will tend to polarize the inductor.

Under the ideal conditions assumed in the analysis all even-harmonic components are blocked from the control circuit for which L_c would require to be infinite. Under these conditions the useful power gain is given by

$$G_p\,(\max) = [G_i\,(\max)]^2 \frac{R_L}{2r_{cw} + r_c}$$

where r_{cw} is the resistance of each control winding, and r_c that of the inductor. As the inductance of L_c is reduced, however, the effective power gain decreases because an increasing proportion

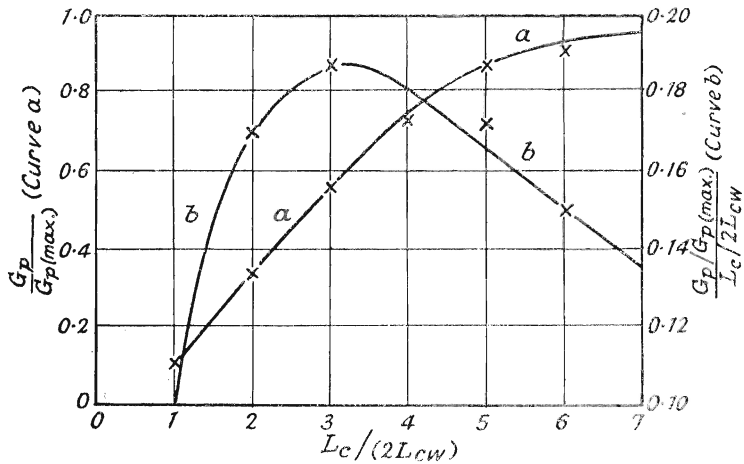


Figure 11.12. Effect of series choke on power gain.
(*a*) $\frac{G_p}{G_p(\max.)}$ plotted against $L_c/(2L_{cw})$.
(*b*) $\frac{G_p/G_p(\max.)}{L_c/(2L_{cw})}$ plotted against $L_c/(2L_{cw})$.

of the energy in the pick-off circuit gets dissipated in the control circuit. The required value of the series inductance L_c is dependent only on the total inductance of the control windings measured under d.c. conditions (i.e. with the a.c. supply switched off). The curve in *Figure* 11.12*a* shows the actual gain G_p plotted as a fraction of its maximum value G_p (max) against the inductance ratio $L_c/2L_{cw}$. Here it is seen that when the series inductance is equal to four times that of the control windings the power gain is 0·7 of its maximum, thus under these conditions

$$G_p = 0{\cdot}7\,[G_i(\max)]^2\,\frac{R_L}{2r_{cw} + r_c}$$

where G_i (max) the ideal current gain is defined either by equation 11.9 or 11.10.

11.14 Time-Constant

The even-harmonic transductor itself has very little inherent time lag. This is because the cores are completely saturated during most of the cycle and therefore the control current can increase practically instantaneously during the saturation interval, since it is not accompanied by flux changes. Unfortunately, however, the series inductance L_c introduces an appreciable delay which is in general somewhat longer than that obtained with an equivalent transductor. Therefore the time-constant τ_t will be equal to L_c/R_c.

The optimum value of L_c is that which gives the maximum power gain for a given time-constant. Thus it is required to know $\frac{G_p}{\tau_t}$ as a function of L_c,

where $$\frac{G_p}{\tau_t} = \frac{G_p}{L_c}\,R_c.$$

If the control circuit resistance is maintained constant then since G_p (max) and L_{cw} are also constant the ratio:

$$\text{power gain/time-constant}$$

will be generalized if it is multiplied by $\dfrac{2L_{cw}}{R_c G_p(\max)}$,

$$\text{thus } \frac{G_p}{\tau_t} \text{ is proportional to } \frac{G_p/G_p\,(\max)}{L_c/2L_{cw}}.$$

This ratio is plotted against $L_c/2L_{cw}$ in *Figure* 11.12*b* where the optimum value of $L_c/2L_{cw}$ is somewhere between 3 and 4. The

time-constant of an even harmonic transductor operating from a 50-c/s supply and having a power gain of 500 is found to be of the order of one second, which is about four times as long as that of an orthodox transductor operating under similar conditions.

If a shorter time-constant is desirable in order to give increased bandwidth, then this may be achieved by reducing the inductance L_c and hence the gain.

The additional time-constant due to the load and smoothing capacitor is of course in cascade with the transductor time-constant τ_t.

11.15 Comparison with Orthodox Transductors

It will by now be appreciated that in many respects the even-harmonic transductor is closely similar to the conventional auto-excited transductor. This is suggested by the form of the current gain equations and the modification introduced by the smoothing capacitor.

In the case of the orthodox transductor, the two cores are never both saturated in the same half-cycle, and this gives rise to a flux time-lag as described in Chapter 3. The time-lag introduced in this way increases the power amplification. In the even-harmonic transductor both the cores are saturated together for a part of each half-cycle and the time-lag is therefore one half-cycle of the supply with a correspondingly reduced gain. The time-constant is increased in the present instance by the additional series inductance, which is nevertheless necessary to give a correspondingly greater power amplification.

Push-Pull Circuits for Even-Harmonic Transductors

11.16 Centre-Tapped Load

One circuit capable of giving a push-pull output is shown in *Figure* 11.13. In this case the load must be centre-tapped, one-half of it operating for positive signals and the other half for negative signals. The gain stability of this circuit is about the same as that of the single output circuit and the zero stability is somewhat better. The circuit is suitable as a first stage for a sensitive magnetic amplifier. The following stage may be a conventional transductor amplifier with centre-tapped control windings.

Since, for a given sign of signal, one-half of the load is quiescent, the operation is essentially that of a single unit.

Final balance of the circuit may be achieved by means of the small presetting potentiometer P shown in *Figure* 11.13.

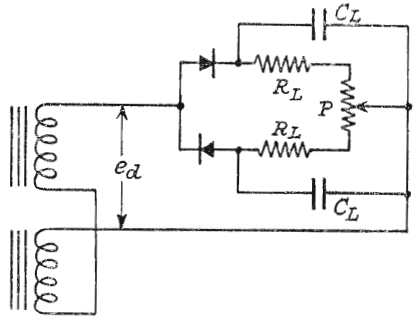


Figure 11.13. Push-pull circuit with centre-tapped load.

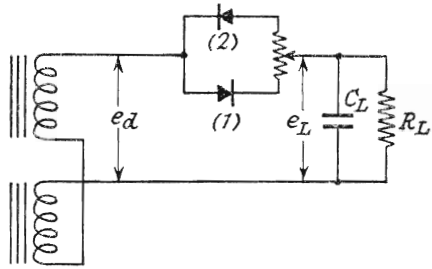


Figure 11.14. Push-pull circuit using balanced rectifier.

11.17 Balanced Rectifier Circuit

The push-pull circuit shown in *Figure* 11.14 is similar to that of *Figure* 11.13, but the zero-stability is appreciably better than the centre-tapped arrangement ; the power gain, on the other hand, is somewhat reduced.

Two conditions are necessary for the correct operation of this circuit. Firstly, the voltage pulses e_d must be carefully adjusted in relation to the non-linear characteristic of the rectifiers near the origin in order to give maximum sensitivity. Secondly, if the signal direction causes the capacitor C_L to charge up via rectifier (1), the voltage levels must be such that appreciable discharge cannot occur via rectifier (2) in the following quiescent period. These two points will now be considered separately.

The characteristic of the parallel rectifier combination is shown

in *Figure* 11.15*a*. For zero signal the two cores require to be unbalanced so that fundamental components of the voltage e_d appear across the pick-off windings. The voltage pulses applied to the rectifier will be equal to $e_d - e_L$, but since e_L is small these pulses are given very nearly by e_d, as shown in *Figure* 11.15*a* and plotted to a vertical time scale. The resulting current i_d which is plotted on the abscissa axis to a horizontal time scale contains no d.c. component. If a signal is applied, even-harmonics appear in the waveform of e_d as shown in *Figure* 11.15*b*, and

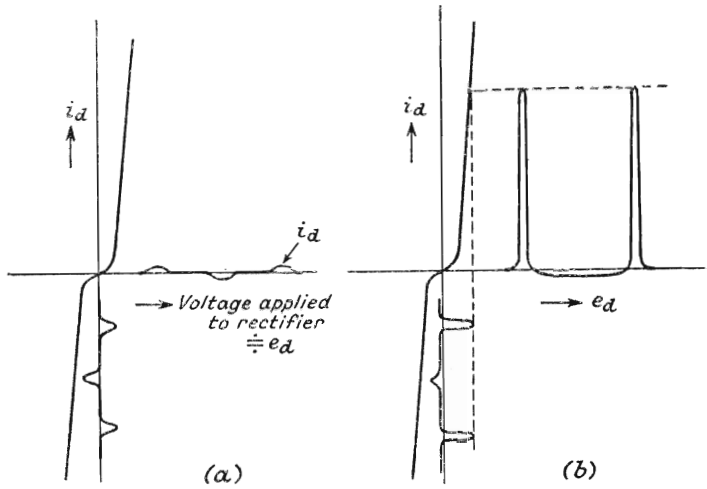


Figure 11.15. Diagrams showing operation of balanced rectifier circuit.
(*a*) At zero signal.
(*b*) Effect of asymmetry produced by a signal.

although the waveform is unsymmetrical it should be noted that it contains no d.c. component. The resulting current now exhibits a d.c. component, however, due to the unsymmetrical operation of the rectifier near the origin. The direction of the d.c. component depends upon the signal direction since the latter determines whether it is the positive or negative peaks of e_d which increase. Thus the pulses of voltage e_d which exist for no signal require to be carefully adjusted in relation to the rectifier characteristics to give maximum sensitivity for the smallest control signals.

The second point, relating to discharge of the load capacitor, may be considered by referring back to the circuit in *Figure* 11.14. When a voltage pulse appears across the pick-off windings, the

forward voltage tending to drive a charging current through rectifier (1), for example, is given by

$$e_r = e_d - e_L$$

and during the following quiescent period the reverse voltage across rectifier (1) is equal to e_L. This reverse voltage acts in the forward direction for rectifier (2), however, and therefore a certain amount of discharge can occur through this rectifier and the pick-off windings. Now, since the rectifier resistance is actually a function of voltage, it is important to ensure voltage levels such that the forward resistance of rectifier (1) during charging periods is small and the forward resistance of rectifier (2) during the following quiescent periods is large.

If it is assumed that the rectifier forward resistance is related to the voltage e_r by an expression of the form

$$r_r = ke_r^{-n},$$

during charging periods the forward resistance of one rectifier is given by

$$r_r \text{ (charge)} = k(e_d - e_L)^{-n}$$

and during the following quiescent period the forward resistance of the other rectifier is

$$r_r \text{ (discharge)} = ke_L^{-n}$$

therefore

$$\frac{r_r \text{ (discharge)}}{r_r \text{ (charge)}} = \left[\frac{e_d - e_L}{e_L}\right]^n$$

for satisfactory operation this ratio should be large compared with unity for which it is required that

$$e_d \gg 2e_L$$

and

$$n \gg 1.$$

Satisfactory performance is therefore governed by the level to which the voltage pulses are adjusted. In addition the material used for the non-linear element must be such as to give a large value for n which implies that the slope of the characteristic must change very rapidly near the origin.

If the signal polarity is reversed the two rectifiers exchange their operation and hence they must have the same characteristics.

The combined characteristic of two metal rectifiers is symmetrical and can be produced by a single silicon carbide resistor which should give better zero stability since differential drift in the two rectifiers is eliminated. Silicon carbide elements, however,

give lower sensitivity and in general the parallel rectifier arrangement is preferred.

It has been found that if the load is not truly passive and voltages are induced in the load from some other source, the operation of the circuit is liable to be upset. These conditions might arise if the load were a stage of normal transductor amplification with induced voltages in the control circuit. The centre-tapped arrangement is therefore generally preferred as a pre-amplifier for auto-excited transductors. The centre-tapped amplifier may, however, be preceded by a stage of amplification using the balanced rectifier circuit in order to give improved zero stability if this is required.

Practical Details Relating to Even-Harmonic Transductors

11.18 Power Gain and Zero Stability

Of the three circuits discussed in the foregoing pages the single output unit undoubtedly gives the greatest power gain, and in the case of a small instrument transductor operating with mumetal toroids at a supply frequency of 50 cycles per second, the gain can be over 1,000. The zero stability, on the other hand, is not so good as with the balanced circuits, and the figure usually obtained is between 10^{-10} and 10^{-11} watt. It is interesting to note, however, that this is about two orders better than that obtainable even with a pair of push-pull auto-excited transductors.

In the centre-tapped arrangement discussed in section 11.16 the zero stability is appreciably improved and is normally rather better than 10^{-11} watt. When using the balanced rectifier circuit the power gain is generally as low as 300–400, but the zero stability is of the order of 10^{-12} watt. These figures refer to the power gain when the bandwidth is very low, in which case the time-constant is usually of the order of $\frac{1}{2}$ second, which corresponds to a signal cut-off frequency of about $\frac{1}{3}$ cycle/second. Improved frequency response can be achieved by sacrificing power amplification.

The problem as to which circuit to use for a given pre-amplifier may best be illustrated by two examples.

(1) The second stage of amplification is a magnetic amplifier having a zero stability referred to its input of 10^{-9} watt, and for a first stage of amplification the choice lies between two even-harmonic transductors, one having a centre-tapped output and

the other having the balanced rectifier output. In the former case the zero stability is 10^{-11} watt and the power gain 1,000, while in the latter case the zero stability is 10^{-12} watt while the gain is 300.

In both cases there are two figures to be considered for the zero drift, the drift of the second stage referred to the input of the first and the inherent drift of the first stage. The larger of these two must be taken as the zero drift.

When using the centre-tapped output arrangement the drift referred from the second stage is

$$\frac{10^{-9}}{1{,}000} = 10^{-12} \text{ watt},$$

while the drift of the first stage is 10^{-11} watt. Since the latter figure is the larger by a factor of 10, this must be taken as the zero drift.

If the balanced rectifier circuit is used the drift referred from the second stage is $\frac{10^{-9}}{300} = 3{\cdot}3 \times 10^{-12}$ watt, while the drift of the first stage is 10^{-12} ; the former figure of $3{\cdot}3 \times 10^{-12}$ is taken to be the zero drift and this shows a factor of 3 : 1 over the centre-tapped output circuit. Therefore, from the point of view of zero drift, the balanced rectifier circuit is preferable.

(2) The second stage of amplification is a magnetic amplifier having a zero stability of 10^{-7} watt.

In this case the pre-amplifier requires a high gain in order to reduce the comparatively large drift referred from the second stage and by putting in the appropriate figures it will easily be seen that the centre-tapped arrangement is preferable.

11.19 Impedance Range

Although the dynamic impedance of any amplifier may be either increased or decreased by the use of negative feedback, the input *resistance* is the important consideration when matching for the best zero stability (see section 11.21).

The input and output impedances are limited by the difficulty of winding coils with a large number of turns of very fine wire without introducing parasitic effects such as excessive self-capacitance. The practicable range of input impedances generally lies between 0 and 5,000 ohms, while the load impedance might be as high as 10,000 ohms.

11.20 Screening and Balancing the Even-Harmonic Transductor

The resultant component of the earth's magnetic field is sufficient to cause appreciable polarization of the cores. For this reason the figures given for zero drift necessarily apply only when the unit is enclosed in a high-permeability box. This is particularly important if the even-harmonic transductor is incorporated in equipment which is liable to be moved from place to place and therefore subject to varying external magnetic fields. The effect of the earth's field must also be taken into consideration when the cores are being balanced before final assembly. In practice, the amplifier is first balanced approximately with the full supply voltage applied to the a.c. coils (a), the pick-off circuit being open and the control current, i_c, zero (*Figure* 11.3). The out-of-balance voltage peaks across coils (d) are viewed on an oscillograph and the unit is rotated in the earth's field until the positive and negative peaks are similar. Laminations may then be removed from one or other of the cores until the oscilloscope indicates a minimum voltage. If toroidal cores are being used, then this preliminary balance must be achieved by using a matched pair of toroids.

Final balance is secured when the circuit is connected up, but with zero control current. A microammeter is connected in series with the load ; the resistor r_a, across one or other of the a.c. windings, is then adjusted until the meter reading is reduced to a minimum.

When the amplifier is balanced it should be clamped up and sealed in a can.

Magnetic Amplifiers in Cascade

11.21 Use of Negative Feedback

Once the zero drift has been reduced to the required level by the appropriate choice of input amplifier, the overall gain of a number of amplifiers in cascade may be stabilized by the use of negative feedback. The type of feedback used depends largely on the purpose for which the amplifier is required. Thus if the input to the amplifier is a voltage from a thermocouple and the output is required to drive a recording instrument dependent upon the load current, the ratio of $\frac{\text{output current}}{\text{input voltage}}$ should be stabilized by feeding back a voltage proportional to the load

current and comparing this with the input voltage. This not only stabilizes the system against changes of gain in the amplifier, but also includes the input and output resistances in the closed loop, thereby making the effect of temperature changes on these resistances less significant. This type of feedback is shown schematically in *Figure* 11.16*a*. Since in this case the output

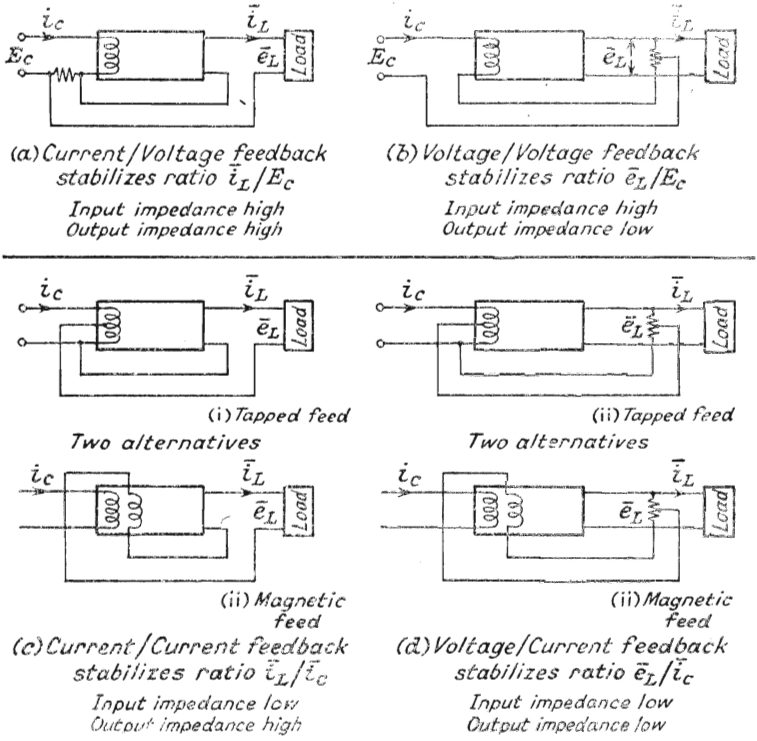


Figure 11.16. Diagrams showing various types of feedback and summarizing properties of each circuit.

current is stabilized against load impedance changes the device has a high output impedance ; it also has a high input impedance.

Alternatively, the load voltage may be stabilized by feeding back a proportion of the load voltage as shown in *Figure* 11.16*b*. This has already been discussed for the single transductor in section 9.11 with reference to the response time. In addition, this type of feedback tends to make the load voltage independent of the load and hence the output impedance of the feedback amplifier is low.

In both of these cases the feedback is applied to the input as a voltage in series with the signal, thereby tending to make the input impedance high.

The feedback can be applied to the input as a current, however, by means of the parallel tapped feed shown in *Figure* 11.16c. This gives the amplifier a low input impedance. Alternatively, current feedback can be applied to an extra winding coupled magnetically to the control winding (see also *Figure* 11.16c). The four possible types of feedback are shown in *Figure* 11.16, where the properties of each are tabulated.

It is sometimes considered advantageous to use the transductor circuits without self-excitation, thereby employing the inherent feedback described in Chapter 4 in order to get stable characteristics, and although this system is very simple, since it obviates the need for auto-excitation rectifiers, it is open to two objections. It will be remembered that the inherent feedback tends to stabilize the ratio of the mean transductor alternating current $\bar{\imath}_a$ to the control current $\bar{\imath}_c$. In the first place, interest is usually centred on the direct load current $\bar{\imath}_L$ and this is only equal to the alternating current if the load rectifier is perfect and the load is resistive.

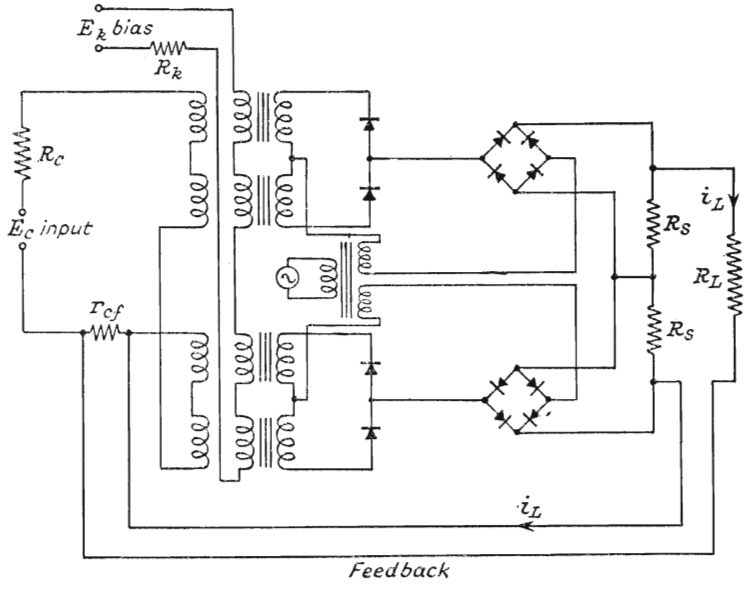


Figure 11.17. Push-pull amplifier with voltage mixing and overall negative feedback which stabilizes the ratio of $\bar{\imath}_L/E_c$.

The first objection is therefore that the load rectifier is outside the feedback loop. The second objection lies in the fact that the control circuit resistance is also outside the feedback loop. The transductor is generally operated from a voltage device such as a thermocouple, and if the control circuit resistance varies the control current will vary ; it is therefore desirable if possible to stabilize the amplifier against variation of the control circuit impedance. The preferred arrangement is to use an auto-excited transductor as the basic amplifier and then to employ negative feedback in order to stabilize the ratio of the load current to the control e.m.f. This ensures that both the load rectifier and the control current resistance are included in the overall feedback loop. An example of such an arrangement is shown in *Figure* 11.17 for a push-pull magnetic amplifier with voltage summing. In this case the load current is fed back as a voltage across the feedback resistor r_{cf} and opposing the control voltage. Such an arrangement reduces the sensitivity to temperature variations acting on the control circuit.

Assuming the feedback resistor is small compared with the control circuit resistance R_c and the load R_L, the resultant control e.m.f. in the control circuit is equal to

$$E_c - \bar{\imath}_L r_{cf}$$

and the mean control current is therefore given by

$$\bar{\imath}_c = \frac{E_c - \bar{\imath}_L r_{cf}}{(R_c + r_{cf})} \quad . \quad . \quad . \quad (11.13)$$

Also, assuming that the characteristics of the load rectifier are included in the current amplification expression G_i, we have

$$\frac{\bar{\imath}_L}{\bar{\imath}_c} = G_i$$

and from (11.13) this gives

$$\bar{\imath}_L = \frac{G_i/(R_c + r_{cf})}{1 + r_{cf}G_i/(R_c + r_{cf})} E_c \quad . \quad . \quad . \quad (11.14)$$

If the gain G_i of the auto-excited transductor is sufficiently large, then

$$\bar{\imath}_L = E_c/r_{cf} \quad . \quad . \quad . \quad (11.15)$$

In other words, the output/input relationship is dependent only upon the feedback resistor. This feedback resistor can be closely controlled by making it of manganin wire having a comparatively small temperature coefficient of resistance.

11.22 Dynamic Stability of Multi-Stage Amplifiers

The previous remarks on negative feedback apply equally well to multi-stage amplifiers, in which case, however, dynamic instability may occur due to the cascading of a number of time lags in a closed loop (Chapter 7). In these circumstances it may be possible to " shape " the open loop frequency response suitably by feeding back derivatives of the output. Otherwise, although the final response is more sluggish, it is usually possible to eliminate instability by making one of the time-constants large compared with the others. This is generally arranged in practice by making the time-constant of the output stage very large compared with that of the pre-amplifiers.

11.23 Optimum Control Circuit Impedance in the Presence of Negative Feedback

In the case of a magnetic amplifier fed from a voltage source having a finite resistance it is easy to show that in common with other power-operated devices, the maximum transfer of power from the source to the amplifier occurs when the winding resistance of the amplifier is made equal to the source resistance.

If negative feedback is applied, such as in *Figure* 11.16*a* and 11.17, the effective input impedance of the amplifier, the dynamic impedance, is increased, and the question then arises as to whether this dynamic impedance should be matched to the source, or the winding resistance matched to the source as previously.

In order to minimize errors arising from zero drift it has been shown [11(c)] that the winding resistance should still be matched to the source. This may be seen immediately from the following physical argument. If we consider an amplifier with negative feedback, as in *Figure* 11.16*a*, the input to this amplifier will be a voltage, and if any zero drift occurs then it will appear in the output as an error in measuring the input voltage. Let us suppose that the input is initially adjusted to give zero output. Since the output is zero there is nothing fed back to the input. Let us now suppose that over a period of one hour, say, the output drifts to some finite value. The zero drift is then given by the amount which the input must be changed in order to restore the output to zero, this being the error involved in measuring the input voltage. If this change is made, then when the output has been restored to zero there is still no negative feedback actually applied since the output is zero again. Thus, so long as the output is maintained

at zero, changes of the input voltage take place as though there were no feedback, these changes measuring the voltage error. The problem now is to arrange for these changes of input voltage to transfer sufficient power to maintain zero output for a minimum change of signal voltage and, as we have seen, this is done by matching the input impedance of the amplifier to the source. But throughout the operation no feedback has been applied and therefore the input impedance of the amplifier is given by that of the control windings and not the effective impedance due to feedback. Thus, in order to minimize errors in amplification of the input voltage, the control windings must have the same resistance as the source as is the case when no feedback is employed.

It should be noted that the source resistance includes the feedback resistance r_{cf}. Thus if the control source has a resistance r_{cs}, the optimum condition applies when

$$\text{resistance of control windings} = r_{cf} + r_{cs} \quad . \quad (11.16)$$

This will now be illustrated by a practical numerical example.

A particular three-stage amplifier with negative feedback designed for thermocouple work has a sensitivity of 5 mA output for 4 mV input. The following information will be required :

(*a*) The winding resistance r_{cw} which will give the smallest errors in temperature measurement due to amplifier zero drift if the source of e.m.f. is a thermocouple having a total resistance, including leads of 10 ohms.

(*b*) The actual error in temperature measurement caused by drift under the optimum conditions determined in (*a*) if the zero drift in the magnetic amplifier is known to correspond to an input power of 10^{-12} watt assuming the thermocouple sensitivity is 40 mV per 1,000° C. :

(i) With an amplification setting of 5 mA output for 4 mV input with negative feedback ;

(ii) With an amplification setting of 5 mA output for 40 mV input with negative feedback.

From equation 11.15 the feedback resistor is given by :

$$\text{(i)} \quad r_{cf} = \frac{E_c}{i_L} = \frac{4}{5} \text{ ohms} = 0{\cdot}8 \text{ ohms}$$

$$\text{(ii)} \quad r_{cf} = \frac{40}{5} \text{ ohms} = 8{\cdot}0 \text{ ohms}$$

Thus for condition (i) we have the optimum winding resistance

$$r_{cw} = r_{cs} + r_{cf} = 10{\cdot}8 \text{ ohms}$$

and for (ii)

$$r_{cw} = 18 \text{ ohms.}$$

The drift may be expressed in terms of the control current thus :

$$\bar{i}_c^2 r_{cw} = 10^{-12} \text{ watt}$$

and the voltage generated by the thermocouple corresponding to this is given by

$$E_c = (r_{cw} + r_{cs} + r_{cf})\bar{i}_c$$

and under the optimum conditions (equation 11.16)

$$E_c = 2r_{cw}\bar{i}_c$$

eliminating $\bar{i}_c$ we get

$$\frac{E_c^2}{4r_{cw}} = 10^{-12}.$$

therefore the drift voltage $= 2\sqrt{r_{cw}} \times 10^{-6}$ volts

for case (i) $r_{cw} = 10{\cdot}8$ ohms, therefore drift voltage $= 6{\cdot}55\ \mu$V.

The temperature sensitivity is 40 μV per degree and the error due to drift is therefore

$$\left(\frac{6{\cdot}55}{40}\right)^\circ \text{C.} = \frac{1^\circ \text{ C.}}{6}$$

for case (ii) $r_{cw} = 18$ ohms.

Therefore the drift voltage $= 8{\cdot}5\ \mu$V, and the error in temperature measurement is

$$\left(\frac{8{\cdot}5}{40}\right)^\circ \text{C.} = \frac{1}{4{\cdot}7}{}^\circ \text{ C.}$$

11.24 Summary

Drift in d.c. amplifiers consists of zero drift and gain drift. The latter can be reduced by means of negative feedback but the former is primarily dependent upon the mechanism of amplification and is normally expressed in definite units, e.g. volts, watts, etc. The zero stability of push-pull magnetic amplifiers is approximately 10^{-8} watt and the gain stability about 5% or 10% without negative feedback, depending upon the supply voltage and frequency variations to which the amplifier is subjected. Linearity and gain stability can be improved considerably by the appropriate negative feedback, but improved zero stability can only be achieved by using a stable pre-amplifier so that the drift referred

to the input is reduced by a factor equal to the gain of the pre-amplifier. The total gain may be reduced to the required value by negative feedback over the constituent stages.

In order to obtain minimum errors due to zero drift, the resistance of the control windings should be matched to that of the control source. This condition still holds good in the presence of negative feedback.

A number of transductor circuits are capable of operating at input power levels as low as 10^{-11} or 10^{-12} watt. In such circuits the load current is obtained by rectifying the even-harmonic components of current flowing in the pick-off windings and then pulse lengthening these currents by means of a reservoir capacitor across the load. With these circuits the power gain obtainable is of the order of 1,000.

The following equations summarize the more important results obtained in this chapter :

(1) Maximum current amplification for an even-harmonic transductor with $L_c = \infty$

(*a*) large signals $$G_i(\max) = \frac{2}{\pi}\frac{\omega M_{cd}}{r_x}\left[\frac{1}{1+\dfrac{\bar{\imath}_c}{2\pi\bar{\imath}_{cs}u\sqrt{K-1}}\left(\dfrac{R_L}{r_x}\right)}\right]$$

(*b*) small signals $$G_i(\max) = \frac{2}{\pi}\frac{\omega M_{cd}}{r_x}.$$

(2) Useful power amplification for even-harmonic transductor with $L_c = 8L_{cw}$

$$G_p = 0{\cdot}7\,[G_i(\max)]^2 \cdot \frac{R_L}{2r_{cw} + r_c}$$

where r_c is the resistance of the inductor L_c.

CHAPTER 12

MAGNETIC MODULATORS

It has been seen that for magnetic amplifiers the zero stabilty ranges from about 10^{-5} watt for the single transductor down to 10^{-12} watt for even-harmonic transductors using a balanced output circuit. It has also been shown that the main limitations are caused by the rectifiers used in these circuits, and it therefore follows that if their effect can be eliminated the low-level sensitivity of the amplifier will be increased.

Experimental evidence of the stability which can be obtained by using magnetic cores without metal rectifiers has been provided by the work of Williams and Noble,[12(a)] in which a d.c. amplifier employing a magnetic modulator input operated at a noise level of 10^{-18} watt over a bandwidth of 3 cycles/sec.

The types of amplifier described in this chapter in general employ electronic amplifiers preceded by magnetic modulators with a range of zero stabilities extending from 10^{-12} watt down to the lower limit of 10^{-18} watt obtained by Williams and Noble.

The low-level sensitivity of an electronic d.c. amplifier may be greatly improved if the d.c. or low-frequency input is used to modulate a carrier. The carrier is then amplified and the modulation detected after amplification. If the modulator zero is stable and the signal frequencies are well below the carrier frequency, this arrangement can be made much less liable to zero drift than a normal electronic amplifier with direct coupling or a magnetic amplifier employing metal rectifiers.

There are two reasons for this ; firstly the amplifying process is independent of rectification, the latter occurring after the carrier has been amplified. Secondly, the bandwidth of the carrier amplifier only requires to extend over the range defined by the sidebands corresponding to the signal and carrier frequencies. If

the signal frequency is small compared with that of the carrier, the amplifier frequency range is well beyond that of the signal and therefore most of the noise arising in the amplifier will be outside the signal bandwidth.

There are many types of modulator which may be used with electronic carrier amplifiers, e.g. relay or commutator choppers, carbon piles, vibrating capacitor devices and others. The magnetic modulator, however, has the advantage of being mechanically static and it is therefore more able to withstand shock or vibration and is less liable to wear. The main operating requirements are that its zero stability should be good and that it should be able to distinguish between positive and negative input signals.

12.1 Causes of Zero Drift

It has already been shown in previous chapters that when a transductor is d.c. polarized, various waveform changes take place in the core m.m.f.s. These changes correspond to a modulation and, in the case of the transductor amplifier, for example, our interest is centred on the modulation of the mean load current, i.e. the mean value of the odd-harmonic components in the core m.m.f.s. As a modulation we could equally well have considered the peak value of the load current, or the saturation period or even the phase angle of a particular harmonic referred to some fixed datum. All these quantities are dependent upon d.c. polarization and the various magnetic modulators of this type differ only according to the particular quantity which is considered to be modulated.

It was shown in Chapter 11 that when the twin-cored transductor is polarized with direct current both odd- and even-harmonic voltages are generated. The latter provide a more suitable modulation of the signal because, as we have seen, the zero drift is smaller and also because the even-harmonic voltages are sign conscious with regard to the input.

The following factors are liable to cause zero drift, either directly or by the introduction of a zero error :

(*a*) It is not possible to balance the two cores exactly, and this will give a zero error in some types of modulator.

(*b*) If the cores are not perfectly balanced and even-harmonics are present in the a.c. supply voltage waveform, this will give a zero error in all types of magnetic modulator since the even-harmonics from the supply will not be distinguishable from those due to a signal.

(*c*) The earth's magnetic field or any other stray fields will polarize the cores to some extent unless they are adequately screened. This effect would be indistinguishable from an input signal and would cause a zero error particularly liable to drift if the orientation of the device were changed. This effect is in fact utilized in certain types of " flux-gate " magnetometer.

(*d*) Thermal e.m.f.s generated at the junction of dissimilar metals (e.g. circuit connections) will produce parasitic signals which will drift with a variation of temperature.

(*e*) When a large signal is applied to the modulator the cores retain a certain amount of flux polarization after removal of the signal. This " memory " effect is attributed to remanence ; it causes a zero error which may be removed by subsequent demagnetization.

If the cores are driven well into saturation by the a.c. supply, memory effects are far less significant, the supply voltage is often made large for this reason although the difficulty of balancing the cores is greatly increased.

Memory effects generally arise in the centre of the cores due to incomplete penetration of the a.c. fluxes arising from skin effects. These become significant at frequencies over 500 c/s with ·004-in. mumetal laminations. Cores of small cross-section give better results in this respect, and demagnetization, when necessary, should be carried out at a low frequency in order to ensure complete penetration of the core by the demagnetizing fluxes.

(*f*) If the modulator is biased, e.g. from d.c. ampere-turns provided by additional polarizing coils, a drift in the gain of the modulator will alter the effect of the bias. This will cause a drift which is independent of the input and will therefore be equivalent to a zero drift.

12.2 Peak-Height Modulator

This is a type of even-harmonic modulator in which the peak value of the second-harmonic voltage components e_d (*Figure* 11.2) is used as a measure of the d.c. polarization.

The operation may be understood by referring to the circuit in *Figure* 11.2 and the magnetization curve of *Figure* 12.1. If the two cores, such as those shown in *Figure* 11.2, are assumed to be exactly matched, the d.c. input signal is zero and the supply

voltage is sufficiently large to drive the cores well beyond the knee of the magnetization curve (*Figure* 12.1). Under these conditions the output voltage e_d is zero at all times.

If there is a d.c. signal, however, the cores become unbalanced, one operating at a point defined by the m.m.f. F_A and the

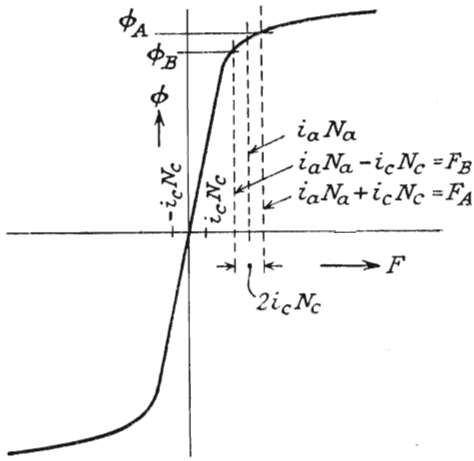


Figure 12.1. Magnetization curve showing fluxes and m.m.f.s in magnetic modulator.

other F_B, the difference between these points being the m.m.f. $2F_c = 2\bar{i}_cN_c$ due to the signal. As the fluxes increase over the knee of the magnetization curve an e.m.f. is induced in the pick-off windings. The magnitude of this e.m.f. depends upon the sharpness of saturation and the difference between the fluxes ϕ_A and ϕ_B, i.e. the signal magnitude.

Figure 12.2*a* shows the output from an ideal peak-height modulator in which the cores are perfectly balanced ; in this case the input is d.c. *Figure* 12.2*b* shows the output for a low-frequency sinusoidal signal. Normally unbalance exists and the two cores saturate at slightly different times, even for no signal, thereby giving rise to odd-harmonic peaks of the kind shown in *Figure* 12.2*c* ; this represents the output for zero input. *Figure* 12.2*d* shows the effect of a signal producing an even-harmonic output superimposed on the odd-harmonic unbalance.

Demodulation is usually carried out after amplification of the carrier by means of some form of polarity-sensitive peak rectifier, a typical arrangement for which is shown in *Figure* 12.3. The

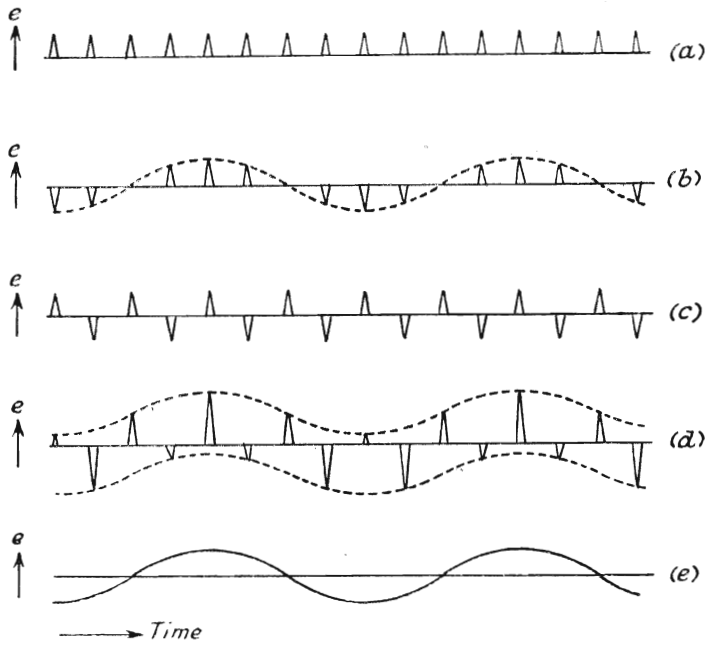


Figure 12.2. Operation of modulator under various conditions.
(*a*) Output for constant input (ideal case).
(*b*) Output for sine-wave input (ideal case).
(*c*) Odd-harmonic output for zero input (effect of unbalance found in practice).
(*d*) Output for sine-wave input with odd-harmonic unbalance.
(*e*) Output corresponding to (*d*) after detecting and smoothing.

rectifier valve A gives a positive output proportional to the positive peaks, while B gives a positive output proportional to the negative peaks. Thus the mean output from the detector $V_A - V_B$ is a reproduction of the input. *Figure* 12.2*e* shows the output after rectification and smoothing.

A number of errors arise in this type of modulator and it is

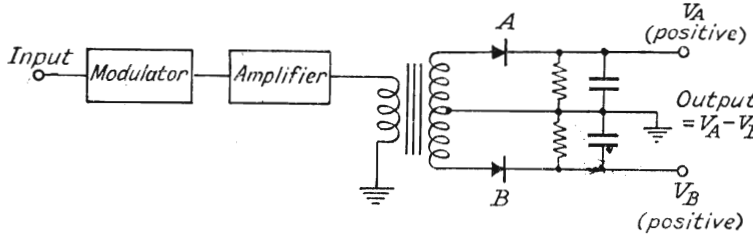


Figure 12.3. Modulator system showing circuit of one type of peak rectifier.

therefore necessary to take certain precautions in order to minimize their effects.

(1) Imperfect balance of the cores gives rise to the following difficulties :—

(*a*) If even-harmonics are present in the supply these will appear in the unbalanced component of the output and will be indistinguishable in their effect from a d.c. signal. It is therefore necessary to ensure that the supply waveform contains a sufficiently small proportion of even-harmonics to make the zero error sufficiently small. In practice this may be achieved by using a rotating machine as the source of supply rather than an electronic oscillator.

(*b*) The odd-harmonics (*Figure* 12.2*c*) which appear for zero input are inevitably applied to the valve amplifier. The gain of this amplifier must therefore be limited to such a value that these harmonics when amplified are not so large as to cause non-linear operation or damage the rectifiers which demodulate the carrier. The amount of unbalance therefore places an upper limit on the permissible gain of the carrier amplifier.

(*c*) Detection of the output obtainable for small input signals involves the subtraction of two large quantities, V_A and V_B, to give a small one. This is an undesirable feature for a number of reasons ; for example, in the electronic amplifier the amplification of the positive peaks from the modulator resulting in the voltage V_A must be exactly equal to the amplification of the negative peaks giving V_B. Otherwise a zero error will appear when these two quantities are subtracted. Such an effect arises due to amplifier distortion and may generally be kept to within tolerable limits by the use of negative feedback.

(2) Memory effects occur if the modulator is not driven well into saturation by using a high supply voltage. The latter, however, increases the zero signal unbalance, *Figure* 12.2*c*, leading to the difficulties mentioned in 1 (*b*) and (*c*) above. It is therefore necessary to compromise between these factors.

(3) The earth's field has a considerable effect on the cores and it is important to shield the modulator unit in a high-permeability box.

A circuit for one type of modulator is shown in *Figure* 12.4.

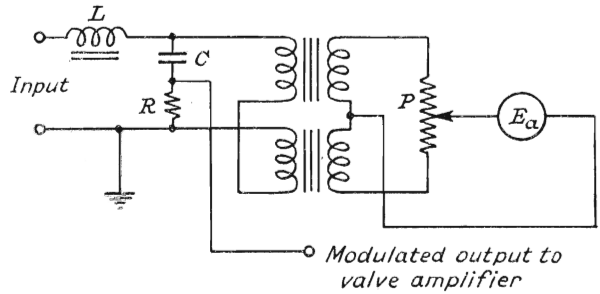


Figure 12.4. Peak-height magnetic modulator circuit using normal iron-core construction.

The signal is applied through a high impedance while the modulated carrier appears across the resistance R. The capacitor C prevents low-frequency signal components from appearing as a ripple in the modulator output and the inductance L prevents the high-frequency carrier components from being dissipated in the input circuit, confining them to the series circuit consisting of the output resistance R and the blocking capacitor C. The two non-linear inductors are balanced by means of the potentiometer P.

A particular feature of this circuit is that the control windings also serve as the output windings, thereby increasing the available winding space on the core.

This type of modulator has a zero stability of about 10^{-13} watt when using mumetal-cored saturable reactors.

A circuit which is basically similar to this but differs in its form of construction is shown in *Figure* 12.5. In this case the transductor arrangement consists of two coils, A and B, each wound on a core consisting of a fine mumetal wire.

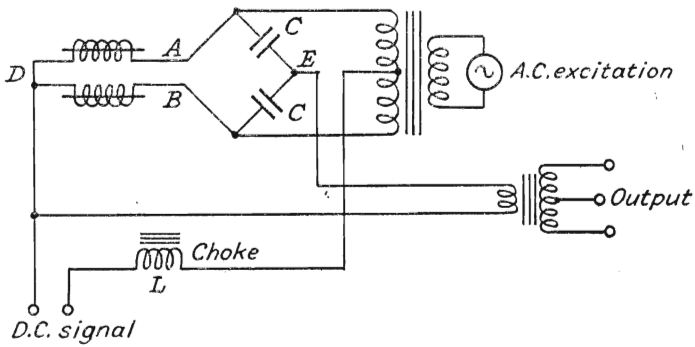


Figure 12.5. Flux-gate modulator.

This arrangement is similar to the flux-gate magnetometer, in which a pair of mumetal wires each carrying a.c. excited coils are arranged beside one another in an external field which polarizes the wires. Even-harmonics appear in the current waveform due to polarization from the external field. These even-harmonics give a measure of the field strength.

In the modulator, however, the two coils are arranged astatically, i.e. one is reversed in relation to the other in order to make the arrangement comparatively insensitive to external magnetic fields.[12(b)] Since the two cores must obviously occupy slightly different positions in space, any field gradient which changes as the modulator is moved about will give rise to a zero drift. It is therefore necessary to enclose the modulator in a magnetically screened can in addition to making the arrangement astatic.

The flux-gates are connected in two arms, A and B, of a bridge circuit, which is balanced by means of the capacitors C in the adjacent arms.

The bridge is supplied from a centre-tapped transformer and the d.c. signal is applied through an inductance L to the centre-tap of this transformer and to the common point D. The modulated output is then taken from the points D and E through another transformer.

If the bridge is balanced and no d.c. control applied, then no output appears, but if the mumetal wires are polarized by means of a direct current applied at the input, the bridge becomes unbalanced and gives an even-harmonic output. As in the previous example of the peak-height modulator, the present arrangement cannot be perfectly balanced and, when no signal is applied, odd-harmonic components appear in the output ; the presence of a direct current in the coils causes even harmonics to be superimposed on the odd-harmonic standing output. Thus in principle this circuit is identical to that previously described, differing only in the method of balancing and the type of core used. The instrument is claimed to have a zero stability of 10^{-15} watt over long periods.

The signal current can be arranged to flow in isolated windings, but this requires extra space, thereby separating the cores and making the system more sensitive to field gradient. Because of this need for common a.c. and control windings, the input impedance is more limited than when the coils are wound on a closed iron circuit. On the other hand, because the area of the wires is

small, the flux effectively penetrates the core and thereby makes the device comparatively free from memory effects due to remanence.

12.3 Second-Harmonic Modulator

This type of modulator is less liable to zero drift than any other, primarily because only the second harmonic is selected from the output of the circuit in *Figure* 11.2 and amplified. A standing output due to core unbalance is therefore only possible if second-harmonic components are present in the supply. The output from the modulator is applied to an electronic amplifier selectively tuned to the second harmonic and the output from the amplifier is then fed to a phase-sensitive rectifier. Thus two results are immediately achieved, firstly the standing zero error is practically eliminated, thereby reducing zero drift due to changes in the zero error to almost negligible proportions. Secondly, the amplifier and rectifier are not loaded by unwanted odd-harmonic components and the amplifier gain may therefore be much higher than is applicable to the peak-height type of instrument. The energy available in the second harmonics corresponds to an appreciable proportion of the total even-harmonic energy, and therefore the loss of sensitivity by rejection of the other harmonics is not excessive.

In the second-harmonic type of modulator, the following possible sources of zero drift remain :

(*a*) Thermal e.m.f.s at junctions of dissimilar metals.
(*b*) Second-harmonic impurity in the supply.
(*c*) Memory effects due to remanence after an excessive signal.
(*d*) Stray fields.

A number of second-harmonic modulators have been used, but the experimental circuit developed by Williams and Noble [12(*a*)] probably represents the ultimate performance possible so far as zero drift is concerned. The general arrangement is shown in *Figure* 12.6.

The modulator unit (*Figure* 12.6) consisted of two inductors toroidally wound on mumetal cores. The a.c. excitation was supplied from a 1,500 c/s sine-wave oscillator and this was applied to the two coils (*a*) through the balancing potentiometer P and the equal capacitors C_1. The windings (*c*) carried the d.c. input signal applied at AB, the high-frequency components being by-passed by the capacitor C_2 and applied to the transformer T

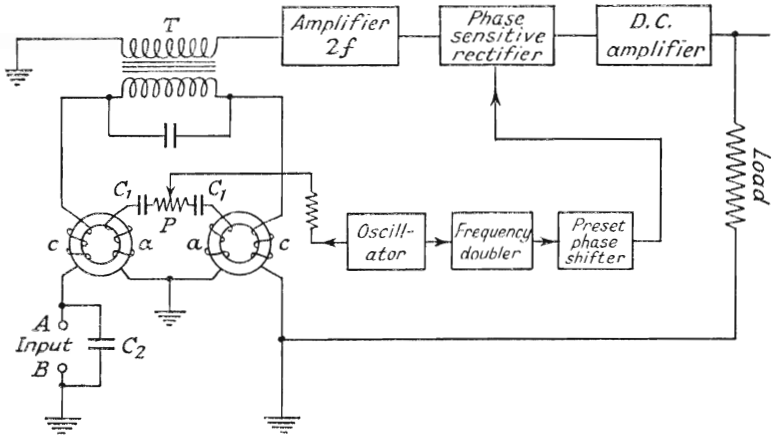


Figure 12.6. Second-harmonic magnetic modulator.
(*After Williams and Noble.*)

which was tuned to 3 Kc/s, the second harmonic of the supply frequency. The output from the transformer was then applied to a selectively tuned amplifier. The amplifier output was fed to a phase-sensitive rectifier, the reference frequency being obtained from the a.c. supply through a frequency doubler and pre-set phase shifter which compensated for amplifier phase shift.

The following precautions were taken in order to minimize the effects of zero drift :

(*a*) *Thermal e.m.f.s.* These being of a low frequency, were eliminated from the a.c. windings (*a*) by means of the blocking capacitors C_1, while those arising in the control circuit were reduced by avoiding joints of dissimilar metals wherever possible.

(*b*) *Second harmonic impurity in the a.c. supply.* The oscillator providing the a.c. supply was carefully designed to reduce second harmonic content to a minimum.

This was achieved by generating a carefully adjusted square wave and then filtering out all frequency components but the fundamental. Thus, due to the inherent symmetry of the square wave, the second harmonic content was low even before filtering.

(*c*) *Memory effects.* Owing to the fact that the output was selectively tuned to the second harmonic, balancing of the cores was less important than in the peak-height device. Therefore the system was balanced adequately although the cores were being driven well into saturation, thereby reducing memory effects due to large signals.

(*d*) *Stray magnetic fields.* The effect of stray magnetic fields was reduced by careful screening and keeping the modulator stationary in the earth's field.

This magnetic modulator was built primarily for the purpose of investigation, and it was claimed to have a zero stability of 10^{-18} watt over a bandwidth of 3 c/s. A more practical instrument [12(c)] was subsequently designed along the same lines and was reported to have a zero stability of about 10^{-17} watt.

12.4 Comparison of Magnetic Modulators and Relay Choppers

When using contact modulators the zero stability varies considerably with operating conditions and the ultimate stability appears to be about 10^{-19} watt,[12(d)] which is one order better than that reported by Williams and Noble for the experimental second-harmonic modulator. Under normal operating conditions using readily available materials relay choppers have a voltage stability of 10–100 μV. When operating from an input impedance of 1 MΩ this corresponds to a zero drift of 10^{-14} to 10^{-16} watt. Thus vibrating relay modulators have comparable performance with magnetic modulators, but in particular the relay modulator is probably the more suitable for high source impedances, say above 20 KΩ, while the magnetic modulator is better in terms of power stability when the source impedance is low. Irrespective of these considerations, the relay chopper has the advantage of requiring less complicated circuitry to achieve a given zero stability while the magnetic modulator possesses the advantage of being mechanically static and therefore probably more reliable.

12.5 Applications of Magnetic Modulators

Magnetic modulators are used chiefly in conjunction with electronic circuits as sensitive d.c. amplifiers. Amplifiers of this type are applicable to industrial measurements and control, and also as d.c. amplifiers in analogue computers, medical equipment and laboratory instruments.

Two specific examples will now be given of magnetic modulator applications, firstly a wide-band d.c. amplifier for use in analogue computers, and secondly a stable d.c. amplifier suitable for industrial measurements.

(*a*) *Wide-band d.c. amplifier.*[12(e)] D.C. amplifiers for analogue computers often require to operate satisfactorily in the audio-frequency range as well as at very low frequencies. The need to

operate at low signal levels suggests the use of a modulating system which would, however, limit the possible bandwidth. On the other hand, direct-coupled audio amplifiers operating over a wide band are inescapably associated with large drift potentials which arise at the grid of the first valve, and this gives a low signal to noise ratio at very low frequencies.

A composite circuit has been developed by Goldberg [12(f)] in which bandwidth and zero stability are both achieved in one hybrid amplifier. A conventional direct-coupled electronic

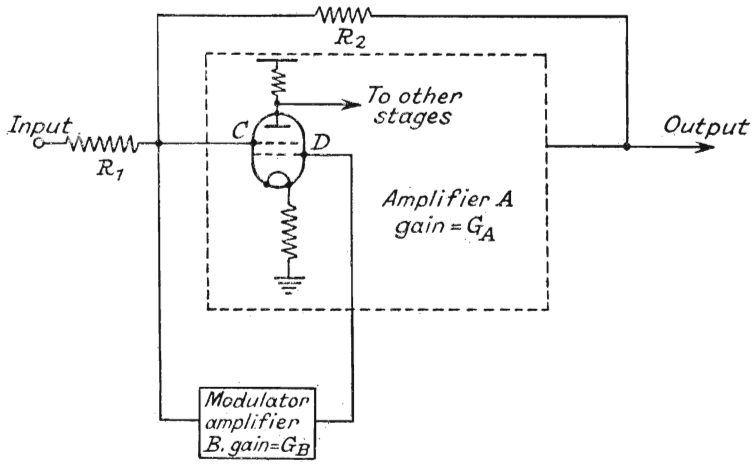


Figure 12.7. Hybrid amplifier having low zero drift and large bandwidth.

amplifier operates up to the maximum audio frequency required and in addition its zero is continuously monitored and maintained constant by a stable d.c. modulator amplifier which operates only over a limited frequency range. A block diagram of the system is shown in *Figure* 12.7.

In the conventional direct-coupled amplifier A the valve in the first stage has two control grids, C and D. Input voltages at C are amplified and appear at the output increased by a factor G_A, the gain of the amplifier.

A modulator-type amplifier B has its input also connected to C and its output to the other control grid D of the direct-coupled amplifier; the gain of the modulator amplifier to d.c. signals is G_B.

Signals of a very low frequency appear at the grid C and are amplified first by B and then by A via the control grid D. Signals are also amplified in A alone via the control grid C. The total gain

to low frequencies is therefore $(G_A + G_A G_B)$. It is important to note that these frequencies are all effectively pre-amplified in B, which is comparatively free from drift and therefore, assuming no drift occurs in B, the signal to drift ratio is increased by a factor corresponding to the gain G_B of the modulator amplifier.

High-frequency signals by-pass B and are amplified in A alone, the gain now being approximately equal to G_A.

Typical values for G_A and G_B at zero frequency are 10^4 and 10^3 respectively, so that at low frequencies the overall gain $G = G_A G_B$ is approximately 10^7, while at high frequencies the gain is reduced to 10^4. The overall gain is stabilized by means of the feedback resistors R_1 and R_2, so that the closed loop gain becomes nearly equal to $\frac{R_2}{R_1}$.

In one particular application of this kind a peak-height modulator of the type shown in *Figure* 12.4 is employed.

The carrier frequency is 400 c/s and the overall gain of the modulator amplifier unit is sufficient to improve the zero drift of the main amplifier by a factor of 200. The input impedance of the main amplifier is 2 MΩ and the closed loop voltage gain is unity. A photograph of the modulator amplifier is shown in *Figure* 12.8.

(*b*) *Flux-gate modulator amplifier.* A flux-gate-type modulator has been employed in a practical d.c. amplifier for the amplification of small e.m.f.s from devices such as thermo-couples, strain gauges, photo-cells, etc. The overall voltage gain is about 10^4, the input impedance is 2,000 ohms and the output impedance 1,000 ohms. The zero stability is approximately 1 micro-volt and the gain is stable to within 1%. A photograph of this unit is shown in *Figure* 12.9.

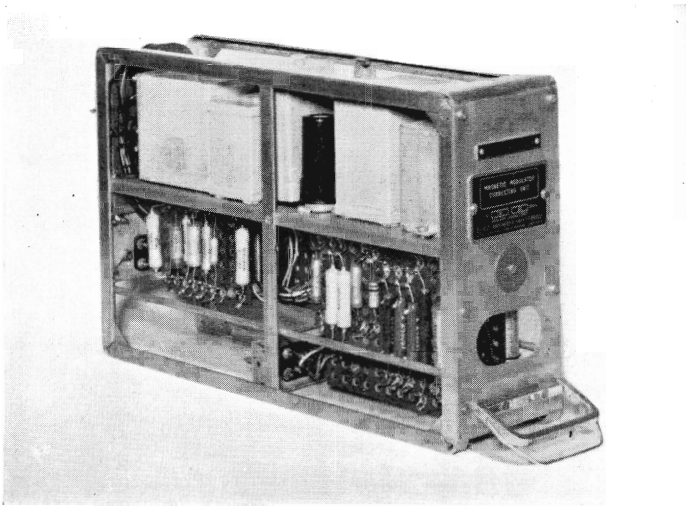


Figure 12.8. Magnetic modulator amplifier for zero-stabilizing a wide-band d.c. amplifier.

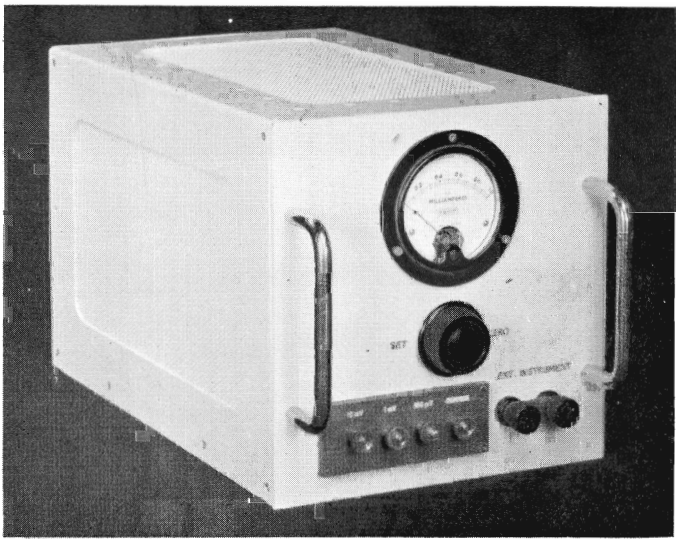


Figure 12.9. Amplifier using flux-gate modulator.

Figures 12.8 *and* 12.9 *by courtesy Elliott Brothers* (*London*) *Ltd.*

[*Facing p.* 358.

CHAPTER 13

CONSTRUCTION AND DESIGN OF MAGNETIC AMPLIFIERS

13.1 Introduction

In previous chapters the mode of operation of magnetic amplifiers was discussed and the transient and steady-state characteristics considered. It has also been stressed that these characteristics only apply under certain idealized conditions and that even empirical data based on practical observations is of limited value owing to the variation in properties of a given type of core material from one batch to another. Uncertainties of this kind influence the design of transductors and the final amplification may differ by as much as 50% from the predicted value. It is therefore usual when designing to aim at an amplification somewhat greater than the required value and arrange for final presetting during manufacture (section 5.7).

In addition to the average slope of the characteristic, however, it is sometimes found that the shape of the characteristics varies, particularly at the upper and lower ends. When two transductors are required to form a push-pull pair it is desirable that their characteristics should be matched as closely as possible. Careful matching of the cores by selection is of course possible. It is usually sufficient, however, to ensure that laminations are selected from one production batch. Toroidal cores may, of course, be wound in pairs which have already been matched.

As in similar problems of this kind, when designing for a given specification the procedure tends to be governed by experience. Nevertheless the information contained in this book has been used as a basis for magnetic amplifier design. This chapter, therefore, contains a summary of the salient factors relating to design and

construction and gives some guide to the methods which have been found adequate for most purposes.

Summary of Pre-Design Considerations

When considering the application of magnetic amplifiers to a specific problem it is usually desirable first to consider the relative merits of the various possible alternatives. In order to do this the more important practical features of the transductor are summarized in the following sections as a guide to determining whether or not a magnetic amplifier is the most suitable in comparison with other types of amplifier.

13.2 Amplification

When a transductor is used as an amplifier the gain is not normally considered in terms of voltage or current ratios but rather as a power amplification, since the latter is more closely related to the response time. Furthermore, it is usually possible to quote a typical figure for the power gain of a specific type of amplifier without reference to the windings or circuit impedances. Thus when using nickel-iron cores and selenium rectifiers, a small single-stage amplifier with auto-excitation cannot in general be expected to operate with reasonable stability for power gains greater than 10^5 although higher figures than this have been reported for magnetic amplifiers operating under laboratory conditions. When larger units are required it may be necessary on economic grounds to use cheaper core materials such as silicon-iron, in which case the power amplification seldom exceeds 5×10^3.

13.3 Time-Constant

It has been shown that the time-constant is approximately proportional to the power amplification and inversely proportional to the supply frequency. For example, the time-constant of an auto-excited transductor having an amplification of 10^3 in one stage and operating from a supply frequency of 400 cycles per second would be 25–50 milliseconds. Where rapid response is required high-frequency supplies are clearly an advantage when available, but in practice the frequency is limited by iron losses to something less than 5 Kc. if reasonable stability of the characteristics is to be maintained.

More rapid response is achieved for a given power gain by cascading two amplifiers, but it must be remembered that in no

circumstances can the response time be less than one half-cycle of the supply frequency.

Rapid response is not always required, however, and in some circumstances the slow response may be emphasized either to provide smoothing or to help stabilize a closed-loop control system.

13.4 Efficiency

As was shown in Chapter 2, the efficiency of a magnetic amplifier is governed primarily by the ratio of the load resistance R_L to the total a.c. circuit resistance R_a. The true efficiency also depends upon the form factor of the current in the a.c. windings (section 2.12), but the mean power efficiency R_L/R_a is a convenient quantity by which to estimate both the power consumed from the mains and the power which will be dissipated in the amplifier itself. Since transductors consist essentially of a reactive controlling impedance, their efficiency is higher than corresponding electronic devices in which the control is due to a variable resistive element.

As was shown in Chapter 2, the ratio R_L/R_a for single transductors varies from 50% for small units operating at low frequencies (say 50 c/s) up to 95% for very large units or for transductors operating at high supply frequencies.

13.5 Power Output in Relation to Size and Weight

The minimum possible size of small transductors is usually limited either by difficulties of manufacture or by the requirements of the electrical characteristics.

For supply frequencies of 400 cycles per second, or more if advantage is taken of the high efficiency, a small amplifier up to about 10 watts output compares favourably with an equivalent electronic amplifier with its power pack. In these circumstances, however, the type of load can play an important part in determining the overall size of the equipment. In order to appreciate this let us consider three types of load each requiring a push-pull output, the supply voltage available being appropriate to the load impedance. Firstly, suppose that the load has no centre-tap ; in this case summing resistors are necessary (Chapter 10). The summing resistors lead to considerable wastage of power and the transductor may have to control an output power seven or eight times greater than that specified for the load. In addition, an isolating transformer is necessary in order to avoid short-circuiting

the load rectifier. The combined effect of these two requirements leads to a unit which is somewhat large considering the ultimate output requirements.

Secondly, if the load is centre-tapped, summing resistors are unnecessary and therefore a considerable reduction in total power output is achieved with a correspondingly reduced size ; the isolating transformer is still required, however. Finally, if the load consists of two separate windings coupled by an iron circuit, as may be the case with a split field motor where the transductor load is the pair of isolated fields, then both the summing resistors and the transformer might be dispensed with, resulting in a unit which is very attractive both from size and weight considerations.

In larger units, say over 100 watts, size is governed by temperature rise due to heat dissipation and it is possible to make a direct comparison with transformers.

Let us consider two equal transformers each rated to supply a secondary load of 1 kW. at V volts and I amps. (r.m.s.). The following assumptions are made ; firstly that ampere-turn balance holds between the primary and the secondary on full load, secondly that these windings occupy equal spaces on the core, and thirdly that they have the same mean length of turn.

If these two transformers are connected as a series transductor the maximum a.c. voltage will be 2 V. The maximum load current will be limited by heating of the overall unit. Now, when used as a transformer, the copper losses and iron losses are ideally equal, but when used as a series transductor without self-excitation, at maximum output the iron losses are very nearly zero since the flux swing is small. Furthermore, the maximum output current is almost sinusoidal, and since the copper losses can be doubled, the maximum current will be very nearly equal to $\sqrt{2}\,I$. If the efficiency is 80%, then the maximum r.m.s. voltage applied to the load will be $2 \times 0{\cdot}8$ V ; thus the maximum volt-amperes in the load (rectified values) will be equal to

$$\frac{2 \times 0{\cdot}8\ \text{V}}{1{\cdot}11} \times \frac{\sqrt{2}\,I}{1{\cdot}11} = 1{\cdot}84\ VI.$$

In practice, the maximum output may be slightly less than this, due partly to slight distortion of the load current at maximum linear output and partly to the small iron losses which must exist in practice. A more conservative figure is therefore about $1{\cdot}6\ VI$.

In the case of an auto-excited transductor, the results are

somewhat different. Firstly, due to the increased m.m.f. ratio, the space occupied by the control windings can be considerably reduced and 10% of the total space will be adequate for large output units in which the gain can be maintained at a conservative figure. Thus the proportion of area occupied by the a.c. windings may be increased from 0·5 to 0·9 and therefore the resistance of the a.c. windings can be reduced by a factor $\frac{5}{9}$. If the maximum r.m.s. load current is now I_1 then, due to the routing action of the auto-excitation rectifiers, this will only flow once every two half-cycles, i.e. for half the time, so that the heating effect may be equated to the heating effect due to the previous current I as follows :

$$I_1^2 \times \tfrac{5}{9} \times \tfrac{1}{2} = I^2$$

giving

$$I_1 = 1{\cdot}9\, I.$$

When using auto-excitation, linear operation is only achieved up to about one-half maximum output for silicon-iron cores, and therefore the maximum load voltage will be only about 0·5 V ; furthermore, under these conditions the maximum current cannot be increased by the factor $\sqrt{2}$, since an appreciable iron loss will also be present and a more reasonable factor would be about 1·25. Finally, the form factor of the waveform under these conditions is equal to about 1·4, and therefore the maximum rectified volt-amperes in the load are now equal to

$$\frac{0{\cdot}5\ \text{V}}{1{\cdot}4} \times \frac{1{\cdot}9\, I \times 1{\cdot}25}{1{\cdot}4}$$

$$= 0{\cdot}6\, VI.$$

These considerations bring out the following important points : firstly that the series transductor without self-excitation controls approximately 80% of the power controlled by an equivalent transformer of the same rating, and secondly that the auto-excited transductor will only handle about 30% of the power of the transformer. If greater non-linearity can be tolerated as, for example, in a closed-loop system, then this figure might be increased to 40%, but it is clear that the series transductor without self-excitation is capable of controlling two or three times as much power as the auto-excited version for a given size. For this reason it is therefore sometimes preferable to employ a small pre-amplifier followed by a low-gain output stage which can dispense with auto-excitation and thereby reap the advantages of greater power-handling capacity for a given size.

13.6 Impedance Range

The maximum input and output impedances are limited by the manufacturing difficulty of winding coils of many turns of fine wire and the corresponding insulation problems which arise in the a.c. windings when high voltages are present. Input impedances can be as high as 10,000 ohms and coils of 70,000 ohms have been wound for special purposes. Such coils are, however, expensive and difficult to wind, and impedances greater than 5,000 ohms are uncommon. When the output impedance is either unusually high or unusually low it may be more economical in terms of cost or efficiency to wind the transductor for a practicable output impedance and then to match the impedance through a transformer before rectification. This would apply particularly when an isolating transformer is required ; in such cases the matching transformer could also serve this purpose.

The dynamic input or output impedance may of course be increased at the expense of amplification by the use of negative feedback.

13.7 Stability

The stability of magnetic amplifiers has already been discussed at length in Chapter 11, and only a brief summary will be given here. Short-term drift is usually due to changes of supply voltage, frequency and ambient temperature, while long-term drift arises due to ageing of components such as metal rectifiers.

Variation of ambient temperature influences the rectifiers, and the resistance of the windings ; it also affects the magnetic properties of the core due to differential expansion of the core and clamps. The latter point is particularly noticeable in the cut C type cores, which are clamped by means of encircling straps, a simple butt joint being formed at the point of contact.

In large single transductors drifts of the order of $\pm$ 20% of full-load current are possible and usually tolerable. In smaller units a gain stability of about 5% or 10% is typical and can be improved by the use of negative feedback.

Zero drift ranges from 10^{-5} watt for the larger transductors down to 10^{-12} watt for small instrument amplifiers (see Chapter 11).

13.8 Waveform

Since the controlling element of a transductor is due to a saturable core, the waveform of the unrectified load current is

considerably distorted. This consideration must be borne in mind particularly when considering the form factor, which varies from a large value down to 1·11 as the output increases from zero to maximum.

13.9 Lamination Noise

Transductors are usually more noisy than transformers of the same rating, and in large units it is sometimes advisable to provide resilient mounts and sound insulation.

13.10 Warming Up

Magnetic amplifiers are generally available for use immediately after being switched on ; there is, therefore, no need for delay switches or other protective gear on starting.

13.11 Reliability

Since transductor amplifiers are static devices, they contain no parts which are liable to wear, nor do they require maintenance. They closely resemble transformers in construction and are equally reliable. When used with metal rectifiers or relays, however, their reliability is governed primarily by these components.

In many applications the transductor is used because reliability considerations outweigh the initial cost and other disadvantages that might be avoided by the use of less reliable equipment.

13.12 Cost

The initial cost of small magnetic amplifiers is due to the need for special core materials, rectifiers and possibly complex windings. Larger units are more directly comparable with transformers on the basis of the remarks made in section 13.5.

The low maintenance costs, however, pay for the capital outlay after a reasonable period.

The development cost of small units also tends to be high, due to the limited scale of activity and lack of mass-produced components such as clamps, bobbins, etc. It should be borne in mind, however, that a special power supply is seldom necessary, and this may help to reduce the initial expense as compared with that of electronic equipment.

Transductor Cores

13.13 Requirements for Cores

Many attempts have been made to arrive at a " figure of merit " for transductor cores.[13(a)] In general, however, this is

somewhat difficult, because a simple criterion fails to take account of all the relevant factors while a complicated one requires special measurements which are just as difficult to take as a complete transductor characteristic. In addition, no criterion can possibly give the complete answer, since the relative importance given to each consideration in the figure of merit cannot be in accordance with all possible requirements.

The requirements for a transductor core will therefore be specified herein by summarizing the desirable features and the particular way in which each one influences the overall performance.

(*a*) *Sharpness of* **B.H.** *Curve.* The non-linearity in the magnetization curve is the basic reason for transductor operation. Unless this non-linearity is pronounced the linear-operating range of the transductor will be small and amplification low ; also the standing current for zero signal will be appreciable, thereby leading to problems of zero drift as discussed in Chapter 11.

(*b*) *Slope in unsaturated region.* If the slope of the magnetization curve is large in the unsaturated region this leads to a high gain and small standing current with the resulting stability of the characteristics.

(*c*) *Iron losses.* Excessive iron losses cause reduced gain together with non-linear operation and increased zero drift. They do not seriously limit the thermal rating because when the iron losses are a maximum the copper losses are negligible and the two are never a maximum at the same time.

(*d*) *Saturation flux density.* This should be as large as possible in order to enable the a.c. windings to support a large supply voltage for a given number of turns.

(*e*) *Relative importance of various factors.* Unfortunately the less expensive core materials are notably inferior in other respects ; thus silicon-iron, which is relatively cheap, has a comparatively low permeability together with a saturation which is not particularly sharp. It also suffers from the disadvantage of having higher losses than the nickel-iron cores, the latter, however, are much more expensive.

When dealing with 70% nickel-irons (mumetal, permalloy C, etc.) the permeability is high, the losses are low and the knee of the characteristic is relatively sharp. The cost of these materials, however, is very high and the saturation flux density is only about one-half of that for silicon iron ; this tends to increase the

weight of mumetal required for a given output which results in a further increase of costs. Mumetal is therefore generally confined to smaller units or first stages requiring a high gain and good zero stability. The 50% nickel-irons (H.C.R. Deltamax, Orthonal) have a high saturation induction but the losses are higher than in mumetal ; the gain and stability of transductors made with these materials is therefore inferior to that obtained with mumetal-cored units.

13.14 Types of Core

Early transductors employed cores assembled from laminations of the type shown in *Figures* 13.1*a* and *b* ; these are commonly

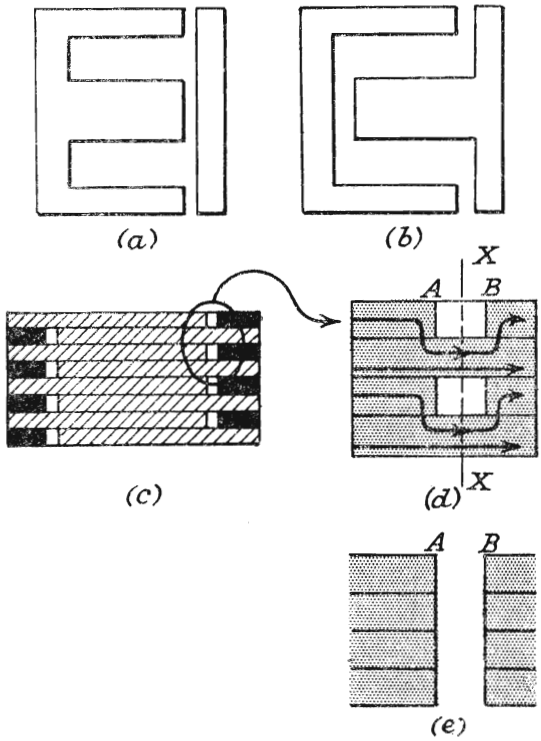


Figure 13.1. Diagrams showing interleaved assembly of core laminations.
(*a*) *E* and *I* type.
(*b*) *T* and *U* type.
(*c*) Method of interleaving.
(*d*) Flux distribution at joints.
(*e*) Effect of saturation at joints.

used in transformers, but when used for transductors they give rise to a pronounced non-linearity halfway up the characteristic. The reason for this is as follows : when the core is assembled with interleaved joints (*Figure* 13.1*c*), the joints at places where adjacent laminations butt together, as shown in *d*, carry the total flux over an iron section XX which is one-half the area of the main body of the core since only one-half the number of laminations are present over this short length of the core path. Therefore, when the flux in the main limbs reaches one-half the saturation value, the flux in the path from A to B (*Figure* 13.1*d*) causes the iron in this region to become saturated. When this occurs the reluctance of this gap becomes comparable with that of a complete air gap (*Figure* 13.1*e*), so that from half the saturation flux to full saturation the characteristic approaches that of a gapped core such as that shown by curve (*b*) in *Figure* 13.4. As the gap is closed by forcing the adjacent laminations together longitudinally the slope of the characteristic may increase as shown by curve (*c*) (*Figure* 13.4). In practice, of course, the curve can never be made to coincide with (*a*), which is the magnetization curve for a core of uniform sectional area, since a gapped core never has such a low reluctance as a continuous core and therefore with this type of lamination a non-linearity is always present.

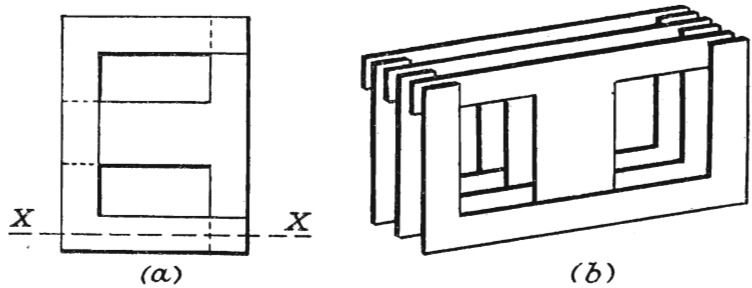


Figure 13.2. Lamination assemblies using lap joints.

Better characteristics are obtained if the gap AB is increased to a very large value ; this results in curve (*d*) which, although only having one-half the saturation flux, at least possesses the necessary sharp knee.

This effect can be achieved by re-arranging the laminations to have complete lapping of joints giving a core assembled as in *Figure* 13.2. The flux density at which saturation occurs is

determined by the minimum sectional area of the core and, referring to *Figure* 13.2*b*, it is seen that although the depth of the core is that of six laminations, the effective iron area, as determined by the minimum area, is only three laminations.

The stacking factor of a core is thus the ratio of the effective

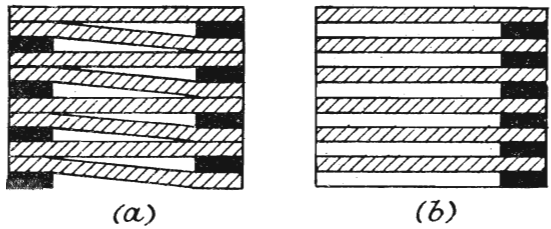


Figure 13.3. Lamination assemblies using lap joints.
(*a*) Stacking factor = 67%.
(*b*) Stacking factor = 50%.

core area to the sectional space occupied by this core area, and in the present case this is equal to 50%. In practice it will be even smaller, due to the need for a coating of insulating material between each lamination. If the coating is ·001 in. thick and the laminations are ·004 in. thick the stacking factor for this type of

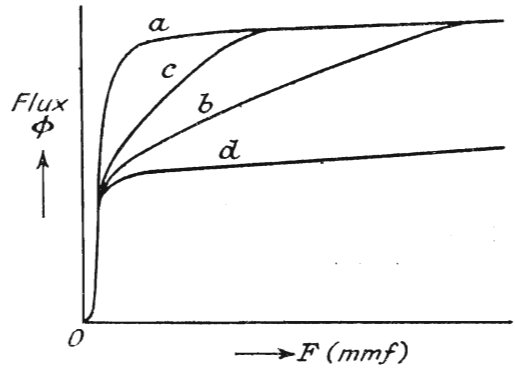


Figure 13.4. Magnetization curves for laminated cores.
(*a*) Continuous core, (*b*) and (*c*) interleaved laminations, (*d*) lapped joints as in *Figure* 13.2 and 13.3(*b*).

assembly will be about 40%. An alternative method of assembly is shown in *Figure* 13.3*a*, in which the laminations are slightly distorted. Usually, however, the distortion is very small and well within the elastic limit so that no harm is done. Furthermore, as is seen by comparison with *Figure* 13.3*b*, which is a section

XX through *Figure* 13.2*a*, an improved stacking factor is obtained by this method. In *Figure* 13.3*a* the actual depth corresponds to that of twelve laminations while the effective area is given by eight laminations. In this case, neglecting insulation, the stacking factor is equal to 67%.

An improved design of lamination has been standardized by the Ministry of Supply for transductor work. With this lamination full advantage is taken of the core area at all parts of the

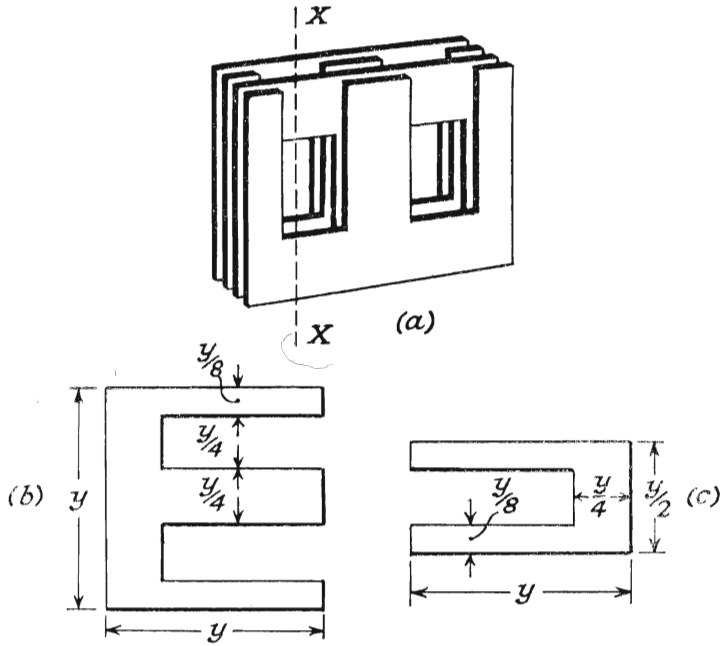


Figure 13·5. Standard transductor laminations for lapped joints. Stacking factor = 100%.

characteristic. Two alternative shapes of these laminations are shown in *Figures* 13.5*b* and *c*. They are assembled as shown with the backs pointing first in one direction and then in the other direction (*Figure* 13.5*a*), and they differ from the normal *E* or *U* type of lamination only in that the back of the laminations has double the width of the outer limbs. Thus if we consider a section through *XX* (*Figure* 13.5*a*), although the number of paths (laminations) passing through this area is halved, the width is doubled and therefore the overall core area is maintained at the same value over the whole of the flux path.

Such a core has a stacking factor which is about 0·8, due to the thickness of the insulating coating on each lamination. These laminations are available in four different sizes. Table 1, at the end of this chapter, lists the dimension y (*Figure* 13.5) for each of the sizes. It should be noted that when using laminated cores in any of the ways discussed above, the magnetic flux must pass from one lamination to the next through the interfacial insulation.

The thickness of the insulation may be as much as ·001 in., which is comparable with that of a simple butt joint. The reluctance of the flux path, however, is proportional to the ratio $\frac{\text{length of path}}{\text{cross-sectional area}}$. The cross-sectional area of the interface is comparatively large and thus the effect of an insulating layer on the total core reluctance is much less than the effect of a butt joint.

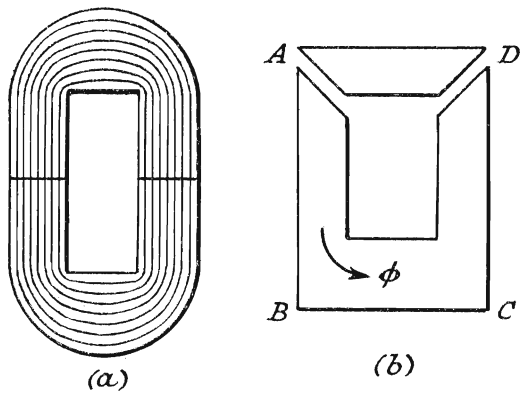


Figure 13.6. Types of core for grain-oriented irons.
(*a*) Split " C " core.
(*b*) Mitred laminations.

When the magnetic material has strong directional properties such as grain-oriented silicon-iron (Crystalloy and Hypersil), the reluctance of magnetic paths at 90° to the direction of rolling is comparatively high ; orthodox laminations made of these materials are therefore unsatisfactory. Sometimes split C type cores are used ; these cores are spirally wound and bonded up tightly after which they are sawn in half and the sawn surfaces then ground into matched pairs as shown in *Figure* 13.6*a*. The following type of core has been found more suitable for transductors however. Laminations of the type shown in *Figure* 13.6*b*

are used, consisting of U's and I's interleaved as for normal transformer construction with the I's occurring at alternate ends. When the path in a U lamination changes direction as at B, the flux transfers to the alternative I lamination which is placed over the top and is being magnetized along the easy direction. Thus the flux path is along the limbs of the U when traversing paths such as AB or CD but along the I's when the paths are BC or DA ; the bases of the U's thus form high reluctance paths in parallel with the I's. Since the bases of the U's are virtually ineffective, the I's must have double the width of the limbs of the U's in order to avoid saturation of the I members.

The square loop materials such as H.C.R. have 45° directional properties, and for this reason the normal type of transductor lamination (*Figure* 13.5) is satisfactory.

Toroidal cores give the highest amplification and greatest stability in all cases however. The effective slope of the magnetization curve is greater than that of a flat laminated core and individual cores are more consistent. The chief difficulties lie in the cost of winding, the poor winding space factor, particularly in small units, and the rather awkward shape which does not readily lend itself to easy clamping or fitting compactly into equipment where space must be conserved.

Continuous laminations of the type shown in *Figure* 13.7*a*

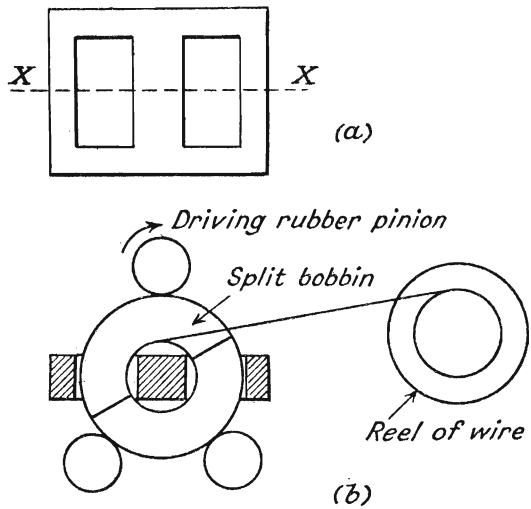


Figure 13.7. Continuous centre-limbed core showing method of winding.

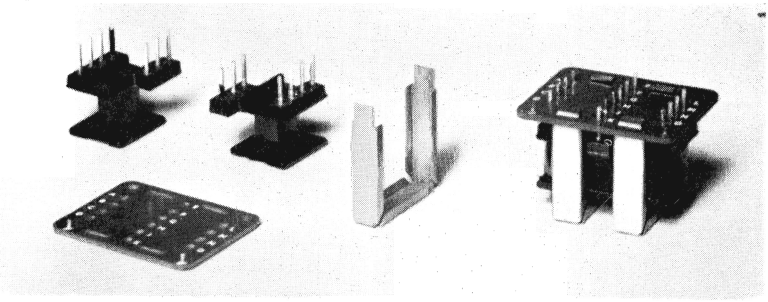


Figure 13.8(*a*). Small transductor assembly on " E " type laminated core.

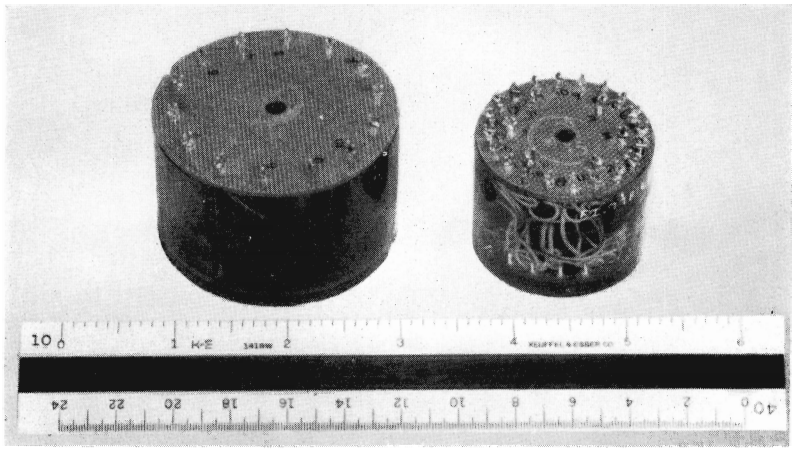


Figure 13.8(*b*). Encapsulated magnetic amplifier coil assemblies used in a missile guidance system.

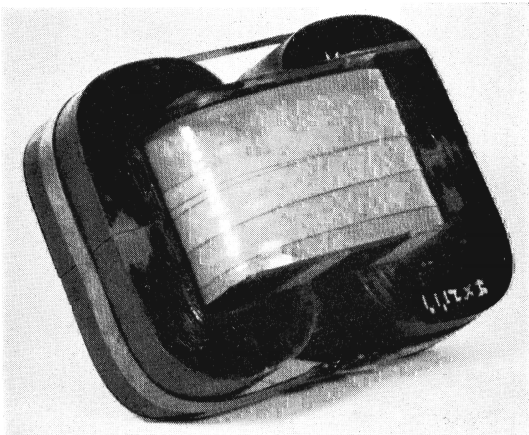


Figure 13.9. 500-watt transductor wound on cut type " C " cores.

Figures 13.8(*a*) *and* 13.9 *by courtesy Elliott Brothers (London) Ltd.* *Figure* 13.8(*b*) *by courtesy Ford Instrument Co., New York.*

[*Facing p.* 372.

are sometimes used ; these give rise to good transductor characteristics but are expensive to wind. The following method of winding is the most effective.[13(b)] A pre-formed bobbin is split and then reassembled over the core as shown in *Figure* 13.7*b*. The bobbin is then rotated on rollers and the wire fed on as shown in the figure. This method of coil assembly is not sufficient improvement over the laminations in *Figure* 13.5 to warrant its extensive use. The photographs in *Figures* 13.8 and 13.9 show various transductor assemblies using small laminated cores and cut " C " cores respectively.

13.15 Composite Cores

Cores composed of two different materials often give rise to useful properties.[13(c)] A particular example of this is a core in which there is one closed iron circuit formed of *U*'s and *I*'s, the *U*'s being of silicon iron and the *I*'s being of nickel iron (mumetal).

At flux densities below the saturation of the *I*, this limb has a

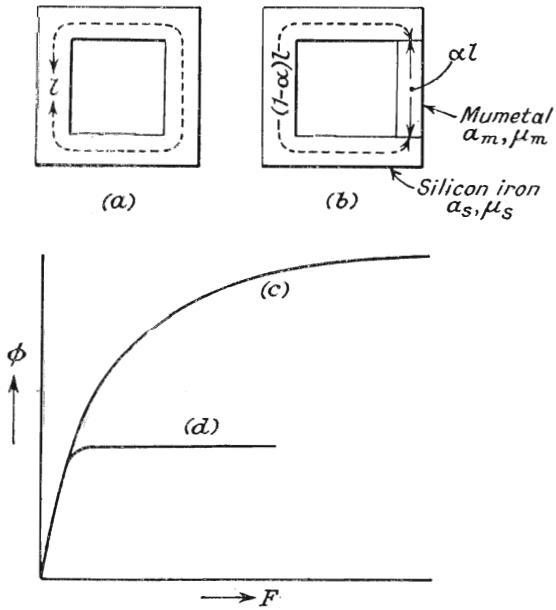


Figure 13.10. Magnetization curve obtained for composite core.
(*a*) Silicon iron core.
(*b*) Composite core.
(*c*) Characteristic of silicon iron core.
(*d*) Characteristic of composite core.

higher permeability than the U, but after the I limbs have saturated, their reluctance becomes very large and virtually determines the total reluctance of the circuit.

If we consider a closed core of silicon iron (*Figure* 13.10*a*) and plot the relationship between flux and m.m.f. for this core, the resulting curve (*c*) is obtained. If the iron circuit is broken, however, and a small length of easily saturable material (mumetal) is placed in the gap (*Figure* 13.10*b*), then at the value of m.m.f. for which the mumetal saturates, the reluctance of the circuit is virtually decided by that of the bridge and the curve shows a sharp saturation at this value of m.m.f. ; the resulting curve is therefore given by (*d*) in *Figure* 13.10. If the sectional area and permeability of the materials are denoted by a and μ respectively, with suffixes as shown, then the total reluctance S_t of the magnetic circuit is equal to the sum of the two individual reluctances thus :

$$S_t = \frac{\alpha l}{a_m \mu_m} + \frac{(1-\alpha)l}{a_s \mu_s}.$$

where l is the total length of magnetic path and α the fraction of this length occupied by the bridge. The total reluctance S_t may be written in the form

$$S_t = \frac{\alpha l}{a_m \mu_m}\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\frac{a_m \mu_m}{a_s \mu_s}\right] \quad . \quad . \quad . \quad (13.1)$$

If the ratio of the areas $\frac{a_m}{a_s}$ is made sufficiently small it is theoretically possible for the total circuit reluctance to be determined solely by that of the bridge, and since the bridge is of mumetal, the whole core will tend to exhibit the properties of mumetal although the greater part of the core is of silicon-iron. The main advantage of this arrangement is that a core built in this way is comparatively cheap due to the small amount of nickel-iron involved ; at the same time, adequate winding area is provided by closing the magnetic circuit with a comparatively inexpensive material.

The disadvantage of this type of core is that in order to obtain a low reluctance the silicon-iron limbs must have a large sectional area to compensate for their comparatively low permeability and greater length of path ; the ideal core, therefore, tends to be somewhat bulky. In practice, however, reasonable results are obtained with a core in which the U is double the area

of the I portion or even is of the same area. Experimental magnetization curves in *Figure* 13.11 are plotted for this latter condition. Curve (*a*) shows the A.C. magnetization curve for a silicon-iron core, curve (*b*) is plotted for a mumetal core, while curve (*c*) is plotted for a composite core in which the mumetal bridge has the same sectional area but approximately one-quarter the length of the core composed entirely of mumetal.

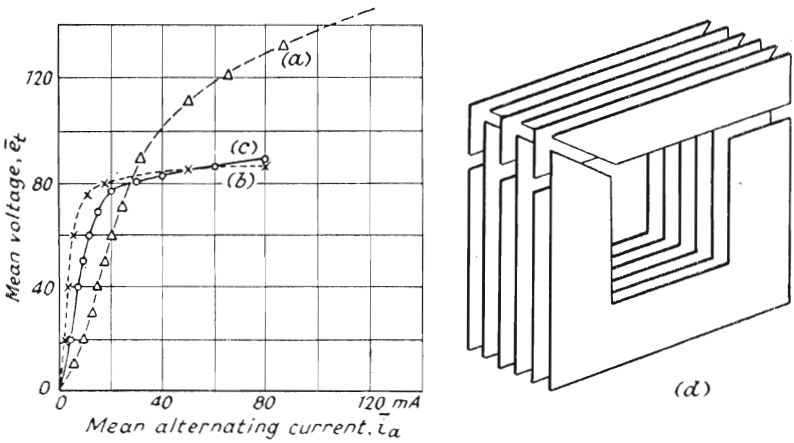


Figure 13.11. A.C. characteristics with composite core.
(*a*) A.C. characteristic of silicon iron core.
(*b*) A.C. characteristic of mumetal core.
(*c*) A.C. characteristic of composite core.
(*d*) Assembly of laminations in composite core.

The only reason for having a large length of path is to provide adequate winding area ; this consideration therefore fixes the useful path length. Thus, when computing the **B**/**H** curve for the composite core the total length of the magnetic circuit is taken to be the same as in the homogeneous mumetal core although the actual length of mumetal is only one-quarter of this value.

A practical form of construction for this type of core is shown in *Figure* 13.11*d*, where the I's are mitred to fit the U's at one end, the mitre occurs on alternate sides in successive laminations as shown.

13.16 Core Geometry

(*a*) *Core area.* The core area, denoted by the symbol a, refers to the effective area of iron embraced by one a.c. winding. The insulating medium usually contributes appreciably to the total

core area and is of course ineffective. Allowance for insulation can be made by assuming an average coating thickness as specified by the manufacturers and then allowing for this when measuring the total depth of core stack. The depth of core stack cannot be obtained accurately by assuming each lamination to have a certain thickness and then counting laminations. This is because lamination thicknesses are liable to vary by as much as 20% from the nominal value. Direct measurement of the core by means of a ruler is subject to an error of about 10% for smaller core sizes. In the case of a laminated core an accuracy of about 4% can be obtained by weighing the core and determining the total volume from the specific gravity. Dividing by the surface area of a lamination gives the effective depth of stack from which the effective area can easily be obtained. Since the weight of insulation is negligible in comparison with that of iron, the error due to neglecting it is unimportant. Furthermore, the surface dimensions of a lamination are accurately known to much closer limits than are necessary. In the case of a toroid the total volume may also be obtained by weighing; the effective area is then given if the volume is divided by the total length of path (i.e. the mean of the outer and inner circumferences). The effective core area determines the supply voltage, and this relation, determined by equation 2.10, is a comparatively accurate one; it is therefore worth while to have a knowledge of the core area to within about 5%.

(*b*) *Length of magnetic field.* The mean length of magnetic path is usually determined by straightforward measurement; its effective value varies slightly, but since it occurs in the amplification equations, and these are only approximate, its accurate assessment is seldom justified.

(*c*) *Mean length of turn.* The mean length of turn of each winding is required in order to determine the winding resistance. For a given winding it depends upon the average of the length of each turn in the first layer to the length of each turn in the last layer. It can usually be assessed to an accuracy of about 15%, which is usually adequate. The various paths taken when assessing the mean length of turn are defined in *Figure* 13.12; it is seen that the mean length of turn for the a.c. and control windings depends to some extent on the relative space occupied by these windings. Also that the total length of control winding is slightly less in the single unit construction (*a*) than in the

double-unit construction (*b*). It is less still, however, in the three-limbed construction (*c*), and the lower weight of the copper in the last, together with the compact arrangement, often makes it preferable for large transductors. When using the single-unit construction the mean length of turn for the control winding is

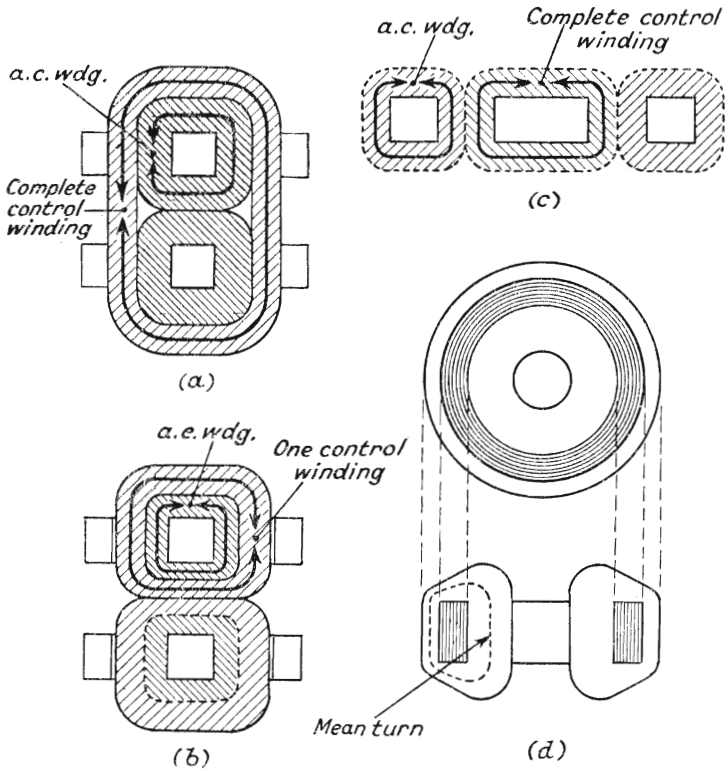


Figure 13.12. Mean lengths of turn for various types of core.
(*a*) Single-unit construction.
(*b*) Double-unit construction.
(*c*) Three-limbed construction.
(*d*) Typical section of windings on toroidal core.

approximately twice that of each a.c. winding and may be taken as being so for practical purposes.

When using toroidal cores, the crowding together of the windings as they go through the centre of the toroid leads to a non-uniform shape of the winding section as shown in *Figure* 13.12*d*.

(*d*) *Available winding area and space allocation of windings.* Ultimately the available winding area for a core can be increased only by increasing the magnetic path length, since the core must embrace the windings in a plane at right angles to the core section. In general, however, bobbins, insulation and imperfect winding usually account for nearly half of the available winding area. In order to assess the number of turns which will pack into a given total area the following procedure is adopted. Measure the total available area and subtract the estimated space taken up by the bobbin (if used). Of the remaining space allow a factor of 0·2 to 0·3 for imperfect winding and interlayer insulation and then

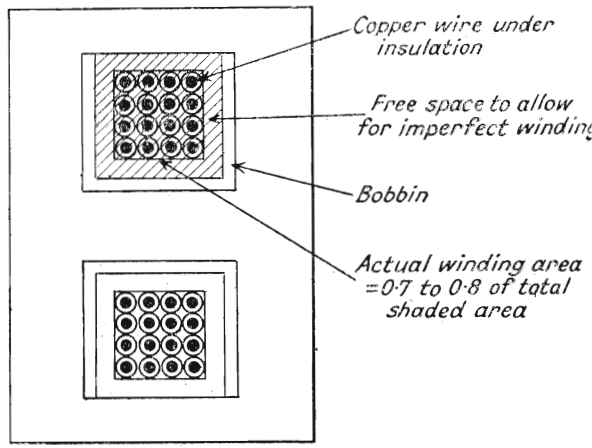

Copper wire under insulation
Free space to allow for imperfect winding
Bobbin
Actual winding area =0·7 to 0·8 of total shaded area

Figure 13.13. Section of windings showing wastage due to imperfect winding and insulation.

assume that the *insulated* turns lie evenly in the remaining area as shown by the drawing in *Figure* 13.13.

There is no fixed rule which governs the proportion of space allocated to the control and a.c. windings. In transductor designs the relative space occupied by the control and a.c. windings is usually governed by the individual turns and impedance values required.

In large units a high efficiency is more important than a high gain, and in such cases it is permissible to allow the control windings only 10% of the total winding area provided the transductor is auto-excited.

In cases where neither self- nor auto-excitation is used, however, ampere-turn balance exists between the control windings and the

a.c. windings. Therefore, assuming the a.c. winding area is fixed by thermal rating, the control windings will require to occupy at least the same area in order to avoid local heating.

13.17 Core Material Constants

The design of a magnetic amplifier usually proceeds by combining the various known constants for the core with the generalized control characteristics for the material under consideration.

In magnetic amplifier technology the magnetic parameters of the core have a rather specialized significance. Thus while the saturation flux density of mumetal is between 0·7 and 0·8 webers per square metre, this value is much too high to use in magnetic amplifier design work for which a value between 0·5 and 0·6 is more appropriate (section 2.6).

The saturation density gives a fairly precise relationship between the mean supply voltage, the core area and the a.c. windings. In addition, the current amplification depends, as we have seen, upon the mutual inductance M_{ca} which is proportional to the slope of the magnetization curve over the working range. For a particular core this is given by the total core reluctance S, but it may also be defined in more general terms by the gain permeability (p. 104). This is assessed by measuring the control characteristics of a large number of transductors using cores of the same material but different dimensions and hence deriving a suitable design figure for the gain permeability.

In design work some knowledge of the final shape of the amplifier characteristic is also desirable. This may be specified in terms of a linearity ratio which is defined as the proportion of the total range over which the amplification never decreases below a certain fraction of the maximum amplification. Alternatively, it may be assessed for a particular problem by inspection of the generalized control characteristics.

The latter method of predicting the linearity of an amplifier characteristic is the more useful owing to the fact that linearity requirements vary from one application to another and, therefore, if the linearity is specified for any given problem, the expected working range can be assessed from the generalized curves.

Although the saturation density primarily affects the maximum output of a transductor, it also influences the gain and the linearity of the characteristic as described in section 5.11. Bearing this in mind, an optimum design figure for the saturation density has

been found. For most of the commonly used materials, and within certain limits, this fixes the gain permeability and linearity applicable to a given core material.

Approximate values for these design parameters are listed in Tables 2, 3 and 4 for some materials at various frequencies. It will be noted that figures are not available for the silicon-irons at the higher frequencies. The losses are excessively high under these conditions, and the transductor will not operate satisfactorily. The amount of information is therefore limited and not of sufficient value to quote design figures.

Finally, it must be remembered that these constants, particularly the gain permeability, are subject to considerable variation from one batch of material to another, and for this reason the figures quoted for the gain permeability are somewhat conservative, and in general the amplification tends to be greater than the calculated value, thereby giving the necessary margin for final adjustment (sections 5.7 and 5.8).

13.18 Core Size and Maximum Output

One of two factors limits the maximum output which may be obtained from a given transductor. Usually the maximum permissible output is limited by the temperature rise which can be tolerated. In this case the surface area of the coils must be sufficient to dissipate the heat, and the current density in the windings must not be so great as to cause large internal temperature gradients leading to the existence of local hot spots. For small units operating at low frequencies, however, it is not possible to dissipate more than a certain amount of power in the a.c. windings due to the high internal impedance.

The internal impedance limitation may be seen on the basis of the current density in the windings, for if the core size is fixed, the maximum voltage which can be applied to the a.c. circuit is fixed by the number of turns and this determines the internal resistance. If a larger voltage is to be applied to a given transductor, more a.c. turns must be wound in the same available area in order to maintain the correct flux density. Thus the a.c. winding resistance will increase and it will do so in such a way that the current density at maximum output is the same. If, however, the supply frequency is increased the internal resistance applicable to a given core area may be reduced with the result that the current density goes up. Ultimately this will cause

heating ; the current density and temperature rise then become the limiting factors.

Thus there is a limit to the current density which can be produced in any transductor, and for small units at low supply frequencies this limit is below the maximum permissible. Therefore transductors having cores below a certain size cannot overheat. This may be shown for an auto-excited transductor from the following considerations.

The maximum applied voltage is given by

$$\bar{E}_s = 4\mathbf{B}_s a N_a f.$$

If the resistance of the rectifiers is ignored then we have the total circuit resistance R_a equal to $\frac{r_{aw}}{(1-\eta)}$ where η is the mean power efficiency $\frac{R_L}{R_a} = \frac{R_a - r_{aw}}{R_a}$ and r_{aw} is the internal resistance equal to the resistance of one a.c. winding. Hence the r.m.s. load current at maximum possible output is given by

$$1{\cdot}11\ \bar{i}_{as} = \frac{4{\cdot}44\ \mathbf{B}_s a N_a f}{r_{aw}}(1-\eta)$$

(since at maximum possible output the current is sinusoidal its form factor is 1·11).

In an auto-excited transductor this current flows in alternate windings every half-cycle and the power continuously dissipated in the transductor is

$$(1{\cdot}11\ \bar{i}_s)^2 r_{aw}$$

therefore the power in each winding is

$$\frac{(1{\cdot}11\ \bar{i}_s)^2 r_{aw}}{2}$$

and therefore the effective (r.m.s.) value of either coil current is

$$\frac{1{\cdot}11}{\sqrt{2}}\bar{i}_{as} = \frac{4{\cdot}44}{\sqrt{2}}\frac{\mathbf{B}_s a N_a f}{r_{aw}}(1-\eta)$$

and the current density is

$$(1-\eta)\frac{4{\cdot}44}{\sqrt{2}}\frac{\mathbf{B}_s a N_a f}{r_{aw} S_a}$$

where S_a is the wire sectional area.

Also, if b_a is the mean length of turn,

$$r_{aw} = \frac{\rho b_a N_a}{S_a}$$

substituting this in the previous expression, the current density becomes

$$\frac{4{\cdot}44}{\sqrt{2}} \frac{\mathbf{B}_s af}{\rho b_a} (1 - \eta)$$

The mean length of turn b_a cannot be less than $4\sqrt{a}$ and therefore we have

$$\text{current density} < 0{\cdot}79 \frac{\mathbf{B}_s f(1 - \eta)}{\rho} \sqrt{a} \quad . \quad . \quad . \quad (13.2)$$

As an example let us take a mumetal cored transductor excited at 50 c/s for which the core area is 0·5 sq. cm. and the mean power efficiency 50%.

In this case $\mathbf{B}_s = 0{\cdot}55$ webers/metre^2 and, since $\rho = 1{\cdot}6 \times 10^{-8}$ ohms metre, the current density will not exceed

$$\frac{0{\cdot}79 \times 0{\cdot}55 \times 50 \times 0{\cdot}5}{1{\cdot}6 \times 10^{-8}} \sqrt{0{\cdot}5} \times 10^{-2}$$

$$= 4{\cdot}8 \times 10^6 \text{ amps/metre}^2 \text{ (3,100 amps/in.}^2\text{)}.$$

Thus even when this transductor is run at an efficiency of only 50% it is unlikely to overheat. At a frequency of 1 kc./second, the core size to give this condition would be impracticably small and thus at higher frequencies overheating must be prevented by running at a higher efficiency.

The overall temperature rise in a transductor is governed by the rate at which heat is generated and the rate at which it is carried away. The maximum rate at which heat is generated depends primarily upon the copper losses since the iron losses in a transductor at maximum output are usually comparatively small (section 13.5). In the absence of forced cooling by means of oil or air the rate at which the heat can be dissipated depends upon the surface area of the coils, and if a bobbin is used then it is only permissible to consider the cylindrical surface area, as the end surfaces are usually ineffective due to the heat-insulating properties of the bobbin ; this fact is a point in favour of using preformed windings without bobbins, thereby leading to better cooling properties.

In order to give some guide as to the temperature rise, the curve in *Figure* 13.14 shows the expected temperature rise plotted against the power generated internally per square inch of radiating surface.

It is seen that for temperature rises up to about 50° C. the

relationship is linear and corresponds to 1° C. for ·006 watts per square inch.

Transductor Design

13.19 Design Procedure

Having decided that a magnetic amplifier is to be used for a given application, the next problem is to determine the type of circuit which will be most suitable and the core materials and rectifiers to give the best performance taking into account the general need to maintain prices at a competitive level consistent with obtaining satisfactory results.

(*a*) *Circuit.* The decision regarding the circuitry is usually a fairly easy one depending, as we have seen, partly upon the required zero stability and time-constant, but primarily upon the type of output required (e.g. push-pull or single-sided) and the method of connection to the load (e.g. centre-tapped or two-terminal). As has been pointed out, a high power gain invariably demands the use of auto-excitation or self-excitation for which trimming is generally necessary after manufacture. If response time is not important and the required power gain is small, then it is probable that, for large transductors, the best performance in respect of output for a given size and stability of characteristics will be obtained by using a series transductor without self-excitation, particularly if the output requirements are considerable and a low cost is desirable.

When using auto-excited transductors which require to have a gain stability of better than about 5% for $\pm$ 10% voltage and frequency variations it is necessary to use negative feedback over one or more stages of push-pull amplification. In this case the internal gain must be large enough to allow the use of feedback and at the same time retain sufficient overall gain to meet the existing specification; for this reason it is often necessary to employ more than one stage of amplification when negative feedback is used.

(*b*) *Core.* Having determined the overall requirements and settled upon the amount of power gain needed per stage of amplification, the next question is to decide upon the type of core material to be used. If the supply frequency is low (say 50 c/s), then for small instruments having a large gain, H.C.R. or Deltamax cores are generally the most suitable on account of the high saturation flux density. For higher supply frequencies (say over

400 c/s), however, losses become more significant, and if a good zero stability is required then the use of mumetal or even supermalloy is advisable. The problem of choosing between H.C.R. and mumetal is essentially one of weighing up the high saturation flux of the former against the higher gain and better zero stability obtainable with the latter. Grain-oriented silicon-iron cores are generally used for the larger outputs where nickel-iron would be uneconomic, but they are not satisfactory at frequencies over 400 c/s. on account of the iron losses which then become excessive.

(*c*) *Rectifiers.* The type of rectifier to be employed has hitherto depended primarily upon the supply frequency, but the rapid development of silicon crystal junction rectifiers capable of operating at high ambient temperatures and comparatively high frequencies is showing that in magnetic amplifier work these are likely to be superior in all respects to the less consistent selenium or copper oxide types. The load rectifiers, however, are not critical, and again on economic grounds the choice might well fall on the latter types of rectifier, particularly when dealing with large outputs. For auto-excitation however, crystal rectifiers are much better, germanium being quite satisfactory up to 50 or 60° C. and silicon well over 100° C., and probably as high as 150° C.

(*d*) *Efficiency and total resistance* R_a. The design of a complete stage of amplification should now be broken down to the design of two single transductors each operating into a certain a.c. circuit impedance. If the device is a push-pull amplifier then, depending upon the type of summing circuit used, each transductor will be operating into an equivalent load as discussed on p. 275.

In the case of a single-sided transductor or one employing magnetic summing, the total a.c. circuit impedance consists of the load, the winding resistance, the rectifier forward resistances and power source impedance, and this total is arrived at by fixing initially on a reasonable circuit efficiency. As has been pointed out earlier in this chapter and on p. 40, a high efficiency is generally applicable to output stages or transductors operating at a high supply frequency, but small units at 50 c/s. may run at an efficiency as low as 50% and thus reap the advantage of a high power gain from a given core size. Thus, having fixed on a reasonable mean power efficiency η, the total circuit resistance R_a is given by

$$R_a = \frac{R_L}{\eta}$$

where the load R_L has presumably been specified.

(*e*) *Supply voltage and a.c. turns.* Having fixed upon the circuit resistance R_a, the maximum possible value of the mean transductor current $\bar{\imath}_{as}$ is determined from the maximum specified load current and the required degree of linearity which will either be specified or must be assessed for a particular application. Alternatively one can use the generalized control characteristic (section 5.13) applicable to the appropriate frequency and core material. Knowing the maximum required change of working load current, the saturation current $\bar{\imath}_{as}$ is determined, since the latter will bear a fixed ratio to the working current for a given linearity factor. The maximum possible load current may be obtained by either of these methods, and since it flows when the transductor is completely saturated, its value is $\bar{E}_{as}/R_a$. The required supply voltage is therefore immediately obtainable.

Thus the mean supply voltage, the frequency and the saturation density are now known and the product of the core area and a.c. turns may be calculated as a single quantity thus :

$$aN_a = \frac{\bar{E}_{as}}{\mathrm{k}f\mathbf{B}_s} = K_1 \qquad . \quad . \quad . \quad (13.3)$$

where $k = 4$ if the auto-excited (parallel) connection is used

and $k = 8$ for the series connection.

(*f*) *Current gain.* The current gain will be specified either directly or in terms of voltages and impedances, and information regarding the control circuit is obtained from the specified current gain. Thus if the transductor is without self-excitation, the current gain gives the ratio of control turns to a.c. turns directly (making an allowance of 4% to 5% for non-linearity). Otherwise the amplifier is likely to be auto-excited, in which case the standard equation is used, i.e.

$$\text{known current gain} = \frac{1}{\pi}\frac{\omega M_{ca}}{R_a}$$

$$= \frac{2f\mu aN_aN_c}{R_a l}.$$

The product aN_a has already been calculated and the gain permeability μ is a constant for the material (see Table 2), thus the value of N_c/l may now be found.

(*g*) *Current density and wire sizes.* The wire size for the a.c. windings is now estimated after fixing on a safe value for the current density. When considering the maximum current density it is necessary to use r.m.s. values and the form factor will be between 1·11 and 1·56. Small units up to about 5 watts output can generally be rated at 4,000 amps. per square inch continuous rating, or even higher, but for larger output transductors this is much too high and 2,000 amps. per square inch is usually the upper limit.

The permissible current density and the load current requirements fix the minimum wire size, and if the diameter of this wire over the insulation is d, then assuming perfect winding, the area occupied by the a.c. windings will be

$$A_a = N_a d^2.$$

Thus from 13.3

$$A_a = \frac{K_1 d^2}{a} \quad . \quad . \quad . \quad (13.4)$$

At this stage is may be found that whereas the winding area and wire diameter are most conveniently measured in square inches and inches, the core area will be in metric units. Provided the distinction is noted and the various units are kept separate, no trouble need be encountered, as will be seen in the numerical examples.

A core is now chosen having a winding area and sectional area which is consistent with the relation $A_a = K_1 d^2/a$, bearing in mind that allowance must also be made for the control windings and a reasonable overall winding space factor. Having chosen the core, the length of iron circuit l is established and hence the control turns. The resistance of the control windings will be specified either by matching or time-constant considerations. Thus, knowing the number of turns and an approximate value for the mean length of one turn, the gauge of wire is fixed by the control winding resistance requirement, and from this the ideal winding area A_c is calculated. A certain percentage of this space (about 5% or 10%) should be added for bias windings and finally an overall space factor (say 0·7) is allocated. The total required winding area will in fact be given by $(A_a + A_b + A_c) \cdot \frac{1}{0{\cdot}7}$, and if the total winding area of the chosen core is less than this then a new choice must be made. In the case of toroids or C type cores, the complete core must be chosen nearest to the actual require-

ments, but if a laminated core is used the exact core size can be obtained by slight adjustment of the iron area and hence the number of a.c. turns.

(*h*) *Temperature rise.* All the information is now available, and it only remains to evaluate the a.c. turns from the core area and K_1, and to calculate the approximate resistance of the a.c. windings and hence the copper losses in watts. The approximate temperature rise can be estimated from the surface area and the curve in *Figure* 13.14. If the temperature rise is excessive then it implies that the core size is not limited by the current density,

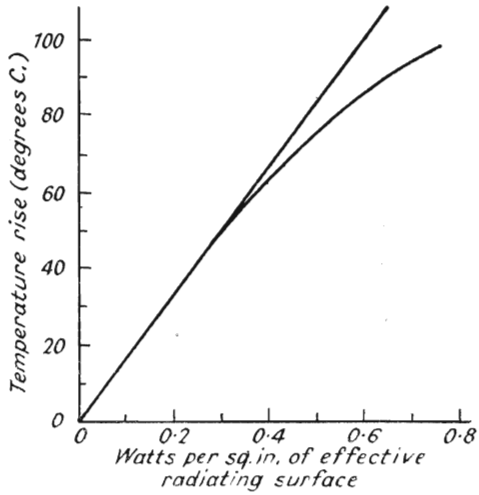


Figure 13.14. Curve relating temperature rise to the heat dissipation per unit area.

but by the overall copper losses and a certain maximum a.c. winding resistance must be the limiting factor ; this will therefore necessitate choosing a larger core and repeating the calculations in (*g*) above. Alternatively, if laminations are being used the core area can probably be increased, thereby reducing the a.c. turns by a certain ratio and their resistance by the square of this ratio since the winding area will remain the same.

Having designed the transductor on the basis of a certain power gain, the time-constant will already have been estimated roughly. The actual frequency response can be considered taking into account the factors discussed in Chapter 8 ; this is done in the last numerical example.

13.20 Design of a Transductor to Handle 750 Watts

A transductor is required to control the voltage of a d.c. machine by varying the field current. The resistance of the load (motor field) is equal to 30 ohms under running conditions and the field current requires to be varied from ·5 amps. up to 5 amps. for a change of control current from zero up to 250 mA. The input impedance should be less than 300 ohms. The mains supply frequency is 50 c/s.

In this case the mean power gain required is approximately equal to

$$\left[\frac{5}{\cdot 25}\right]^2 \times \frac{30}{300} = 40.$$

Since this is not very large and good linearity is obtained by using a simple series transductor without self-excitation, this will be taken as an example of the design of such an amplifier. The total d.c. output required is equal to 750 watts and a large silicon-iron core will therefore be the best proposition.

(*a*) *Estimation of mean supply voltage.* Firstly it is necessary to know the mean supply voltage. Assuming the transductor to have a high efficiency, most of the supply voltage will be developed across the load at maximum output, only a small proportion appearing across the a.c. windings and rectifier. For this reason it is adequate to establish the mean power efficiency R_L/R_a in order to derive the required supply voltage sufficiently accurately.

Therefore, taking a reasonable value of R_L/R_a to be 0·85, gives the total circuit impedance thus :

$$R_a = \frac{30}{0{\cdot}85} = 35{\cdot}3 \text{ ohms} \quad . \quad . \quad . \quad (13.5)$$

Since the transductor is not auto-excited the linearity should be good almost up to maximum output and a factor 0·90 is taken as the ratio of the maximum useful load current to the maximum possible load current. This gives a maximum possible load current

$$\bar{\imath}_{as} = \frac{5}{0{\cdot}90} = 5{\cdot}55 \text{ amps} \quad . \quad . \quad . \quad (13.6)$$

Hence the mean supply voltage must be sufficient to drive this current through the total a.c. circuit resistance :

$$\begin{aligned} \bar{E}_{as} &= 35{\cdot}3 \times 5{\cdot}55 \text{ volts} \\ &= 195 \text{ volts mean} \quad . \quad . \quad . \quad (13.7) \end{aligned}$$

(*b*) *Core area and a.c. turns.* If a Crystalloy core is used the maximum flux density is approximately 1·10 webers/sq. metre (11,000 gauss). From the basic equation for a series transductor (equation 2.10) we therefore have

$$\bar{E}_{as} = 8\mathbf{B}_s a N_a f$$

i.e. $$195 = 8 \times 1{\cdot}10 \times aN_a \times 50$$

giving $$aN_a = {\cdot}442 \text{ turns metres}^2 \text{ (4,420 turns cm.}^2\text{)} \quad (13.8)$$

(*c*) *Thermal ratings and core size.* In order to ensure that the transductor is not overrun, the surface area must be sufficient to radiate the heat generated internally due to copper losses. Local hot spots are avoided by ensuring that the current density in the a.c. windings is not excessive. A reasonable current density for the present design would be 2,000 amps. per sq. in., and for a maximum current of 5 amps. mean (approximately 5·55 amps. r.m.s.) this requires 16 s.w.g. copper wire.

When choosing the core consider the core area and its influence on the winding area (equation 13.4). Bearing in mind that the useful a.c. winding area will at most be only 0·7 to 0·8 $\times \frac{1}{2}$ (total area), a possible core size is selected from a list of sizes. In this case a crystalloy C type core is chosen, No. XZ 371015 (English Electric catalogue), and two such cores will be used for each a.c. winding. From published data the core area $a = {\cdot}00145$ sq. m. (14.5 sq. cm.)

giving from 13·8, $$N_a = \frac{{\cdot}442}{{\cdot}00145}$$

$$= 300 \text{ turns (approx.)} \quad (13.9)$$

With this core the mean length of turn for the a.c. windings will be approximately 8 in.

Thus the total length of wire in the two a.c. windings is approximately

$$\frac{2 \times 300 \times 8}{36} \text{ yd.}$$

which has a resistance of 0·88 ohm. At maximum current the power to be dissipated from these windings is therefore equal to

$$(5 \times 1.11)^2 \times 0{\cdot}88 \text{ watts}$$

$$= 27 \text{ watts.}$$

(*d*) *Winding areas and control windings.* The actual area occupied by the a.c. windings on each core will be

$$300 \times (\text{insulated wire diameter})^2$$

$$= 300 \times (0{\cdot}064)^2 = 1{\cdot}23 \text{ sq. in.}$$

Allowance has been made for the wire insulation and the space lost due to the fact that the wire is circular; thus, assuming the wire is evenly wound, an additional space factor of 0·8 should be adequate. The actual space required for the a.c. windings will therefore be

$$\frac{1{\cdot}23}{0{\cdot}8} = 1{\cdot}54 \text{ sq. in.}$$

The total winding area available on this particular core is 4·8 sq. in., thus the space available for the control windings is

$$4{\cdot}8 - 1{\cdot}54 = 3{\cdot}26 \text{ sq. in.}$$

Assuming the same space factor is applicable to the control windings, the " useful " winding area is equal to

$$3{\cdot}26 \times 0{\cdot}8 = 2{\cdot}61 \text{ sq. in.}$$

The required current gain = 20.
Therefore the control turns are given by

$$300 \times 20 = 6{,}000.$$

Allowing approximately 5% extra for loss of current gain due to slight non-linearity

gives $N_c = 6{,}300$ turns . . . (13.10)

The square sectional area occupied by each turn of the control windings is

$$\frac{2{\cdot}61}{6{,}300} \text{ sq. in.}$$

and the wire diameter is therefore

$$\sqrt{\frac{2.61}{6{,}300}} = {\cdot}02 \text{ in. diameter,}$$

i.e. 26 s.w.g.

Taking the mean length of turn for the control windings to be 16 in., the total length of wire is

$$\frac{6{,}300 \times 16}{36} = 2.800 \text{ yd.}$$

Using 26 s.w.g. copper wire, this has a resistance of approximately

236 ohms . . . (13.11)

Therefore the *useful* power gain is

$$\left[\frac{5}{{\cdot}25}\right]^2 \times \frac{30}{236} = 51 \quad . \; . \; . \; (13.12)$$

(*c*) *Thermal considerations.* The power dissipated in the control windings at maximum input is equal to $236 \times (0{\cdot}25)^2 = 14{\cdot}8$ watts. The total copper losses which occur at maximum output are therefore given by the sum of the power dissipated in the control windings and the power dissipated in the a.c. windings, i.e.

$$27 + 14{\cdot}8 = 41{\cdot}8 \text{ watts.}$$

The approximate temperature rise can now be estimated from the watts dissipated per unit surface area by means of the curve in *Figure* 13.14.

In order to conserve space and facilitate cooling, pre-formed coils should be used without bobbins.

With this particular core the total final surface area will be approximately 180 sq. in., giving a surface dissipation of about 0·23 watt per sq. in., which corresponds to a temperature rise of 38° C.

(*f*) *Rectifier.* A bridge rectifier is required which will pass a current of $5 \times 1{\cdot}11$ amps. without heating and withstand a peak reverse voltage equal to

$$\frac{195\pi}{2} = 310 \text{ volts (peak).}$$

Using double voltage selenium elements, the peak reverse voltage permissible is 42 volts. Therefore eight elements will be required per arm of the bridge. A suitable rectifier element can be selected from a maker's catalogue, e.g. Westinghouse size 2 which are 3 in. × 3 in. elements.

Allowing a forward voltage drop of 1·3 volts r.m.s. per element, there are sixteen elements in the current path, giving a total voltage drop of approximately 21 volts and therefore an effective resistance equal to

$$\frac{21}{5{\cdot}55} = 3{\cdot}8 \text{ ohms} \qquad . \quad . \quad . \quad (13.13)$$

(*g*) *Efficiency.* Having calculated the a.c. winding resistance and the rectifier losses, the mean power efficiency can now be estimated.

Total circuit resistance

$$R_a = 30 + 0{\cdot}88 + 3{\cdot}8 \simeq 34.7 \text{ ohms}$$

giving a mean power efficiency of 87%.

(Strictly speaking, the supply voltage should now be recalculated on the basis of this efficiency, nevertheless the supply voltage

need not be accurate to much more than 5% and hence the figures already obtained are quite good enough.)

(*h*) *Circuit amplification and time-constant.* On the basis of the effects described in Chapter 8, the unself-excited transductor with inductive load approximates dynamically to a second order system having a tendency to oscillatory response. Assuming the load to be purely resistive, however, the response time can be estimated from the circuit amplification. The circuit power amplification is $(R_a/R_c)G_i^2$.

Thus if the control source is assumed to be 200 ohms, for example,

we have $$G'_p = \frac{35}{436} \times 400 \doteqdot 32$$

and the response time will not be less than

$$\frac{G'_p}{4f} = \frac{32}{4 \times 50} \text{ seconds} = 0{\cdot}16 \text{ seconds} \quad . \quad . \quad (13.14)$$

(*i*) *Magnetizing current and iron losses.* The total magnetizing current may be found from the manufacturer's specification, which quotes it as about 0·5 V.A. (r.m.s.) per lb. of core at the working flux density, added to the magnetizing current appropriate to an air gap of ·00125 in. ($3{\cdot}17 \times 10^{-5}$ metre).

Magnetizing current for core

Weight of core is 6·86 lb.

current is equal to $$\frac{0{\cdot}5 \times 6{\cdot}86}{0{\cdot}5 \times 195 \times (1{\cdot}11)^2} = 28{\cdot}7 \text{ mA. (mean)}$$

(since the voltage across each a.c. winding $= 195 \times 0{\cdot}5$ volts mean, equation 13·7.)

Magnetizing current for air gap

field $\mathbf{H} = 1{\cdot}10$ webers/sq. metre $\times \frac{1}{\mu_{\text{air}}}$

therefore magnetizing current due to air gap

$$= \frac{1{\cdot}10 \times \text{gap}}{N_a} \cdot \frac{1}{\mu_{\text{air}}} \text{ amp.}$$

$$= \frac{1{\cdot}10 \times 3{\cdot}17 \times 10^{-5}}{300} \cdot \frac{10^7}{4\pi} = 92 \text{ mA.}$$

Therefore total magnetizing current $= 120{\cdot}7$ mA.

The iron losses will be about ·38 watt per lb. giving an iron loss equal to 2·6 watts per core. Thus the total loss of 5·2 watts will easily be dissipated without undue heating since the copper losses are negligible when the iron losses have this value.

The mean loss current corresponding to this power and considering both cores is

$$\frac{5{\cdot}2}{(1{\cdot}11)^2 \times 195} = 22 \text{ mA.}$$

Thus the total current flowing for zero control is

$$\sqrt{(120{\cdot}7)^2 + (22)^2}.$$
$$= 123 \text{ mA.}$$

(*Note:* In order to obtain the required load current of 0·5 amp. for zero control it may be necessary to provide a small bias winding supplied from a constant d.c. source.)

13.21 Design of Push-Pull Magnetic Amplifier with Negative Feedback

A recording instrument is required to record voltages from — 0·5 volt to 0·5 volt from a source impedance of 50 ohms. The recorder operates over its full scale for an input of 50 — 0 — 50 mA and has an impedance of 500 ohms. A magnetic amplifier is required to amplify the signal voltage to a level suitable for operating the recorder. The accuracy of measurement of the recorded voltage must be to within 1% with an additional allowable zero error not greater than 5 millivolt. (Supply frequency = 400 c/s.)

Assuming the input winding resistance will be approximately equal to 50 ohms, the useful power gain required will be approximately

$$\frac{(\cdot 05)^2 \times 500 \times 50}{(\cdot 5)^2} = 250.$$

This, of course, is very easily obtainable with a single stage of amplification. However, a gain stability of 1% cannot be obtained under normal conditions and therefore, besides using mumetal cores, overall negative feedback will be necessary.

If the control signal were a current from a high impedance source sufficient gain and linearity might be achieved by using series transductors without self-excitation and this would be a reasonable approach. In the present instance, however, the control circuit must be stabilized against resistance variations

due to ambient temperature changes, since with a low impedance signal source variations of resistance will vary the control current. For this reason negative voltage feedback after auto-excitation is a better arrangement. In this case a voltage proportional to the load current is fed back into the control circuit in series with the control voltage.

The zero stability will not be influenced by the negative feedback and that specified corresponds to an input current of

$$\frac{\cdot 005}{R_c} \text{ amp.}$$

$$= \frac{\cdot 005}{100} = 5 \times 10^{-5} \text{ amp. control current.}$$

In a winding of 50 ohms this corresponds to a power level of

$$(5 \times 10^{-5})^2 \times 50 \text{ watt}$$

$$= 1{\cdot}25 \times 10^{-7} \text{ watt.}$$

Since the normal zero stability for a push-pull amplifier is about 10^{-8} watt (Chapter 10), the required accuracy of 5 mV. will be possible.

The circuit diagram of the amplifier will be that shown in *Figure* 11.17 (since no centre-tap is available on the load, the voltage summing circuit will have to be used ; the characteristics of this circuit were discussed in Chapter 10).

From the results in Chapter 11 we have, assuming that the internal current gain is large,

$$r_{cf} = \frac{E_c}{\bar{\imath}_L} = \frac{\cdot 5}{\cdot 05} = 10 \text{ ohms,*}$$

therefore, for maximum sensitivity in respect of zero stability (section 11.23) the total resistance of both control windings will be (50 + 10) ohms, i.e. 30 ohms per winding.

(*a*) *Current gain required.* The required current gain can be estimated from the amount of negative feedback necessary to maintain an accuracy of 1% on the basis of about 10% accuracy without feedback. Thus if the load current per unit control voltage of the overall amplifier is

$$\bar{\imath}_L/E_c = \frac{G_i/(R_c + r_{cf})}{1 + r_{cf}G_i/(R_c + r_{cf})} \text{ (equation 11.14)}$$

* The feedback resistor r_{cf} must be preset since its final value, although approximately 10 ohms, must be fixed by the gain requirements of the amplifier.

we have $\dfrac{\delta(\bar{\imath}_L/E_c)}{\bar{\imath}_L/E_c} = \dfrac{\delta G_i}{G_i}\dfrac{1}{1 + r_{cf}G_i/(R_c + r_{cf})}$ (see p. 309)

now the overall accuracy is required to be 0·01 therefore, assuming that on open loop the accuracy, dG_i/G_i, is approximately 0.10, we have

$$0{\cdot}01 = \frac{0{\cdot}1}{1 + r_{cf}G_i/(R_c + r_{cf})}$$

giving $$r_{cf}G_i/(R_c + r_{cf}) = 9,$$

so that G_i must be *at least* equal to

$$\frac{9 \times 120}{10} = 108$$

(since the control winding is 60 ohms and is matched to the external circuit and source, the total will be 120 ohms).

Thus the required current gain of the amplifier on open loop is determined by the accuracy specified rather than the required closed loop gain. The latter is determined almost entirely by the resistance r_{cf} which must therefore be temperature stable.

(*b*) *Calculation of circuit impedances and maximum transductor current.* The total output required is equal to

$$(\cdot 05)^2 \times 500 \text{ watts} = 1{\cdot}25 \text{ watts}$$

and therefore each transductor will need to be capable of handling a power output between 9 watts and 18 watts, according to whether the internal resistance is large or small (*see Figure* 10.4).

Therefore, in order to limit the overall power requirements, the internal resistance will be made as small as possible and a figure of $r_a = 0{\cdot}1\ R_L$ will be aimed at.

Curves (*a*) in *Figure* 10.4 show that the optimum value of summing resistor R_s is approximately equal to the load for maximum efficiency. When maximum output is being delivered, one transductor is almost cut off and therefore the other transductor is supplying a total load consisting of R_s in parallel with the series combination of R_L and R_s as shown in *Figure* 10.17*b*.

Thus the total transductor current $\bar{\imath}_a$ under these conditions is given by

$$\bar{\imath}_a = \frac{R_L + 2R_s}{R_s}\,\bar{\imath}_L$$

and since $R_s = R_L$ represents a reasonable compromise (section 10.5), we have

$$\bar{\imath}_a \text{ (max)} = 150 \text{ mA}.$$

(*c*) *Mean supply voltage.* The mean supply voltage is now calculated on the basis of the maximum possible current (where maximum linear current is 150 mA.). The total circuit impedance referred to one transductor with the other cut off is equal to

$$r_x + \frac{R_s(R_s + R_L)}{R_L + 2R_s}$$

$$= 50 + \frac{2R_L}{3} = 383 \text{ ohms.}$$

Assuming a linear range extending over one-half the total range the maximum possible output for complete saturation will be 300 mA and hence the mean supply voltage must be sufficient to drive this current through an impedance of 383 ohms, i.e. 115 volts mean.

(*d*) *Core area and a.c. turns.* The product of the core area and the a.c. turns will be found as in previous examples from the flux density equation. In this case, however, the voltage across a summing resistor opposes that causing flux changes in the adjacent transductor and hence a voltage uprating is possible as described in section 10.6.

Since, however, the voltage has been fixed, the voltage uprating factor will result in a smaller area-turns product than for a single transductor.

The uprating factor is equal to

$$1 + \frac{R_s}{R_L + r_x\left(\frac{R_L}{R_s} + 2\right)}$$

$$= 1 + \frac{500}{500 + 50 \times 3} = 1{\cdot}77.$$

Thus we have the actual supply voltage $\bar{E}_a$ given by

$$\bar{E}_a = 1{\cdot}77 \times 4\mathbf{B}_s a N_a f$$

and since saturation density for mumetal is 0·55 webers/m.^{2} (5,500 gauss) this gives

$$aN_a = \frac{115}{4 \times 1{\cdot}77 \times 0{\cdot}55 \times 400} \text{ turns/metres}^2$$

$$= 0{\cdot}075 \text{ turns/metres}^2$$

(750 turns/cm.^{2}).

(*e*) *Control turns and core length.* The current gain of a push-pull pair with voltage mixing (equation 10.6) is equal to

$$\frac{2}{\pi} \cdot \frac{\omega M_{ca}}{R_L + \left(\frac{R_L}{R_s} + 2\right) r_x} = \frac{2}{\pi} \frac{\omega M_{ca}}{650}$$

the required current gain is at least 108

so that $$\frac{4 \times 400}{650} \times aN_a \times \mu \times \frac{N_c}{l} = 108$$

and since $aN_a = 0{\cdot}075$ turns/metres^2

and taking $$\mu = \frac{4\pi}{10^7} \times 10^5$$

this gives $$\frac{N_c}{l} = \frac{100 \times 650 \times 10^7}{4\pi \times 10^5 \times 0{\cdot}075 \times 4 \times 400}$$

$$= 4{,}300 \text{ turns/metre} = 43 \text{ turns/cm.}$$

(*f*) *Rectifier considerations.* In this particular case the load rectifier will consist of a bridge, each arm of which contains a germanium element (e.g. B.T.H. No. G.J. 3–D). Each auto-excitation rectifier will also comprise one of these elements. Taking a forward voltage drop of approximately 0·5 volts mean per element, we have a total circuit resistance due to the rectifier of $\frac{3 \times 0{\cdot}5}{0{\cdot}150}$ ohms $= 10$ ohms. Thus the resistance r_{aw} of each a.c. winding must not be greater than 40 ohms since the total internal resistance r_x must be less than 50 ohms (from (*b*)). Assuming a current density of 2,000 amps. per sq. in. is allowable, a minimum wire size can be calculated on the basis of the maximum r.m.s. transductor current.

$$\bar{\imath}_a \text{ (max)} = 150 \text{ mA.}$$

taking the form factor equal to 1·4

$$\text{the r.m.s. current is } 1{\cdot}4 \times 150 \text{ mA.}$$

and the power generated in one winding will be

$$\frac{(1{\cdot}4 \times 0{\cdot}150)^2}{2} \times r_{aw}$$

since the current flows every other half-cycle. Therefore the r.m.s. current per winding will be

$$\frac{1{\cdot}4 \times 0{\cdot}150}{\sqrt{2}} = 0{\cdot}149 \text{ amp.}$$

for which the permissible wire gauge is No. 34 s.w.g.

(diameter = 0·011 in.).

The total number of a.c. turns per core is $0{\cdot}075/a$

(since $aN_a = 0{\cdot}075$)

or $\dfrac{750}{a}$ where a is measured in sq. cm.

Therefore the total area A'_a in square inches required for the a.c. windings assuming a space factor of 0·7 is

$$\frac{750}{0{\cdot}7a} \times (0{\cdot}011)^2$$

(where 0·011 in. is the diameter of the insulated wire)

$$A'_a = \frac{0{\cdot}13}{a} \text{ sq. in.}$$

Taking lamination size 522 we have $l = 0{\cdot}084$ metres, therefore the number of control turns is $4{,}300 \times 0{\cdot}084 = 360$ turns approximately (since $N_c/l = 4{,}300$). The mean length of turn will be 5 in. approximately, and thus the total length will be

$$\frac{360 \times 5}{36} \text{ yd.} = 50 \text{ yd.}$$

The total resistance must be 30 ohms per transductor, making a total of 60 ohms.

Therefore the resistance per 1,000 yd. will require to be

$$\frac{30}{50} \times 1{,}000 = 600 \text{ ohms.}$$

This corresponds to 36 s.w.g. (diameter = 0·0093 in.), occupying a total area A_c given by

$$A_c = 360 \times (0{\cdot}0093)^2 \text{ sq. in.}$$

The actual area occupied A'_c is equal to $\dfrac{A_c}{0{\cdot}7} \doteqdot 0{\cdot}044$ sq. in.

Now the total winding area available is 0·28 sq. in. If a core area of 0·75 sq. cm. is assumed the total winding area will be $0{\cdot}174 + 0{\cdot}044 \doteqdot 0{\cdot}22$ sq. in. Since the a.c. windings will each have 1,000 turns of No. 34 s.w.g., the resistance of each a.c. winding on the basis of a mean turn length of 2·5 in. is only 25 ohms. This is much smaller than the estimated maximum of 40 ohms and a considerable space is still left available on the core. This extra space can be used to increase the current gain and

thereby the stability with negative feedback by increasing the control turns.

The winding area available for control windings allowing 0·01 sq. in. for bias windings is

$$0{\cdot}28 - (0{\cdot}173 + 0{\cdot}01) = 0{\cdot}097 \text{ sq. in.}$$

This corresponds to a useful area of 0·068 sq. in. which will take 560 turns of No. 34 s.w.g. having a resistance of 28 ohms. Thus the final transductor will be wound on a core of No. 522 laminations having a total area equal to 0·75 sq. cm. and

$$N_a = 1{,}000 \text{ (No. 34 s.w.g.)}$$

and

$$N_c = 560 \text{ (No. 34 s.w.g.).}$$

Final calculations on the temperature rise, etc., now proceed as in the previous example.

13.22 Design of a Magnetic Amplifier Position Servo

A magnetic amplifier is required to operate a d.c. split-field motor (with attached d.c. tacho) for use in a closed loop automatic position controller.

The angular position of the output shaft is transmitted to and compared with that of the input shaft by means of rotary position transmitters (synchros). The proposed layout is as shown in *Figures* 13.15 and 13.16.

It is required that the steady-state error shall be not greater than an angular misalignment of $\frac{1}{3}°$ between the input and output (excluding errors in the position transmitters).

It will be assumed that each component may be represented by a linear transfer function of the form $K.g(D)$, where K is the steady-state gain of the component.

DATA

(i) *Motor field.* Each motor field has a resistance of 1,500 ohms when hot and an inductance of 35 henries. Maximum field excitation corresponds to a current of 80 mA. in one field and zero in the other. Equal and opposite currents of 40 mA. in each field give zero resultant excitation.

(ii) *Motor armature.* Stall torque at maximum field current is 20 oz. in.

Maximum speed is 7,000 r.p.m.

Armature moment of inertia, including tacho and allowance for gear box, is 2 oz. in.^{2}

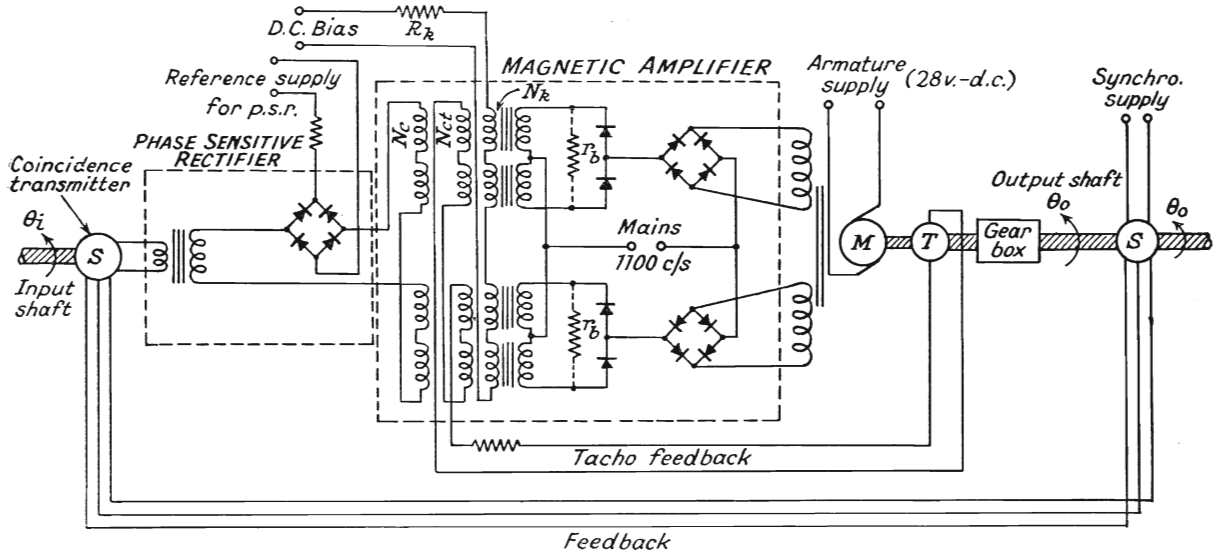


Figure 13.15. Magnetic amplifier position controller (circuit).

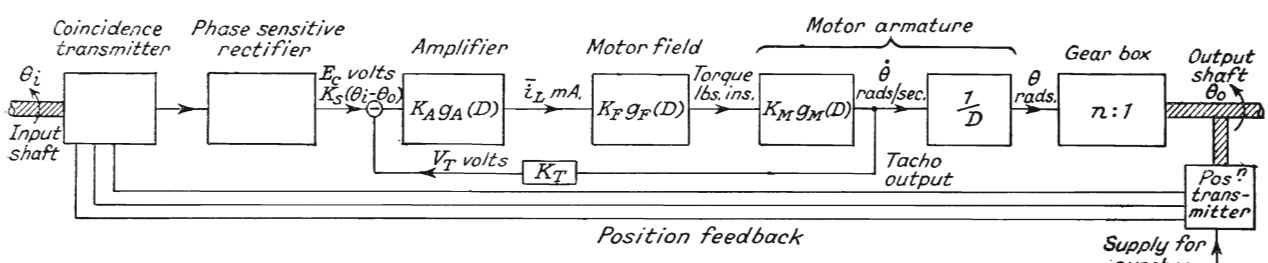


Figure 13.16. Block diagram for position servo showing inputs, outputs and relevant transfer functions.

Armature supply is 28 volts 1 amp. direct current.

Tacho output = 0·19 volts per radian per second.

(iii) *Position transmission system.* Synchros supplied with 100 volts at 1,100 cycles per second.

Output impedance from phase sensitive rectifiers = 600 ohms.

Overall sensitivity of synchros and rectifier

= 0·2 volts per degree angular misalignment (d.c. output).

(iv) *Power supplies.* 180 volts 1,100 c/s for synchros and magnetic amplifier, direct current for armature.

Design of Amplifier

(*a*) *Determination of reasonable efficiency.* When the supply frequency is low it is possible and in fact necessary to design at a low efficiency in order to obtain sufficient gain from a reasonably small unit. In the present design, however, this is not likely to be the case and a high efficiency is desirable. Thus the ratio of the load resistance to the transductor internal resistance r_x should be as large as possible. There are two reasons which make this requirement particularly important in the present application. Firstly, the power available from the high-frequency supply is likely to be limited and thus, if the transductor has a low efficiency a large amount of power will be wasted in heating up the transductor windings. Secondly, if the load resistance is not a large proportion of the total circuit resistance, the supply voltage will be considerably higher than the maximum load voltage with the result that insulation problems might be difficult. In the present example the maximum working load voltage will be at least equal to

$$0{\cdot}08 \times 1{,}500 = 120 \text{ volts mean.}$$

Furthermore, the maximum working load current cannot, for reasonable linearity, be much more than one-half the saturation load current. Therefore the maximum load voltage under conditions of complete saturation will be approximately 240 volts. If it were possible to make the mean power efficiency 50%, for example, then the supply voltage would require to be nearly 500. (It should be noted that although the supply voltage must correspond to the saturation current, $\bar{\imath}_{as}$, the maximum usable current is only about one-half this. Nevertheless the given supply voltage must be used in order to obtain the characteristic and fix the maximum current, half of which will be the limit of linearity.)

Thus, in view of the need to conserve power, prevent overheating and avoid insulation problems, a mean power efficiency of at least 90% will be aimed at.

(*b*) *Determination of required amplifier gain.* The amplifier gain is determined by the specified steady-state accuracy with which the two shafts must be aligned and this in turn is determined, as follows, by the opposing torques due to static friction. The effect of static friction is such as to require a certain minimum armature torque known as the stiction torque before the motor will start. This implies that until the differential field current is greater than a certain value, usually about 15% of the maximum value, static friction will oppose rotation of the armature. The differential field current is produced by the error between the input and output shafts via the synchros and the amplifier, and the magnitude of the field current for a given error depends upon the amplifier gain. It therefore follows that for the maximum allowable error the amplifier output must be just sufficient to start the motor against the stiction torque.

The overall sensitivity of the synchro system with the phase sensitive rectifier is given by

$$K_s = 0{\cdot}2 \text{ volts per degree,}$$

and assuming that 15% of the maximum differential field current will produce sufficient torque to start the motor, we have for a maximum error of $\frac{1}{3}°$, an input signal of $\frac{0{\cdot}2}{3}$ volts which must give a differential output of 12 mA. This corresponds to a differential output in the load of 180 milliamps per volt.

Since the motor field has two independent and isolated coils, the push-pull magnetic summing circuit can be used without an isolating transformer, as shown in *Figure* 13.15. Also, assuming the input of the amplifier is matched to the output of the phase sensitive rectifier, the total input resistance of both transductors in push-pull will be 600 ohms, i.e. 300 ohms each, making a total circuit resistance R_c of 1,200 ohms.

The control current corresponding to $\frac{1}{3}°$ will thus be equal to

$$\frac{0{\cdot}2}{3} \times \frac{1}{1{,}200} \text{ amps.} = 0{\cdot}056 \text{ mA.}$$

and this must produce a change of 6 mA. (mean) in the corresponding field coil giving the required current gain as

$$G_i = 108 \qquad . \quad . \quad . \quad (13.15)$$

The total a.c. circuit resistance can be estimated from the proposed efficiency thus :

$$R_a < \frac{1{,}500}{0{\cdot}9} = 1{,}670 \text{ ohms} \quad . \quad . \quad . \quad (13.16)$$

(*c*) *Estimated time-constant.* An approximate value for the time-constant of the transductor may be estimated from the circuit power amplification assuming a value for the ampere-turns ratio. In this case a high value, say 80, will be chosen since the most important feature is a high gain/time-constant ratio (p. 193). If the total control circuit resistance is considered to be divided equally between the two transductors, the time-constant of the push-pull pair will be the same as that of one unit, since both the resistance and the effective inductance of the control circuit are doubled.

The circuit power amplification for each transductor is

$$(108)^2 \times \frac{1{,}670}{600} = 3{\cdot}3 \times 10^4 \quad . \quad . \quad . \quad (13.17)$$

and for the time-constant τ_A we have

$$\tau_A > \frac{3{\cdot}3 \times 10^4}{2 \times 1{,}100} \times \frac{1}{80} = 0{\cdot}19 \text{ sec.} \quad . \quad . \quad . \quad (13.18)$$

The response time may be reduced by the use of two stages of amplification, but in this case the design of the amplifier will be carried out on the basis of a single stage.

(*d*) *Core material and rectifiers.* The power gain required and the high supply frequency indicate the need for a high permeability low-loss material for the cores. In addition, rectifiers of low self-capacitance are essential if reasonable working conditions are to be maintained with the high-frequency supply. The present design will therefore be carried out on the assumption that a mumetal core will be used and that the rectifiers will be of the germanium crystal junction type.

(*e*) *A.C. supply voltage.* The a.c. supply voltage is fixed by the total circuit resistance and the maximum possible load current.

The limit of linearity which fixes the maximum possible current for a single transductor is estimated for mumetal from the generalized curves applicable to 1 Kc/sec. In the present case the limit can be taken as high as 0·65 of the maximum value for the following reasons :

(i) Since the amplifier is included in a closed loop, non-linearity will be less important than if the loop were open.
(ii) The effects due to static friction only occur at balance when the amplifier is operating near the origin and the amplifier sensitivity would be maintained in this region.
(iii) This relaxation of the amplifier requirement will help to reduce the value of the required supply voltage.

On the other hand, if the maximum output is taken too far up the characteristic, a large error will be necessary to produce full motor torque with the result that the rate of following a sudden change of input will be reduced.

When one transductor is giving its maximum useful output the output from the adjacent transductor will not be quite zero ; it therefore follows that since the resultant output is required to be 80 mA., the actual value of the transductor current at the limit of linearity must be somewhat greater than 80 mA. Assuming that the minimum current is about 5% of the maximum possible value, the actual range of the transductor characteristic which is utilized corresponds to

$$65\% - 5\% = 60\%,$$

and this is equivalent to a mean current swing of 80 mA. Therefore the maximum possible value of the mean load current at complete saturation would be

$$\bar{\imath}_{as} = \frac{80}{0{\cdot}6} \text{ mA.} = 133 \text{ mA.} \quad . \quad . \quad . \quad (13.19)$$

This current flowing in a load resistance of 1,670 ohms requires a supply voltage such that :

$$\begin{aligned} \bar{E}_{as} &= 0{\cdot}133 \times 1{,}670 \text{ volts mean} \\ &= 223 \text{ volts mean} \quad . \quad . \quad . \quad (13.20) \end{aligned}$$

(*f*) *Core area and a.c. turns.* The product of the a.c. turns and the core area can now be calculated knowing the working flux density. In the present example the cores will be mumetal, which has a design saturation density of 0·5 weber/sq. metre (5,000 gauss) at 1 Kc. (Table 3).
Thus since $\bar{E}_{as} = 4\mathbf{B}_s a N_a f$

$$aN_a = \frac{223}{4 \times 0{\cdot}5 \times 1{,}100} = \begin{matrix} 0{\cdot}101 \text{ turns/metre}^2 \\ (1{,}010 \text{ turns/cm.}^2) \end{matrix} \quad . \quad (13.21)$$

(*g*) *Magnetic core length and control turns.* Knowing the required current amplification and the empirical figure for the gain permeability, we may use the results of Chapter 5 to estimate the control turns per unit length of flux path thus :

$$\text{current gain} = \frac{1}{\pi}\frac{\omega M_{ca}}{R_a} = \frac{1}{\pi}\frac{\omega N_a N_c a\mu}{lR_a} = 108$$

$$\text{taking } \mu = \frac{4\pi}{10^7} \times 30{,}000 \text{ (at 1·1 Kc./sec.)},$$

substituting for aN_a from equation 13.21
and putting in the known values of frequency and a.c. circuit resistance gives

$$N_c/l = 2{\cdot}16 \times 10^4 \text{ turns/metre (216 turns/cm.)} \quad . \quad . \quad (13.22)$$

(*h*) *Choice of core, estimation of heating and control windings.* It now remains to select appropriate rectifiers and to choose a core and windings meeting the following requirements :

(i) $aN_a = 0{\cdot}101$ turns/metres^2.

(ii) $N_c/l = 2{\cdot}16 \times 10^4$ turns/metre.

(iii) Resistance of control windings = 300 ohms.

(iv) Temperature rise not to be excessive, say less than 30° C.

This places an upper limit on the permissible resistance of the a.c. windings and also implies a certain minimum gauge of wire to avoid local hot spots.

The first three conditions fix the current amplification, power amplification and time-constant while the fourth condition ensures that the transductor operates under reasonable thermal conditions.

Before the core can be chosen it is first necessary to know what proportion of the total internal resistance is due to the rectifiers, and therefore it is now necessary to choose rectifiers having the correct rating from the various types available. In the present instance the high frequency makes it desirable to use crystal rectifiers of which the germanium or silicon-junction type are, in this case, necessary in order to pass the specified current. The following conditions of operation have to be taken into account :

(i) *Peak inverse voltage.* The maximum possible voltage which could be applied to the rectifier elements in the reverse direction is equal to

$$\hat{E}_{as} = \frac{223 \times \pi}{2} = 350 \text{ volts peak.}$$

(ii) *Continuous rating.* This cannot be estimated exactly since it will be governed largely by the type of signals which the servo is likely to be following. If the maximum linear conditions are considered to represent the continuous requirements then this should allow an adequate margin.

On this basis the continuous reverse working voltage for the load rectifiers is equal to $0{\cdot}133 \times 0{\cdot}65 \times 1{,}500 = 130$ volts mean, and for the auto-excitation rectifiers less than

$$0{\cdot}133 \times 0{\cdot}65 \times 170 = 14{\cdot}6 \text{ volts mean.}$$

(Total internal resistance is $R_a - R_L = 170$ ohms $= r_x > r_{ar} + r_{aw}$.) Similarly, the continuous load current will be equal to 86·5 mA. mean.

It must be remembered, particularly when dealing with current ratings, that, as in the previous examples, the r.m.s. values will be somewhat higher. From these considerations a suitable rectifier is chosen, e.g. Messrs. B.T.H., type G.J. 3–D. Two of these particular units would be required in series for each arm of the load bridge rectifier and a further two for auto-excitation. This corresponds to five rectifier elements in the forward path of the current. A voltage drop of 0·5 volts is allowed per element, and the equivalent resistance of the rectifier may be obtained by dividing the forward voltage by the mean current at maximum output.

$$\text{Equivalent resistance of rectifiers} = \frac{2{\cdot}5}{0{\cdot}086} = 29 \text{ ohms.}$$

Since the total resistance must not be greater than 1,670 ohms and the load is 1,500 ohms, the resistance of one a.c. winding should not be greater than

$$170 - 29 = 141 \text{ ohms} \quad . \quad . \quad . \quad (13.23)$$

The field current of 80 mA. corresponds to a firing angle which is greater than 90°, and the form factor will be between 1.11 and 1·56 so that the maximum r.m.s. load current will probably be about 120 mA. Therefore, if the winding resistance is as great as 141 ohms the power generated in each winding will be

$$141 \times (0{\cdot}120)^2 \times \tfrac{1}{2} = \text{approximately 1 watt} \quad . \quad . \quad (13.24)$$

The surface area necessary to dissipate this heat will dictate the minimum size of transductor provided the wire gauge is sufficiently large to limit the current density to a reasonable value.

Since the field current is unlikely to be at its maximum value for prolonged periods, a current density of 4,000 amps. per sq. in. is permissible if based on the maximum r.m.s. current of 120 mA. This means that the smallest permissible wire gauge for the a.c. windings will be about 37 s.w.g., having a nominal diameter over the insulation of about 0·0084 in.

If the core section is a sq. cm., each of the a.c. windings will have $\frac{1{,}010}{a}$ turns (from equation 13.21) so that, allowing a space factor of 0·7 and assuming the windings are pre-formed without bobbins, the total window area occupied by the a.c. windings will be

$$A'_a = \frac{1{,}010 \times (0{\cdot}0084)^2}{0{\cdot}7\,a} \text{ sq. in.}$$

giving

$$A'_a = \frac{0{\cdot}103 \text{ sq. in}}{a}.$$

Thus if a lamination has a window sufficient to accommodate an a.c. winding area equal to A'_a, then the core section would be given by

$$a = 0{\cdot}103/A'_a \text{ cm.}^2 \qquad . \quad . \quad . \quad (13.25)$$

(Note that the winding area is measured in sq. in. and the core area in sq. cm. ; see p. 386.) This now gives a basis on which to choose a lamination size. For example, referring to the table of standard laminations, it is clear that if size 521 (Table 1, p. 415) is chosen, the a.c. winding area would be less than 0·125 sq. in., giving a core area of the order of 1 sq. cm. and hence a stack depth disproportionately large compared with the width in order to obtain this area of section. Turning to lamination size 522, it appears as though this size will probably meet the requirements.

The total area available is 0·28 sq. in. and the length of magnetic path is 0·084 metres. The number of control turns are therefore given by

$$\frac{N_c}{0{\cdot}084} = 2{\cdot}16 \times 10^4 \qquad . \quad . \quad . \quad (13.26)$$

i.e., $N_c = 1{,}820$ turns. Assuming a mean length of turn of 5 in. gives a total length of wire equal to 253 yd., and since the resistance must be 300 ohms, the wire size for the control winding is fixed at 39 s.w.g., which has an overall diameter of 0·0066 in.

Therefore control winding area

$$A'_c = \frac{(0{\cdot}0066)^2 \times 1{,}820}{0{\cdot}7} = 0{\cdot}113 \text{ sq. in.}$$

Assuming the bias and tacho feedback windings occupy about 10% of the space taken by the control windings, the total space for bias and control coils will be about 0·124 sq. in. Since the total window area on this size of lamination is 0·28 sq. in., the space remaining for a.c. windings is equal to

$$0{\cdot}28 - 0{\cdot}124 = 0{\cdot}156 \text{ sq. in.}$$

and therefore from (13.25) the core area will be given by

$$a = \frac{0{\cdot}103}{0{\cdot}156} = 0{\cdot}66 \text{ sq. cm.}$$

On this basis the number of turns on each a.c. winding may be obtained from equation 13.21 thus :

$$N_a = \frac{1{,}010}{0{\cdot}66} = 1{,}530 \text{ turns} \quad . \quad . \quad . \quad (13.28)$$

assuming the mean length of turn to be 2·5 in., the total length of wire is

$$\frac{1{,}530 \times 2{\cdot}5}{36} \text{ yd.}$$

$$= 106 \text{ yd.}$$

and the total resistance is 70 ohms ; this is only one-half that allowed for and therefore the power wasted in each winding will only be 0·5 watt at maximum output, making 1 watt total. Since the surface area is about 14 sq. in. the heat dissipation corresponds to 0·072 watt per sq. in., which gives a temperature rise of about 12° C. from *Figure* 13.14.

In this case it is clear that the core size has been fixed by current density considerations. If, however, the temperature rise had been found excessive the wire diameter would need to be increased, thereby reducing the coil resistance. This would have necessitated a smaller number of turns in order to accommodate the windings in the available area which in turn would have demanded a proportionately greater core area to maintain the product aN_a at the required value.

Thus the final design specifies a magnetic amplifier consisting of a pair of mumetal-cored transductors working in push-pull into two independent coils as shown in *Figure* 13.15. Adjustment of

the gain is provided by means of the presetting resistors across the auto-excitation rectifiers.

Useful core area = 0·66 sq. cm.

Lamination size = E type I.S.S. No. 522 $1\frac{1}{2}$ in. $\times$ $1\frac{1}{2}$ in.

A.C. turns N_a = 1,530 turns No. 37 s.w.g. enamelled wire.

Control turns N_c = 1,820 turns No. 39 s.w.g. enamelled wire.

It only remains to estimate the overall servo performance and deduce a reasonable control winding for the tacho feedback.

13.23 Servo Response

(*a*) *Tacho feedback loop and overall frequency response.* In order to estimate the response and the required feedback the maximum tacho loop gain consistent with stability is first determined. Referring to *Figure* 13.16 since K_A, K_F and K_M are already fixed, K_T is limited to a value which maintains adequate gain and phase margin round the tacho loop. Having fixed K_T, the ratio $\dot{\theta}/E_c$ is given by

$$\frac{K_A g_A(D) \cdot K_F g_F(D) \cdot K_M g_M(D)}{1 + K_A g_A(D) \cdot K_F g_F(D) \cdot K_M g_M(D) \cdot K_T} \quad . \quad . \quad . \quad (13.29)$$

$$= \frac{K_R g_R(D)}{1 + K_R g_R(D)} \cdot \frac{1}{K_T} \quad . \quad . \quad . \quad (13.30)$$

where $K_R g_R(D) = K_A g_A(D) \cdot K_F g_F(D) \cdot K_M g_M(D) \cdot K_T$.

The total gain round the main loop is then equal to

$$\frac{K_R g_R(D)}{1 + K_R g_R(D)} \frac{nK_s}{DK_T} \quad . \quad . \quad . \quad (13.31)$$

and the gear ratio n is chosen to give a gain and phase margin consistent with a reasonable overall frequency response to a sinusoidal input θ_i.

The first step is therefore to determine the frequency response of the various elements.

(*b*) *Determination of* $K_A g_A(D)$. The steady-state gain K_A of the amplifier is equal to 180 mA. per volt. The amplifier also has a time-constant, however, which determines the nature of the function $g_A(D)$. This is defined by a transfer function of the form $\frac{1}{1 + \tau_A D}$ plus a phase shift α' which is proportional to the signal frequency, and is 90° when the signal frequency is one-half the supply frequency (section 8.6).

The values of gain and phase shift are plotted against the signal frequency (Table A). τ_A is assumed equal to 0·2 seconds* and it will be seen that the additional phase shift due to the half-cycle delay is negligible compared with that due to the normal time-constant.

TABLE A $g_A(D)$

Signal frequency $\times 2\pi = \Omega$	phase lag α	phase lag α'	total lag $(\alpha + \alpha')$	gain dB
0·5	− 5·3°	− 0·013°	− 5·3°	− 0
1·0	− 11·4°	− 0·026°	− 11·4°	− 0
2·0	− 22·5°	− 0·052°	− 22·6°	− 0·7
5·0	− 45·0°	− 0·13°	− 45·1°	− 3·0
10	− 64°	− 0·26°	− 64°	− 6·8
20	− 76°	− 0·52°	− 77°	− 12·5
50	− 84°	− 1·3°	− 85°	− 20·5
100	− 86°	− 2·6°	− 89°	− 26·5

(*c*) *Determination of* $K_F g_F(D)$. The constant K_F is the armature torque per mA. of field current. Since the maximum torque is 20 oz. in. for 80 mA. we have, assuming linearity,

$$K_F = \frac{20}{16} \times \frac{1}{80} = 0{\cdot}0156 \text{ lb. in. per mA.}$$

The transfer function $g_F(D)$ is given by the field time-constant τ_F and is equal to $\dfrac{1}{1 + \tau_F D}$ where $\tau_F = \dfrac{35}{1{,}650} = 0{\cdot}021$ sec. approx.

The gain and phase shift corresponding to $g_F(D)$ are plotted against signal frequency in Table B.

TABLE B $g_F(D)$

Ω	phase lag °	gain dB
0·5	0	0
1·0	1	0
2·0	2	0
5·0	6	0
10	12	0
20	23	− 1·5
50	46	− 3·1
100	65	− 7·3

* In the present design the m.m.f. ratio has worked out at about 90, which gives a time-constant of 0·17 seconds on the basis of equation 13.18 ; the value of 0·2 seconds should therefore make a reasonable allowance for the errors discussed in Chapter 8.

(*d*) *Determination of* $K_M g_M(D)$. The constant K_M is the steady-state angular velocity of the motor shaft per unit armature torque. Now the viscous friction coefficient is the opposing torque per unit angular velocity, and when the motor has reached a constant velocity the applied torque acting in the armature is equal to the opposing torque due to viscous damping. At a maximum speed of 7,000 r.p.m., the applied torque is equal to 20 oz. in., thus the viscous friction torque per unit angular velocity is equal to

$$k_v = \frac{20}{16} \times \frac{60}{7{,}000} \times \frac{1}{2\pi}$$

$$= 0{\cdot}0017 \text{ lb. in./radian per second}$$

and $$K_M = \frac{1}{k_v} = 590 \text{ radians per second/lb. in. torque.}$$

The transfer function $g_M(D)$ defines the transient occurring before the motor shaft reaches a steady velocity, and if it is assumed that the opposing torque is due entirely to viscous friction, the transfer function will be of the form $\dfrac{g_M}{1 + \tau_M D}$

where

$$\tau_M = \frac{\text{Armature moment of inertia (lb. in.}^2\text{)}}{\text{viscous damping per unit angular velocity (lb. in./rad./sec.)}g}.$$

The viscous damping is equal to k_v but must be expressed in absolute units, by multiplying by $g = 12 \times 32$ in./sec./sec., thus :

$$\tau_M = \frac{2/16}{0{\cdot}0017 \times 12 \times 32} = 0{\cdot}19 \text{ second.}$$

This enables the frequency response diagram for the motor to be plotted against Ω as in Table C.

TABLE C $g_M(D)$

Ω	phase lag °	gain dB
0·5	− 5°	0
1·0	− 10·5°	0
2·0	− 20·5°	− 0·7
5·0	− 43°	− 2·5
10·0	− 62°	− 6·0
20	− 75·5°	− 11·7
50	− 84°	− 19·7
100	− 86°	− 25·7

The overall transfer function corresponding to the three individual delays due to the amplifier, motor field and mechanical inertia are now combined by adding the gains and phase lags in Tables A, B and C. The results are plotted against Ω in *Figure* 13.17 to give the overall frequency response round the tacho loop. Since the frequency response diagram is referred to a datum corresponding to a steady-state gain of unity (0 dB) the actual steady-state gain which can be tolerated is found by raising the gain curve up to the point which gives a reasonable margin of stability. In this case it is seen that a vertical shift of 15 dB gives

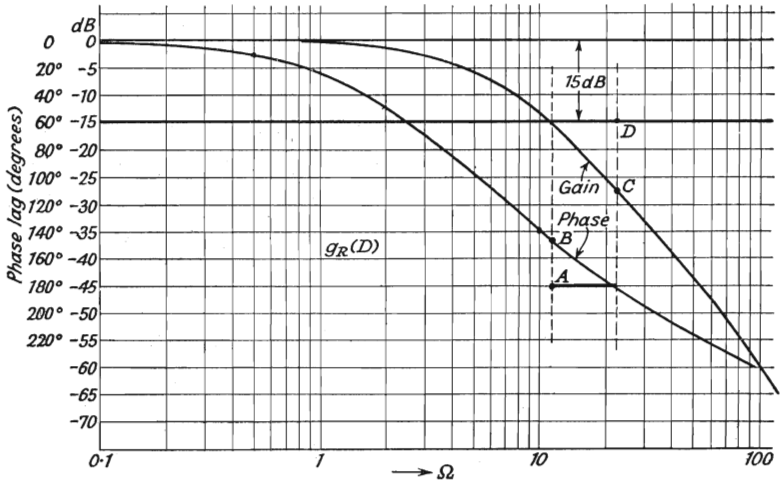


Figure 13.17. Frequency response round tacho loop $g_R(D)$.

a phase margin of 34° (AB) and a gain margin of 12·5 dB (CD). In this case the steady-state gain round the tacho loop is equal to 5·7 (15 dB), and since this is equal to $K_A K_F K_M K_T$

we have $$K_T = \frac{5{\cdot}7}{180 \times 590 \times 0{\cdot}0156}$$

$$= 3{\cdot}45 \times 10^{-3} \text{ volts/radian per second.}$$

This e.m.f. is referred directly to the main control circuit of the magnetic amplifier ; if a separate control circuit is used the resulting ampere-turns from the tacho must be the same as would be produced in the main control winding. Therefore if, for example, the actual e.m.f. available from the tacho were double that specified by K_T above, the control turns would be halved for the same control circuit resistance R_c. In the present instance

the actual tacho voltage available is 0·19 volts/radian per second and therefore for a tacho circuit resistance of 1,200 ohms (equal to R_c) the tacho-feedback control winding turns N_{ct} can be reduced by a factor $\dfrac{0{\cdot}191}{3{\cdot}45 \times 10^{-3}}$ on the control turns N_c.

So that $$N_{ct} = \frac{1{,}820 \times 3{\cdot}45 \times 10^{-3}}{0{\cdot}191} = 33.$$

In practice the value of N_{ct} would be made about 50 or 100 and subsequent presetting carried out during final tests by varying the tacho circuit resistance. The time-constant of these windings will be very small compared with that of the main control circuit due to the large decrease in the number of turns for the same resistance, and hence their slugging effect on the amplifier response will be negligible.

It only remains to determine the gear ratio n from the conditions for stability round the main position control loop. The closed tacho loop may now be regarded as a single element having a transfer function

$$\frac{K_R g_R(D)}{1 + K_R g_R(D)} \frac{1}{K_T} \text{ (see equation 13.30).}$$

The resulting frequency response for a sine wave input may be plotted from *Figure* 13.17 by the use of standard Nicholls charts ;

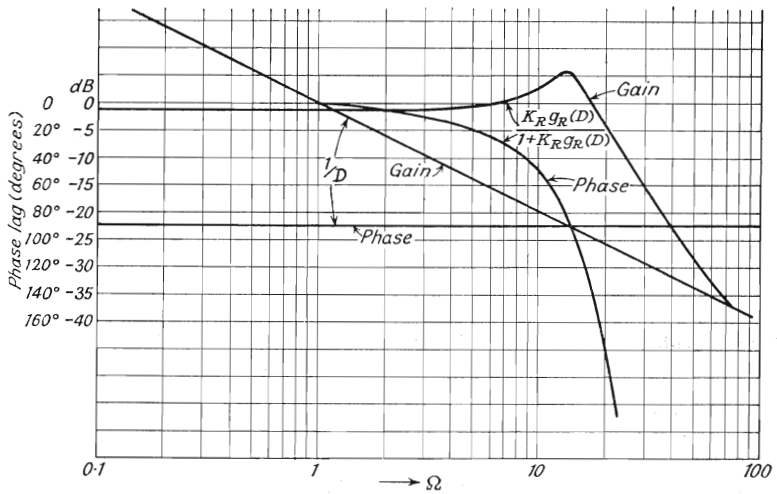


Figure 13.18. Frequency response for closed tacho loop and for pure integrating element.

the result is shown in *Figure* 13.18, where it is seen that the constant $\frac{1}{K_T}$ has not been included since this will merely influence the datum level of the gain characteristic. It is seen from equation (13.30) that if the gain K_R round the tacho loop can be made very large, the output velocity $\dot{\theta}$ is given by E_c/K_T and the motor behaves as a perfect integrator. The frequency response round the main control loop is given by equation 13.31. The dynamic component of this may be plotted by adding the two sets of curves in *Figure* 13.18, namely those for the closed rate loop and those for the simple integration $\frac{1}{D}$. The result is shown in the full-line curves of *Figure* 13.19, and it is seen that a steady-state gain of 12 dB gives a gain margin on closed loop of 5 dB (CD) and a phase margin of 76° (AB). Under these conditions the closed loop frequency response of the overall position servo may be calculated or obtained from the standard charts and is shown by the dotted curves of *Figure* 13.19.

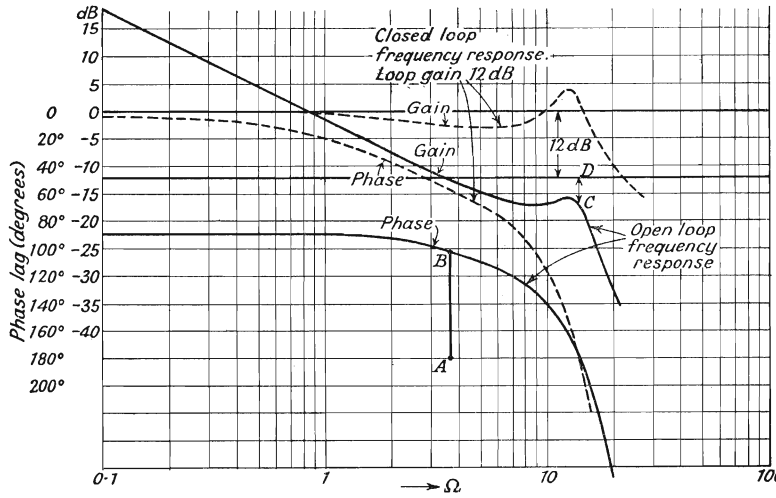


Figure 13.19. Open loop (full line) and closed loop (dotted line) response for overall position servo.

From equation 13.31 we have the steady-state component nK_s/K_T is equal to 4 (12 dB).

Thus $$n = \frac{4K_T}{K_s} = \frac{4 \times 3{\cdot}45 \times 10^{-3}}{0{\cdot}2} = 0{\cdot}069.$$

Therefore the gear box reduction ratio is $\frac{1}{0{\cdot}069}$

$= 15 : 1$ reduction approximately.

It should be noted that the 15 : 1 ratio used purely for stability need not apply to the gearing from the armature to the ultimate load for which a separate requirement may well apply. This ratio applies specifically to the loop gain and as such defines the gear ratio between the armature and the position transmitter, which need not be coupled directly to the main output shaft.

13.24 Design Tables

TABLE 1

Sizes of Standard Laminations

Inter-Service No.	Dimension " y " (*Figure* 13.5).
521 " E " type	1 in.
522 ,,	1½ ,,
523 ,,	2 ,,
524 ,,	2½ ,,

TABLE 2

*Sensitivity Permeability * of Materials at Various Frequencies*

Material	50 c/s	400 c/s	1,000 c/s	2,400 c/s
Mumetal . .	80–120	70–100	30	25
H.C.R. . .	80–100	60–80	25	20
Crystalloy .	25–35	15–25	—	—
Stalloy . .	10–15	8–12	—	—

* These figures multiplied by $4\pi/10^4$ give the value in M.K.S. units, or multiplied by 1,000 give the value in C.G.S. units.

TABLE 3

Design Flux-Density, $\mathbf{B}_s$ *(M.K.S. units)* *

Material	50 c/s	400 c/s	1,000 c/s	2,400 c/s
Mumetal . .	0·55	0·55	0·5	0·5
H.C.R. . .	1·2	1·1	0·9	0·8
Crystalloy .	1·1	1·0	—	—
Stalloy . .	1·0	0·8	—	—

* Values are obtained in C.G.S. units when multiplied by 10^4.

TABLE 4

Linear Operating Range for Auto-Excited Mumetal Transductors*

50 c/s	400 c/s	1,000 c/s	2,400 c/s
0·5–0·65	0·4–0·6	0·35–0·55	0·3–0·5

* Expressed as a fraction of the saturation current $\bar{\imath}_{as}$.

CHAPTER 14

APPLICATIONS OF MAGNETIC AMPLIFIERS

14.1 Review of Applications

MAGNETIC amplifiers have been applied to various problems both in Industry and the Services. The following activities are typical of those in which magnetic amplifier techniques are often applicable.

In process control and the control of industrial machinery such as crane hoists, rolling mills, machine tools, etc., which may not require a response faster than about 0·1 second, the magnetic amplifier is attractive chiefly on account of its reliability and ruggedness. Its additional properties as a low-level d.c. amplifier open up the fields of industrial instrumentation ; for example, the measurement and control of temperature, pressure and light intensity, or as amplifiers for strain gauge measurements in electrical weighing using load cells.

In electrical installations such as power supplies, large computers, etc., magnetic amplifiers have been employed ; sometimes being incorporated with relays as protective gear, in which case a slow response may be required, e.g. for sequence switching. They have also been used to provide stabilized heater supplies for electronic equipment, in which case the slow response is usually immaterial in view of the thermal lags already present.

The voltage and frequency control of generators and the speed control of motors are other important instances where it is often worth while to consider the use of magnetic amplifiers, either as output stages preceded by an electronic amplifier or more usually as completely magnetic systems to give increased reliability and less stringent servicing requirements. The following alternative methods of controlling electrical machines are also available :

(*a*) *Thyratron control.* This generally involves electronic pre-amplification, and although it provides fast control, it is usually considered less reliable than magnetic amplifier equipment.

(*b*) *Amplidyne control.* This system suffers from similar disadvantages to the magnetic amplifier in respect of its time-constant and has the additional weakness of requiring brushes ; it is also probably more costly, but has the advantage of generating its own output.

(*c*) *Carbon pile regulators.* Carbon piles have been used in many cases, particularly in Service equipment, but they have a long time-constant. They also contain moving parts as part of the closed loop system and require careful setting up in addition to frequent servicing. When used for regulating d.c. machines they have the advantage of not requiring an a.c. supply, but in a.c. systems they could probably, with advantage, be replaced by magnetic amplifiers in all cases.

Considering these various systems, it will be seen that the magnetic amplifier may present a favourable alternative to the others, chiefly on account of its reliability. It should also be remembered that, being a reactive controlling element, large powers can be handled for a comparatively low heat dissipation compared with a valve in which the control is resistive.

In Service equipment too the magnetic amplifier is widely applicable. Aircraft instrumentation, position servos for repeating equipment, amplifiers for hydraulic motors actuating control surfaces, gun turrets, etc., represent some typical applications. Similarly, in guided missiles or auto-pilots automatic control is possible with increased reliability by using transductors or magnetic amplifiers.

The rapidly expanding field of nuclear instrumentation finds many uses for magnetic amplifiers, particularly where stable d.c. amplifiers are required in pile instrumentation, or position servos for remote handling or automatic positioning of control rods.

Many of these applications utilize the valuable property of a magnetic amplifier whereby a number of variables may be added magnetically while remaining electrically isolated.

Finally, a discussion of magnetic amplifier applications would not be complete without some mention of transistors. The view has occasionally been expressed that the transistor will replace

magnetic amplifiers in many fields of instrumentation. It is becoming evident, however, that this is true only to a limited extent, particularly in industry, where large power outputs are generally required with stable d.c. pre-amplification. In the main, however, the transistor appears to be opening up new fields for the magnetic amplifier, and by regarding the two devices as complementary, it has been possible to develop combined circuits ideally suited to instrument servos such as auto-pilots, where a junction transistor amplifier provides an initial power amplification with a high speed of response followed by an output stage magnetic amplifier.

This chapter gives an account of some typical applications for magnetic amplifiers in the above-mentioned fields.

14.2 Instrument Amplifiers

D.C. amplifiers with stable and linear amplifying characteristics are widely used in the field of industrial measurement and control.

It is often required to measure non-electrical quantities such as temperature, speed, light intensity, etc. This is generally achieved by converting the quantity to be measured into an electrical signal by means of a transducer,* such as a thermocouple or a photocell, and amplifying the output from the transducer.

The output from the amplifier may then be used either to operate a robust measuring instrument or recorder, or possibly to control the original quantity automatically. The output from the transducer is usually only sufficient to operate a very sensitive instrument, and hence the need for a linear amplifying device free from drift.

Electronic amplifiers are sometimes unsuitable for these purposes for the following reasons :

Firstly, because they are subject to a voltage zero drift which is generally comparable with the output from the transducer.

Secondly, valves and their associated circuits are seldom suitable for prolonged and continuous use under industrial conditions where reliability and minimum servicing requirements are essential.

Many applications demand an accuracy of better than 1% of

* The term transducer should not be confused with transductor ; the former is a general term for a device which converts one form of energy into another as, for example, a microphone in which acoustic energy is converted to electrical energy.

full-scale indication, and even if this corresponds to a signal power level no greater than 10^{-12} watt, then the magnetic amplifier is likely to be suitable. Furthermore, response time is not usually a limiting factor except in closed-loop systems, and therefore magnetic amplifiers are seldom precluded solely on account of their time-constants.

In some cases a transductor amplifier may be used without self-excitation in order to take full advantage of the inherent linearity in its control current characteristics. Better characteristics are always achieved, however, by using self- or auto-excitation and then applying overall negative feed-back. This has the advantage of stabilizing the ultimate input-output relationship and, since the output usually feeds a current-operated device whereas the input is taken from a low-impedance voltage device, stabilizing the output current/input voltage ratio allows for variations in the control circuit resistance. Overall feedback also tends to stabilize the circuit against variation in the rectifier characteristics with age and temperature. By whichever method

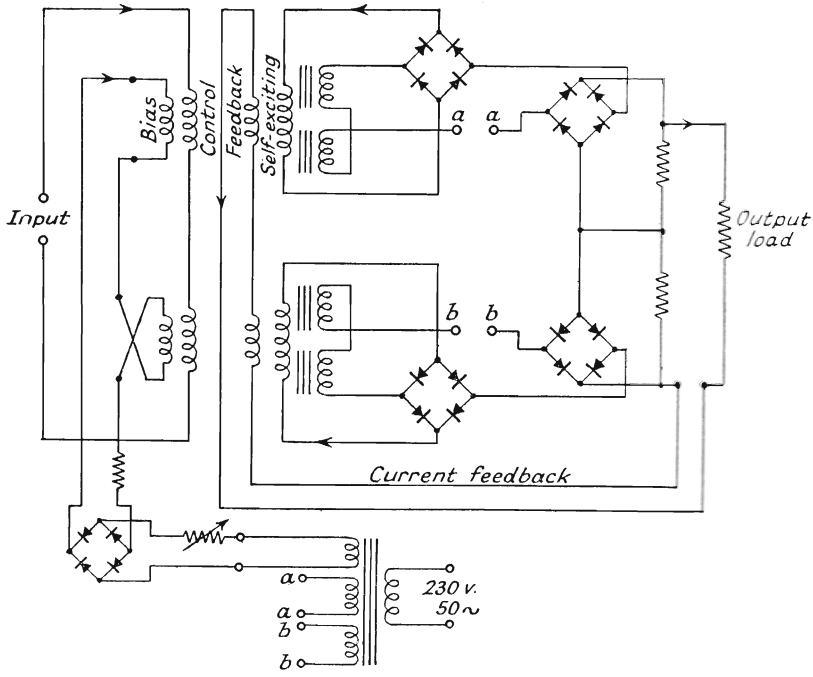


Figure 14.1. Single-stage instrument amplifier.

linearity is obtained, however, it is necessary to sacrifice power gain, as shown in the previous chapters.

Three typical applications of magnetic amplifiers to instrumentation will now be described :

(*a*) *Magnetic amplifier for current telemetering installation. Figure* 14.1 shows the circuit of a single-stage magnetic amplifier which is used to amplify, before transmission over eight miles of telephone cable, the current obtained from a telemetering unit. In this case the unit is required as a current amplifier ; it consists of a pair of push-pull self-excited transductors in which current feedback is applied to extra windings wound over the control windings. The overall power gain is about 150 and the accuracy is within $\pm$ 1% of full-scale output for a supply voltage variation of $\pm$ 5% and a frequency range of 45/52 c/s. *Figure* 14.2 shows a number of these amplifiers in a British Electricity Authority telemetering installation.

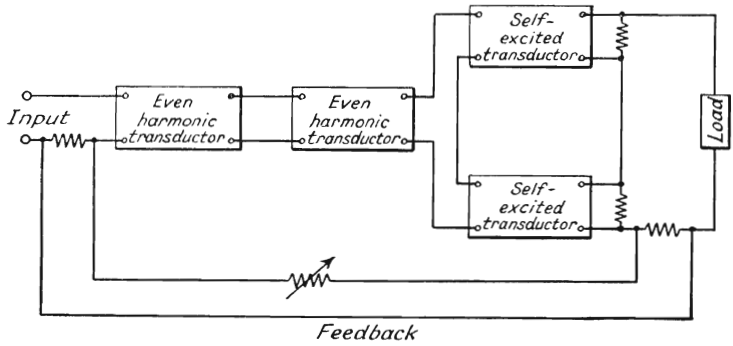


Figure 14.3. Block diagram of low-level three-stage instrument amplifier.

(*b*) *General-purpose low-level amplifier. Figure* 14.3 shows a block diagram of a low-level magnetic amplifier. This consists of two stages of even-harmonic amplification followed by a pair of push-pull transductors. The amplifier is stabilized with overall negative feedback, but in this case the load current is fed back as a voltage into the control circuit. Thus the ratio of load current to signal voltage is independent of variations in the amplifier gain or the control circuit resistance and depends only upon the characteristics of the feedback network.

Due to the large internal gain of the amplifier, it is necessary to ensure that the system is stable when the loop is closed. In

this particular case stability is achieved by making the time-constant of the output stage very large compared with that of the other two stages.

The sensitivity of this amplifier is such that for an input signal of 4 millivolts from a source impedance of about 20 ohms, the load current is $\pm$ 5 milliamps in a load of 500 ohms. The sensitivity can be varied by adjustment of the variable resistance in the feedback network, which should consist of manganin resistors. The gain stability is about $\pm$ 1% for supply variations of $\pm$ 10% in voltage and $\pm$ 5% in frequency. The zero drift is about 10^{-12} watt and the response time approximately 1 second. This amplifier operates satisfactorily from the output of strain gauges, thermo-couples and photo-cells, and gives sufficient output to operate recording equipment. A photograph of the unit is shown in *Figure* 14.4.

(*c*) *Magnetic amplifier for jet pipe temperature measurement.* *Figure* 14.5 shows an amplifier designed to operate from the output of a thermocouple placed in the jet pipe of aircraft engines. The output from the amplifier is recorded on a robust meter, the full-scale deflection of which occurs for 1 mA. The amplifier consists of two stages of even-harmonic amplification with overall negative feedback, and the whole unit will fit into the rear of the instrument case behind the control panel. The maximum input voltage is 35 millivolts from an impedance which may vary over a considerable range depending upon the length of the leads from the thermocouple. The impedance of the instrument is 500 ohms. Such a system permits the use of a robust milli-ammeter, and the voltage feedback into the control circuit tends to make the amplifier independent of the resistance of the signal leads.

14.3 Use of Transductors for Metering Large Direct Currents

This is an application of the transductor without self-excitation, in which it is used essentially as a direct current transformer.

The a.c. windings are placed on two closed cores as described previously, but the space normally occupied by the control windings is left vacant and provides room for a bus-bar to pass through. The bus-bar thus corresponds to a single-turn control winding. The a.c. windings are supplied from the a.c. mains supply and the d.c. measuring instrument is placed in series with the a.c. windings via a bridge rectifier as shown in *Figure* 14.6.

Figure 14.2. Instrument amplifiers in telemetering installation.

By courtesy Elliott Brothers (London) Ltd.

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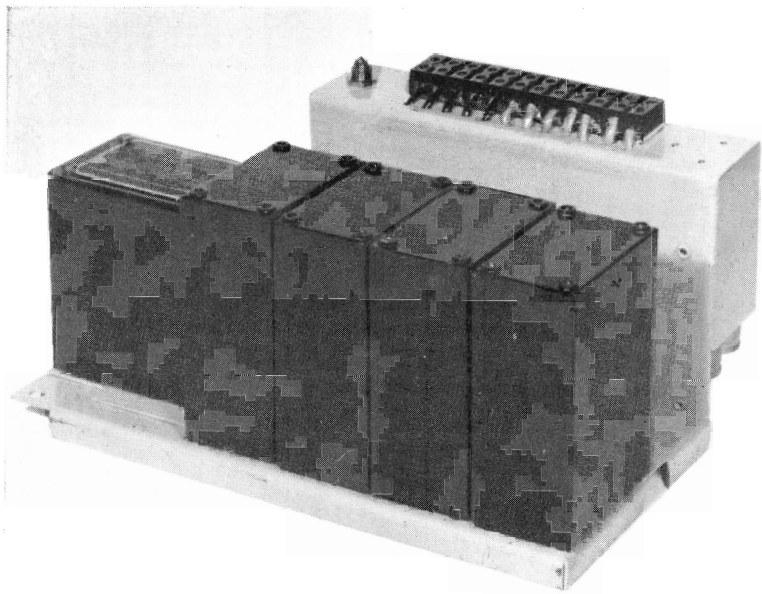

Figure 14.4. Low-level d.c. amplifier.
By courtesy Elliott Brothers (London) Ltd.

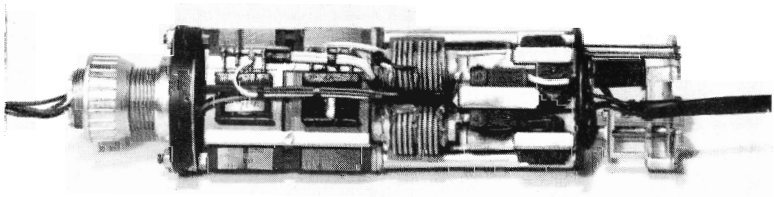

Figure 14.5. Amplifier for jet-pipe temperature measurement.
By courtesy Elliott Brothers (London) Ltd.

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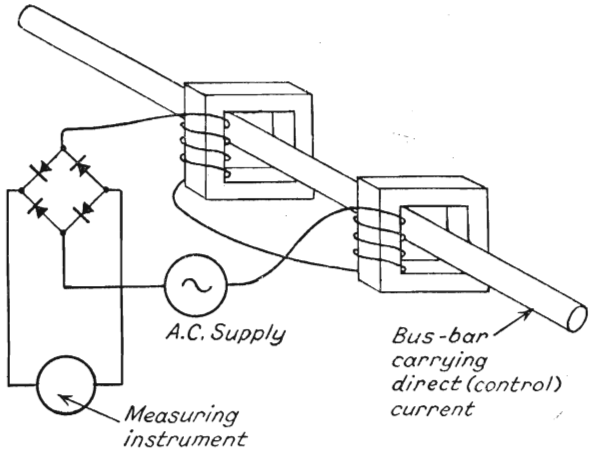

Figure 14.6. Arrangement employed for measuring large direct currents.

A linear relationship exists between the control current and the meter current which ideally depends upon the turns ratio. In this particular case, the control ampere-turns are provided by a very large current flowing in a single turn.

This method of d.c. metering has the following advantages :

(i) It dispenses with the need for heavy shunts having a very low resistance.

(ii) Due to the constant current characteristics of the transductor, the meter current tends to be independent of the a.c. circuit resistance ; therefore a number of meters may be connected in series with one transductor. Furthermore, the resistance of the meter leads from the transductor is much less critical than when a shunt is employed and therefore the meter may, if necessary, be installed at an appreciable distance from the bus-bar.

(iii) Heavy overloads of current need not damage the instrument, since the transductor can be designed so that the cores saturate at any required level of bus-bar current.

(iv) The bus-bar is electrically isolated from the meter, which may therefore be earthed independently.

(v) The transductor used in this way is very economical to produce compared with the cost of a large shunt to do the same job.

An accuracy to within $\frac{1}{2}\%$ or 1% may be obtained for the transductor when the a.c. supply voltage varies within 2% to 3%,

and it is easy to obtain an overall accuracy of measurement within $1\frac{1}{2}\%$.

The linearity of a transductor operating under these conditions is good, and the control ratio is dependent primarily upon the maximum alternating current permissible and also upon the permeability of the core material employed.

If a high degree of linearity and accuracy are required, the cores should be made of mumetal or supermalloy. Since the current gain is proportional to the ratio of control turns to a.c. turns, the instrument may be calibrated by means of a smaller direct current flowing in a calibration control coil consisting of a proportionately increased number of turns.

Such a metering system [14(a)] has been applied to the measurement of large direct currents up to 50,000 amps. for which a corresponding d.c. shunt might weigh 3 cwt. and would have such a low resistance that calibration would be difficult.

14.4 Use of Transductors as Relay Amplifiers

In the applications discussed so far, one of the important features required of magnetic amplifiers has been the need for linearity and overall stability of the amplifying characteristic. When the transductor is used as a relay pre-amplifier, however, these requirements are not generally so stringent and therefore full use can be made of the high gain obtainable with self- or auto-excited transductors. Applications of transductor relays to protective gear have been described by Edgeley and Hamilton [14(b)] in which transductors used as pre-amplifiers, time-delay units, etc., have enabled robust relays to carry out duties which would otherwise require sensitive and highly complex relay units.

The applications described here, however, are of a comparatively simple nature, and if the reader is interested in some of the more advanced techniques, reference should be made to the original paper.

(*a*) *Sensitive transductor relay.* A standard sensitive transductor relay is shown in *Figure* 14.7, in which a self-excited transductor supplied from 12 volts at 50 c/s. amplifies low-power signals to operate the relay. The input power level required for relay operation is of the order of 4μW. If transductors in cascade are employed, then greater sensitivity can be obtained. The limit is determined by the drift level of the amplifier.

(*b*) *Photo-electric lighting controller.* *Figure* 14.8 shows an

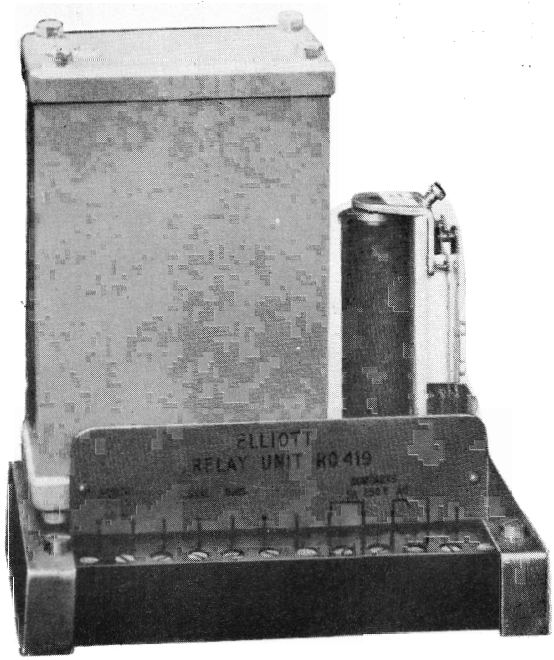

Figure 14.7. Sensitive relay using transductor control.

By courtesy Elliott Brothers (London) Ltd.

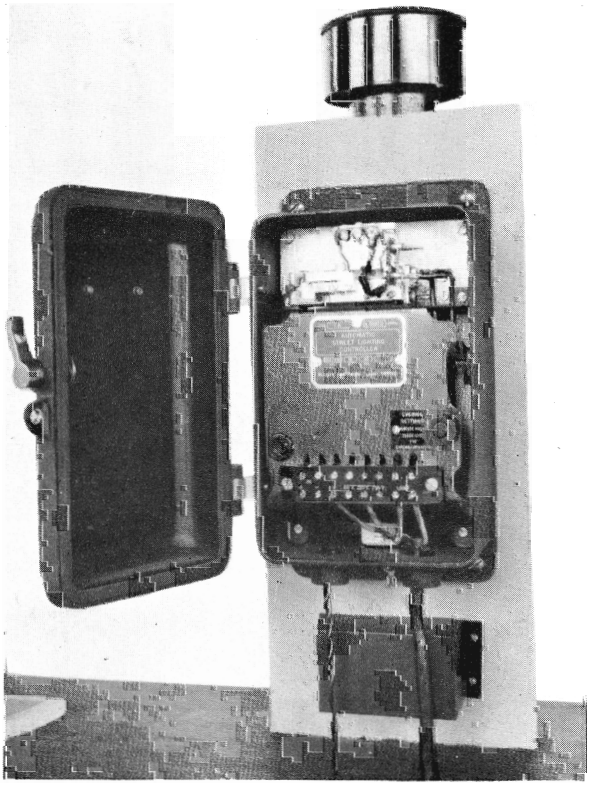

Figure 14.8. Magnetic amplifier lighting controller.

By courtesy Elliott Brothers (London) Ltd.

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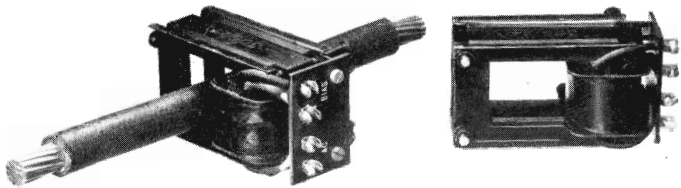
(*a*)



(*b*)

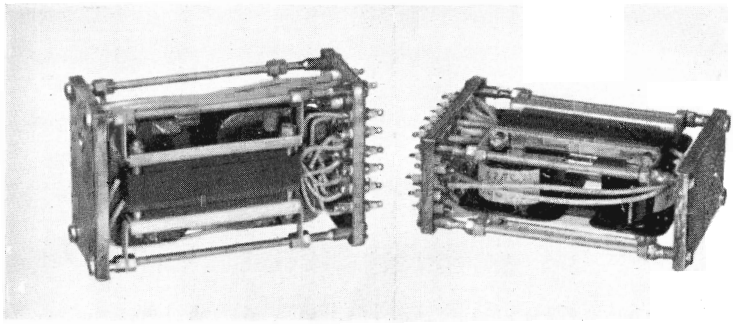


Figure 14.9(*a*) and (*b*). Relay transductors for safeguarding power supplies.

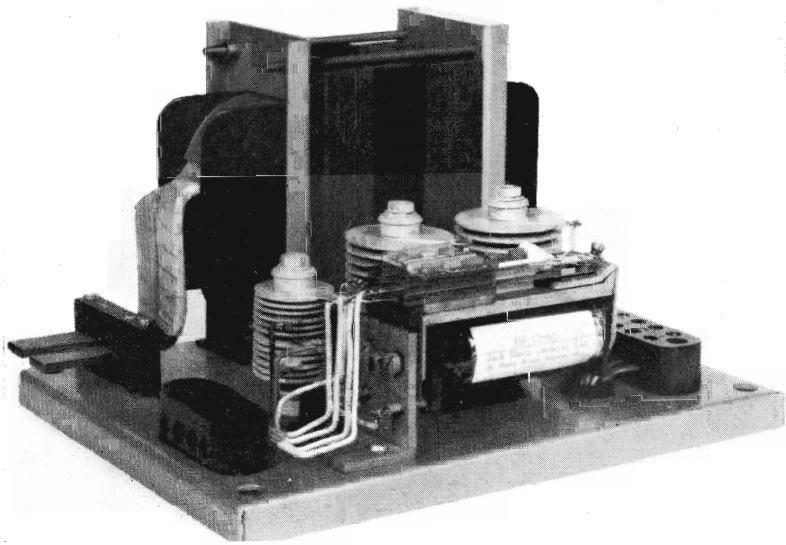


Figure 14.10. Relay amplifier for control of roll grinding.

Figures 14.9 *and* 14.10 *by courtesy Elliott Brothers (London) Ltd.*

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automatic lighting controller. Here the input is obtained from a photocell and amplified to operate a mercury relay. The magnetic amplifier consists of three stages in cascade of which the first two are even-harmonic push-pull amplifiers. The input power level to this amplifier is 10^{-10} watt, while the output power is governed by the capacity of the mercury relay which, in this case, is 2 kW.

(*c*) *Transductor overload protection relay. Figure* 14.9*a* shows a transductor designed to operate a relay for overload protection. The construction is similar to that of the metering transductor in which a bus-bar forms a single turn control winding. If the bus-bar current exceeds 70 amps. the output from the transductor operates a protective relay.

A similar type of unit is shown in *Figure* 14.9*b*, in which the effective control m.m.f. is due to the sum of twelve equal currents flowing in twelve isolated control windings. Each winding carries a current of 1 amp. and the transductor is arranged to close the relay if any one of the currents exceeds 1·2 amp. This multiple protection reduces cost but no indication is given as to which circuit causes the relay to trip.

(*d*) *Magnetic amplifier for wheel-wear compensation in roll-grinding.* In this application a magnetic amplifier is employed as a pre-amplifier for a relay which controls a grinding wheel.

Current fed into the amplifier is directly proportional to the load current of the grinding wheel motor. As this load current falls, due to wear of the grinding wheel, the relay operates and the grinding wheel is fed in to correct for this change. Rolls may be ground, using this method, to accuracies of 0·001 in. The normal supply is 70 volts at 50 c/s. (see *Figure* 14.10).

14.5 Magnetic Amplifier for Position Servo Mechanisms

Magnetic amplifiers may be used as servo amplifiers in position controllers provided the available supply frequency is high enough to ensure a sufficiently fast response from the amplifiers. A typical schematic for such a system is shown in *Figure* 14.11, where it is seen that the general arrangement is similar to that set out in the last numerical example of Chapter 13. Synchros, magslips or potentiometer systems are used to detect the error between the positions of the input and output shafts. This error is then amplified and used to drive the motor, the motor torque being in such a direction as to reduce the error. The amplifier may be entirely magnetic or, in some cases, a magnetic amplifier

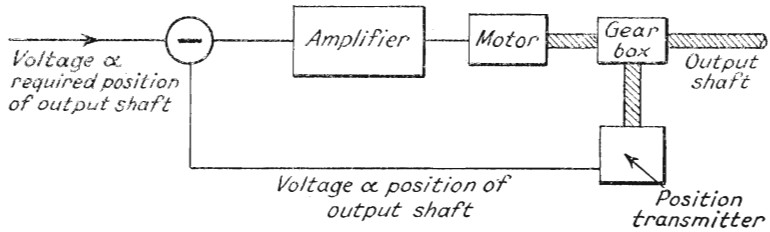

Figure 14.11. Block diagram of position controller.

may be used with an electronic pre-amplifier. The following are typical applications of such servo mechanisms :

(*a*) Remote control or remote indicating systems, especially in service equipment.
(*b*) As torque amplifiers in analogue computers for the control of mechanical movements (e.g. potentiometer shafts and plotting tables) from electrical signals.
(*c*) In automatic pilots for aircraft.
(*d*) In self-balancing bridges and potentiometer recorders.

In nearly all these applications, however, the amplifier time-constant usually decides whether a magnetic amplifier will be suitable. Sometimes, where a high speed of response is essential, the transductors are operated from a high supply frequency and

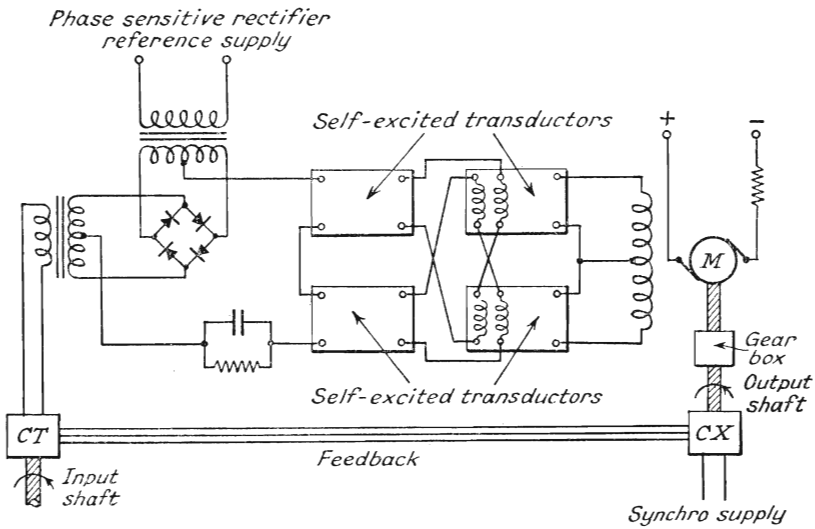

Figure 14.12. Position servo-mechanism using two-stage magnetic amplifier.

stabilizing networks are used to give suitable phase advance. A typical example of such an amplifier is shown in *Figure* 14.12. This is a two-stage push-pull magnetic amplifier operating at a supply frequency of 1,100 c/s. The input stage receives its signals from a phase-sensitive rectifier of the type shown. A shaping network consisting of a parallel R.C. combination is included in series with the control windings to help stabilize the system. The overall power gain is 2×10^5 and an angular error of $\frac{1}{2}°$ is sufficient to give full motor torque, while an error of about $\frac{1}{10}°$ will start the motor against static friction.

The transient response of the amplifier approximates to a simple exponential lag, with a time-constant equal to 0·025 sec. Within the linear range the damping is adequate and the settling time about 0·35 sec. For large errors a small overshoot occurs and the settling time depends upon the maximum speed of the output shaft. A photograph of the unit is shown in *Figure* 14.13.

An alternative scheme is shown in *Figure* 14.14. In this case the magnetic amplifier output stage is preceded by an electronic pre-amplifier. High-power gain in the magnetic amplifier is not necessary and, because the control windings are connected in the anode circuits of a pair of valves, the time response can be reduced to three or four periods of the supply frequency. In the particular example shown, the amplifier controls a d.c. split field servo motor requiring 10 watts maximum power in each half of the field. The transductors, operating at 1,100 c/s, replace two output valves of about the same size. The total saving of power dissipation is approximately 25 watts and the current drawn from the high-tension power pack is reduced from 120 mA. to 25 mA.

Another example of a servo amplifier appears in *Figures* 14.15*a* and *b*. These show two views of a production model of a 10-watt magnetic servo amplifier employed in a shipboard fire control computer.

Magnetic amplifiers are sometimes arranged to give an a.c. output and feed directly into a two-phase servo motor (Chapter 10). A typical example of this occurs in the Bendix Polar Path Compass. *Figure* 14.16 shows a gyro and flux-gate amplifier assembly. Two output stages each consisting of a pair of three-limbed transductors are shown on the extreme right and extreme left of the photograph. These output stages are driven from a demodulator system which follows an a.c. amplifier, and each magnetic amplifier drives a small two-phase servo motor. The four a.c. windings

for each stage are arranged in the form of a bridge as shown in *Figure* 14.17. The a.c. supply is connected to one pair of terminals (*ab*) and the control phase of the motor to the opposite pair (*cd*). In this case it is seen that the transductors are used without auto-excitation.

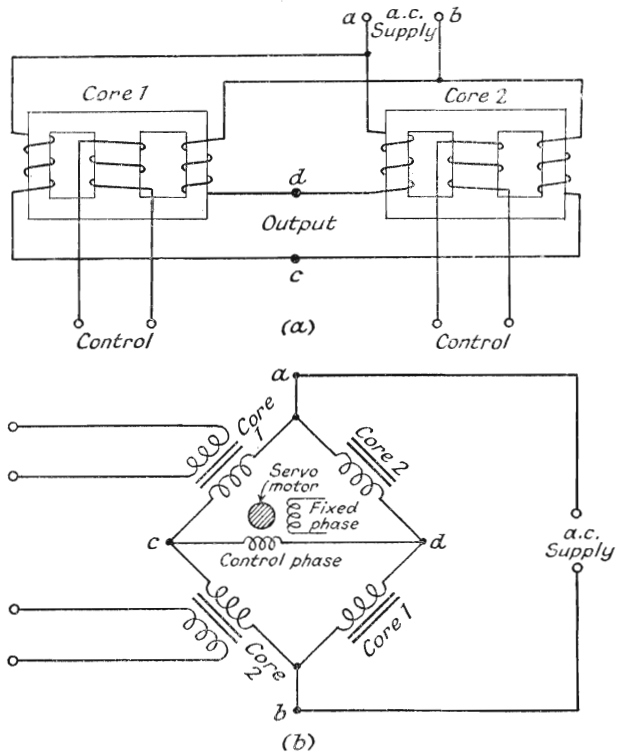

Figure 14.17. Schematic of a.c. output amplifier for polar path compass (flux-gate amplifier).

(*a*) Relative connection of windings on transductor cores.

(*b*) Circuit of transductor bridge (*a*).

By courtesy Bendix Aviation Corporation.

14.6 Magnetic Amplifiers with Transistor Control

It was shown in Chapter 9 that in the transductors having half-cycle response, the output in any half-cycle depends upon the total flux set in the previous half-cycle. In both the half-wave and full-wave circuits control was effected by means of the signal voltage E_c. It has also been shown, however, that the output can be controlled equally well if the voltage source E_c is

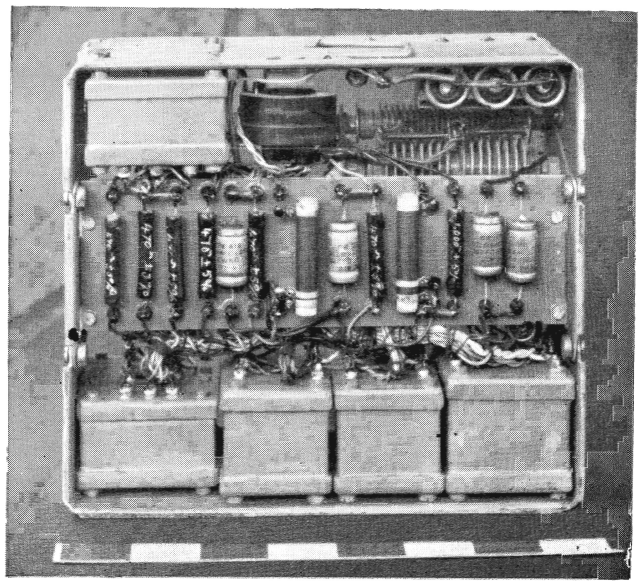

Figure 14.13. 1,100 c/s magnetic amplifier for position servos.

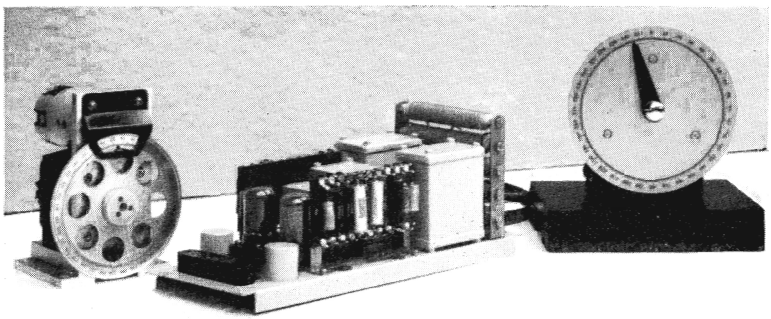

Figure 14.14. Combined magnetic/electronic amplifier for position servo.

Figures 14.13 *and* 14.14 *by courtesy Elliott Brothers (London) Ltd.*

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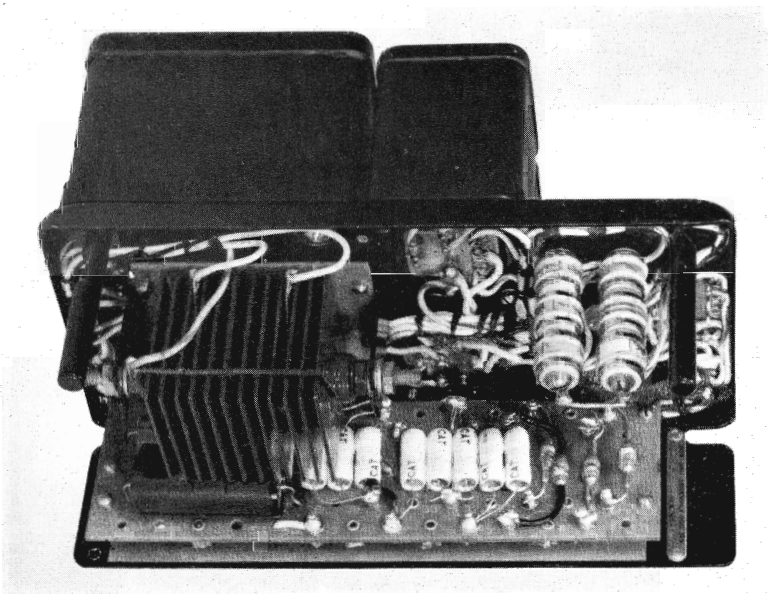
(*a*)



(*b*)

Figure 14.15(*a*) and (*b*). 10-watt servo amplifier.

By courtesy Ford Instrument Co., New York.

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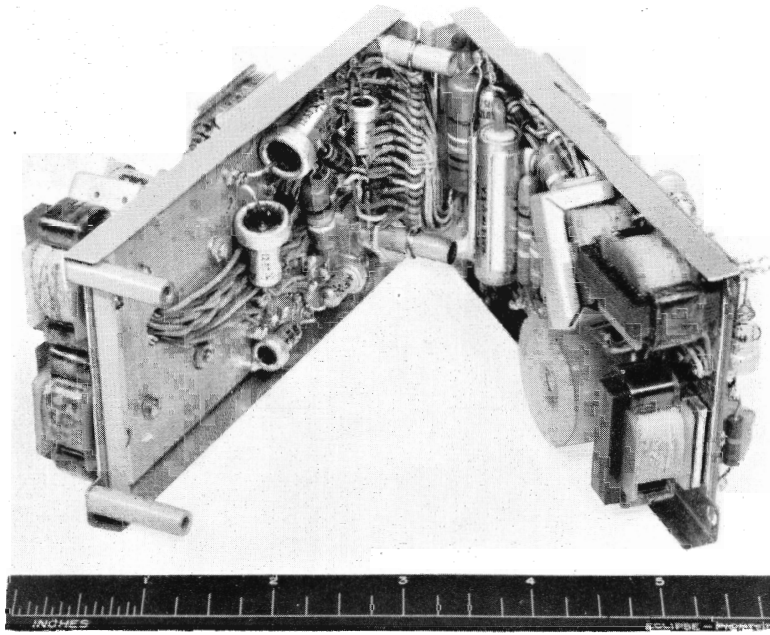

Figure 14.16. Gyro and flux-gate amplifier for polar-path compass.
By courtesy Bendix Aviation Corporation.

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replaced by a variable resistance (*Figure* 9.3*c*). This controls the amount of flux set by varying the proportion of the voltage E'_{ac} which is applied to the control winding during the setting periods.

In *Figure* 14.18*a* a p.n.p. junction transistor is shown connected in the control circuit of a half-wave transductor in place of the resistor. Thus, by varying the emitter current the output is controlled. If the emitter current is zero the collector current

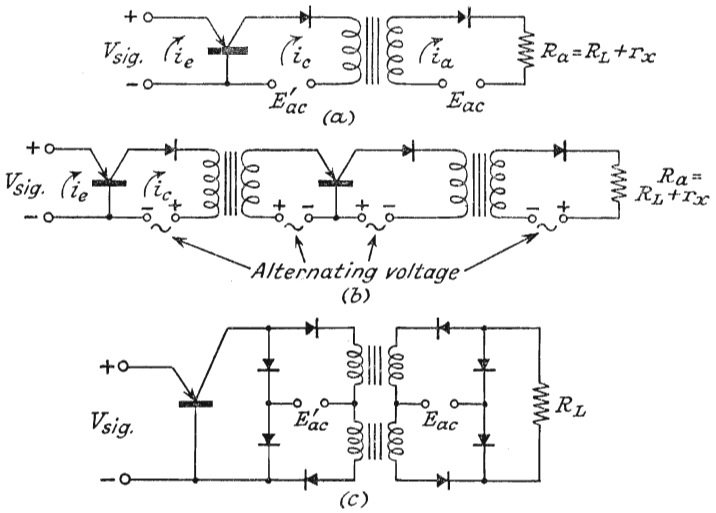

Figure 14.18. Transistor control of transductors.
(*a*) Half-wave circuit.
(*b*) Cascaded half-wave circuits.
(*c*) Full-wave Ramey circuit.
The signs ± indicate the relative phasing of the alternating voltages.

is also (very nearly) zero and hence no setting action takes place and the full output is obtained. In these circumstances the transistor may be regarded as having maximum impedance. If an emitter current is applied, some collector current will flow. The voltage appearing across the control windings will increase as this current increases, thereby causing a greater flux to be set and hence a smaller output will be obtained. Coupling of two such amplifiers in cascade is shown in *Figure* 14.18*b*,[14(c)] and transistor control of a full-wave amplifier is shown in *Figure* 14.18*c*.

In the foregoing circuits the output is a maximum when the input is zero. If an increased output is required with an increasing emitter current the biased circuit shown in *Figure* 14.19 may be used.[14(d)] In this case the control voltage may be an alternating

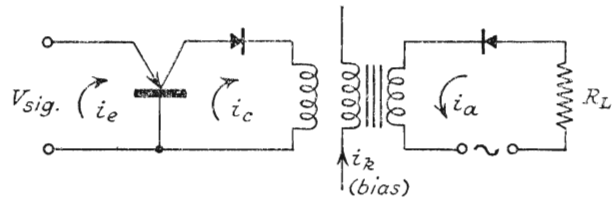

Figure 14.19. Transistor control of half-wave transductor with bias.

carrier having a frequency equal to that of the supply; the signal is then a modulation on this carrier.

14.7 Voltage and Frequency Control

Magnetic amplifiers can be used in many cases where it is required to maintain constant the voltage and/or the frequency of a generator.

In such applications the controlled quantity is compared with some reference and a control signal proportional to the difference between the actual output and the reference is fed to the amplifier and amplified to control either the generator field (for voltage control) or the speed of the prime mover (for frequency control).

The amplification must be sufficiently high that the full output of the regulator is obtained for the maximum specified error between the controlled quantity and the reference.

Motor-generator sets giving outputs up to several kW. are used as power supplies on ships or in aircraft or for specialized installations such as large electronic computors, battery charging equipment, etc.

Voltage control is normally effected by varying the field excitation of the generator ; this can be done either by providing two field windings on the machine—one supplying a constant excitation and the other controlling the machine—or by using a rectifier mixing circuit such as that shown in *Figure* 14.20. In a particular application of the latter alternative the voltage of a 1-kW. generator is maintained at 250 volts $\pm$ 1% for all normal variations of load, mains supply, etc. The full range of excitation corresponds to a field current variation from 0·33 amps. to 0·5 amps. Of this, 0·3 amps. is supplied from the generator and the remaining variable quantity supplied from a magnetic amplifier.

Generator voltage control is usually effected by means of a carbon pile regulator the output of which supplies the field

current. Very often, however, the initial cost of a transductor regulator is acceptable when more accurate regulation and greater reliability are the chief requirements.

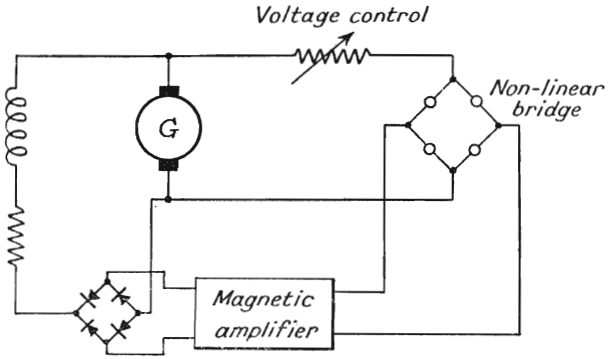

Figure 14.20. Generator voltage regulator.

(*a*) *Frequency regulator for small alternator fitted in German missiles.* One of the most notable applications of magnetic amplifiers to the control of generators was the frequency control of a small three-phase motor-driven alternator which was fitted in the German pilotless missiles.[14(e)] A circuit diagram is shown in *Figure* 14.21.

The frequency reference was provided by the two series-tuned circuits supplied from the output of one of the alternator phases. One of these circuits was tuned slightly above and the other slightly below the required frequency. When the machine ran at the correct speed the rectified currents i_1 and i_2 were equal so that the resulting m.m.f. controlling the transductor via the windings c_1 and c_2 was zero. If the frequency increased, the current in one circuit increased while that in the other circuit decreased so that a resultant m.m.f. was obtained, the magnitude of which was approximately proportional to the difference between the actual frequency and the required frequency. If the frequency fell, the resultant m.m.f. reversed its direction. The a.c. windings (*a*) were placed across one of the alternator phases and in series with a bridge rectifier supplying the field current to the driving motor. The latter current also flowed in the self-exciting windings *f*.

If, for example, the frequency decreased slightly the tuned circuits provided a control m.m.f. which was amplified in the transductor and the output controlled the field of the motor in

such a direction as to cause an increase of speed. The windings (*b*) were feedback windings which stabilized the system by applying an m.m.f. proportional to the rate of change of field current through the capacitor C_b.

The output from the alternator was approximately 150 VA. at 42 volts r.m.s. and 500 c/s. During the first few minutes of the flight the d.c. voltage supply to the motor fell from 31·5 to 29·5 volts, during which time the alternator frequency was maintained to within 0·1% of the demanded value.

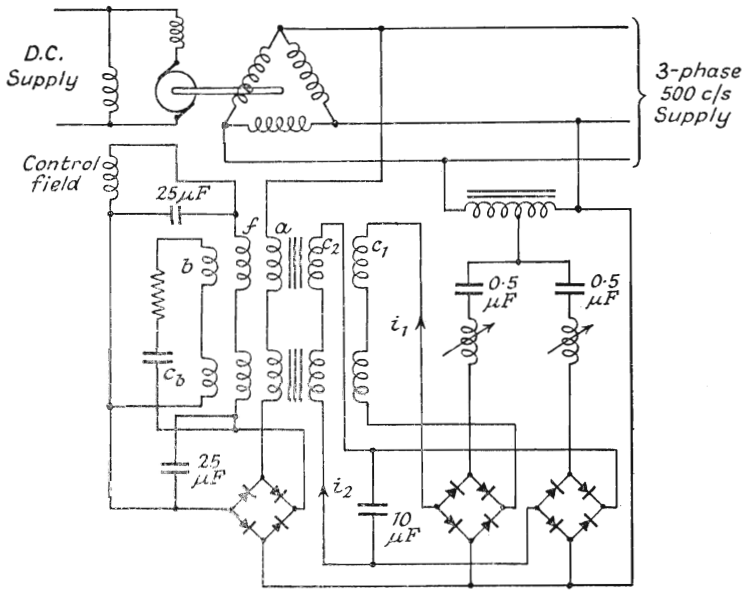

Figure 14.21. Frequency control for German rocket.

(*b*) *Alternator voltage regulator.* The circuit in *Figure* 14.22*a* is that of a magnetic amplifier voltage regulator for a 3-kW. 3-phase alternator supplying 210 volts per phase at 400 c/s for general-purpose laboratory supplies. The voltage reference in this case is obtained from the three saturable chokes, one in each phase. Each choke is arranged to saturate when the voltage is just below the required value. Small changes of voltage above the saturation value cause a large change of magnetizing current. All three currents i_1, i_2 and i_3 are rectified and applied to the control winding of an auto-excited transductor ; another control winding carries the rectified current i_4 flowing in linear resistors R.

The curve in *Figure* 14.22*b* shows the total control m.m.f. due to i_1, i_2 and i_3 plotted against voltage while the straight line shows the control m.m.f. due to the current i_4. If the alternator voltage is correct the resultant control m.m.f. is zero, but for a

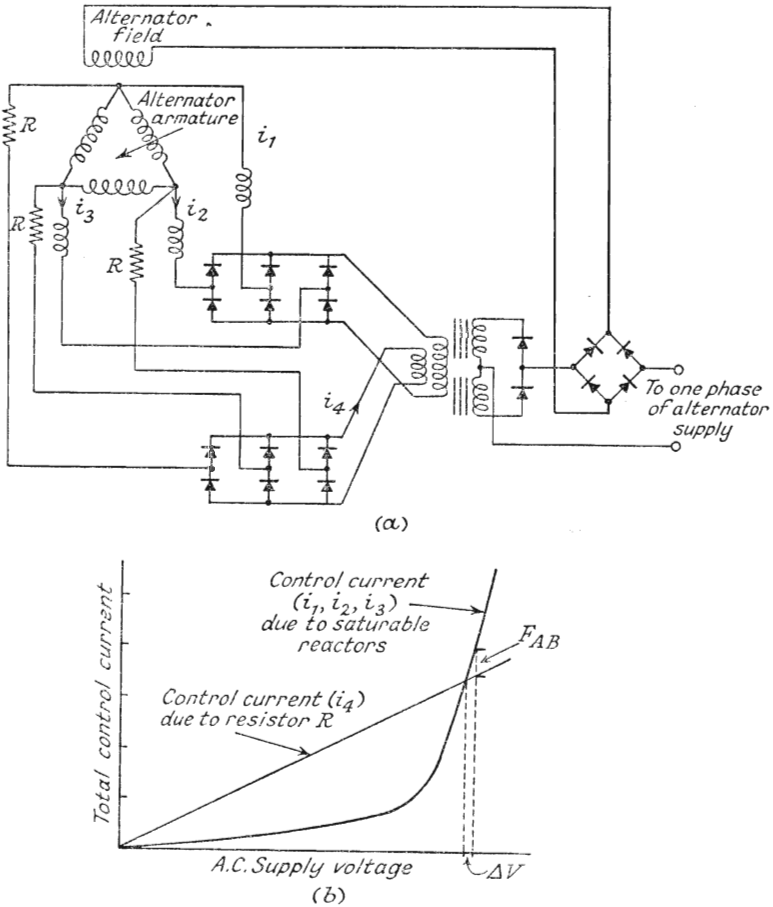

Figure 14.22(*a*). Magnetic amplifier voltage regulator for 3-kW. alternator having a 3-phase output.

(*b*) Total control m.m.f., due to saturable reactors and resistors, plotted against phase voltage of alternator.

change of voltage ΔV, the control m.m.f. is equal to F_{AB}. This m.m.f. varies the output from the transductor which in turn varies the alternator field in a sense which tends to maintain the alternator voltage constant. With this arrangement the output

voltage was maintained to within $\pm 1\frac{1}{2}\%$ for the full load change and the full range of temperature variation. One attractive feature of this application lies in the fact that the whole unit, including the reference, is magnetic, and hence it runs satisfactorily for indefinite periods without the need for servicing or adjustment.

(*c*) *Voltage regulation of large machines* (*magnestat regulator*). This type of regulator, as applied to the voltage control of generators, comprises a voltage-sensitive circuit and a magnetic

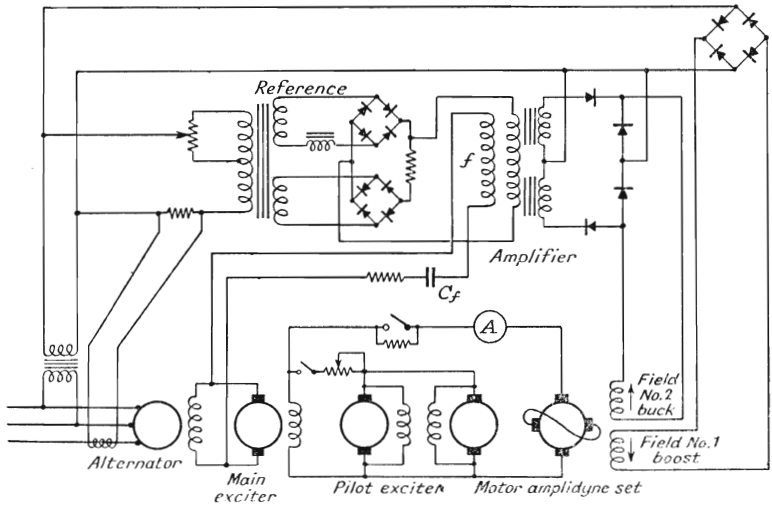

Figure 14.23. Magnetic amplifier/amplidyne voltage control of 30-mW. 3-phase alternator.

By courtesy British Thomson Houston Ltd.

amplifier, which together control the generator excitation in response to deviations in the controlled voltage. The magnetic amplifier supplies the exciter field power through the medium of an amplidyne in the case of large generators. For small generators the amplidyne is sometimes omitted, the exciter field being supplied directly from the magnetic amplifier.

The circuit diagram in *Figure* 14.23 applies to a 30-MW. alternator. The magnetic amplifier is controlled by means of a voltage reference circuit of the same type as that described previously.

The output of the amplifier feeds a bucking field on the amplidyne which in turn controls the field of the main exciter and hence that of the alternator. A feedback circuit (f) stabilizes the

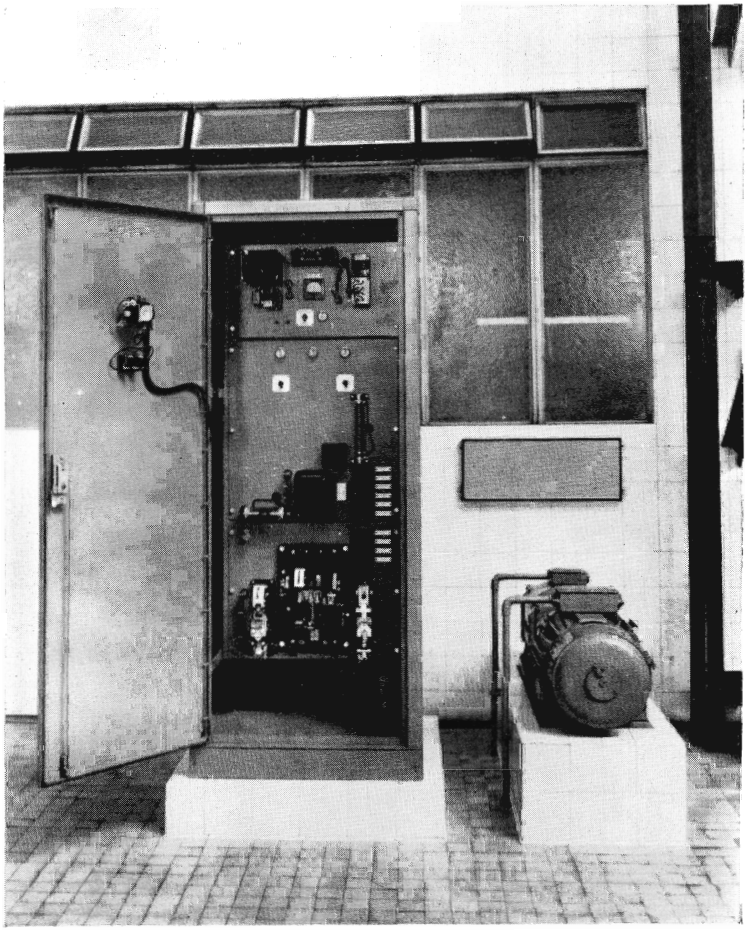

Figure 14.24. Magnetic amplifier voltage regulator (Magnestat).
By courtesy the British Thomson-Houston Co. Ltd.

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system by transient feedback of the main alternator field voltage, i.e. rate of change of field current.

A photograph of the unit is shown in *Figure* 14.24.

14.8 Speed Control of Electrical Machines

The automatic speed control or torque control of motors is often required for machine tools, crane hoists, etc. The control can be effected by rotary amplifiers, grid-controlled rectifiers or, in the case of a.c. machines, by variable transformers or induction regulators.

The transductor or magnetic amplifier can also be applied to the speed control of either a.c. or d.c. machines and is generally suitable at power levels up to a few kilowatts. For power levels in the megawatt range it is preferable on economic grounds to use a rotary amplifier as the ultimate controller ; the rotary amplifier may itself be controlled by means of a transductor amplifier.

Electrical signals representing the required and actual speed or torque are compared, the error signal then being amplified to operate the controller.

The speed of d.c. motors may be controlled either by variation of the field current or by varying the supply to the armature, with the field giving a constant excitation. The advantage of the latter is that a wide speed range is practicable, full torque being obtained down to zero speed. In addition, the response time is more rapid than if the field is used as the controller. On the other hand, the power output required from the amplifier is considerably less when supplying the field, since a certain amount of amplification is obtained in the motor itself.

It will be clear that the various methods of speed control will be very similar to frequency controls for motor generators. If accurate speed control is required, it is necessary to measure the speed by means of an a.c. or d.c. tacho-generator. Otherwise an approximate measure of the speed is given by the generator e.m.f. as in the following example.

(*a*) *Motor speed control.* A simple and robust speed control for a d.c. machine is shown in *Figure* 14.25. This arrangement obtains a measure of the speed by measuring the armature voltage V_A and comparing it with a constant direct voltage in order to give the error signal. Owing to the voltage drop v in the brushes, and also the armature resistance drop $i_A R_A$, however, the armature voltage does not give a very accurate indication of

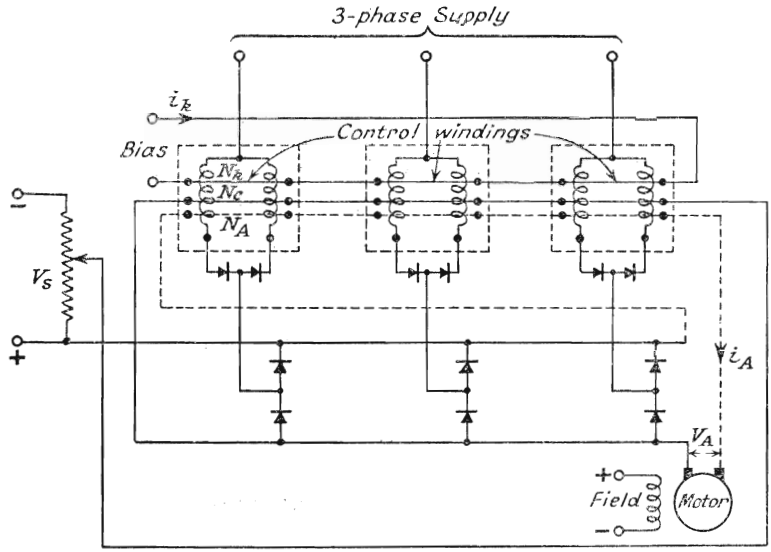

Figure 14.25. Circuit of motor speed control using 3-phase magnetic amplifier.

the speed and a better one is given by the e.m.f. Thus an additional control winding N_A (shown dotted) carries a current proportional to the armature voltage drop and therefore the error signal, i.e. the total control m.m.f. is more nearly proportional to the motor speed. This may be seen as follows :

From *Figure* 14.25, the total control m.m.f. A_c is given by

$$A_c = \frac{V_s - V_A}{R_c} N_c + i_k N_k + i_A N_A$$

where i_A is the armature current.

Now since the armature voltage is given by

$$V_A = E_A + i_A R_A + v,$$

the control m.m.f. may be written as

$$A_c = \left[V_s - i_A R_A - v - E_A \right] \frac{N_c}{R_c} + i_A N_A + i_k N_k.$$

Thus if N_A is made equal to $\dfrac{R_A N_c}{R_c}$, the control m.m.f. will be proportional to the error between the reference voltage V_s and the e.m.f. E_A, the latter being proportional to the speed.

The circuit shown in *Figure* 14.25 applies to a three-phase system, each phase containing an auto-excited transductor. In

this application the closed loop is stabilized by making the transductor time-constant large compared with that of the motor. This is done by winding short-circuited coils consisting of single straps of copper foil over the control winding. Thus the response of the motor to demanded changes of speed tends to be sluggish. Using this regulator, the speed is maintained constant to within $\pm$ 2·5% from no-load to full-load torque. In addition the speed may be set to any value from zero to maximum. A photograph of the unit is shown in *Figure* 14.26.

If more accurate control is required it is necessary to use a separate tacho-generator to measure the speed, as in the following example.

(*b*) *Magnetic amplifier for rate gyro speed control.* The output from a rate gyroscope is dependent for its accuracy upon the gyro rotor maintaining a constant speed. In one application to fire control equipment the speed of the gyro rotor is maintained at a constant value by using a magnetic amplifier speed-control system.

A small a.c. tacho-generator is fitted to the gyro rotor to give an alternating voltage output whose frequency is proportional to the speed of the gyro. The frequency reference consists of two series-tuned circuits similar to those discussed in section 14.7 (*a*). The current in these circuits is rectified and applied to two control windings on an auto-excited magnetic amplifier. The output stage consists of a three-phase transductor, one a.c. winding being placed in series with each supply phase to the gyro motor, which is a three-phase squirrel cage induction motor, the gyro rotor forming the squirrel cage armature.

The accuracy of speed control depends primarily on accurate discrimination and high gain and stability in the tuned circuits, which consist of constant permeability dust iron toroids and silvered mica capacitors. The overall accuracy of speed control was $\frac{1}{3}$% for a supply voltage variation of $\pm$ 10% and a frequency variation of $\pm$ 2%. The whole equipment operated from a supply frequency of 333 c/s.

An interesting feature of this application is that the time-constant of both amplifiers, although amounting to something of the order of one second each, is extremely small compared with that of the gyro rotor, the time-constant of which was several minutes. For this reason, the system was extremely stable even for a very high amplifier gain. It is true that a sudden change

in the reference (i.e. the demanded wheel speed) would be followed by a prolonged transient ; this was immaterial, however, since the wheel speed was always required to be maintained at the same value. Short-term variations of supply voltage and frequency, on the other hand, produced no appreciable transient due to the smoothing effect of this large time-constant.

A photograph of the speed control unit installed in the computing equipment is shown in *Figure* 14.27.

(*c*) *Fast response amplifier for motor speed control.* In general, if the maximum speed of response is required from a motor speed control, the amplifier time lag must be reduced to a value which

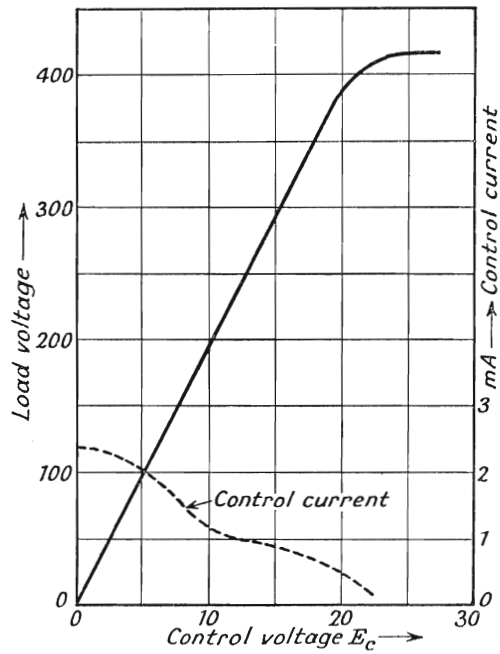

Figure 14.28. Control characteristic of 3-phase high-speed amplifier for motor control. (*After Maine.*)

is small compared with that due to the motor inertia. If, however, the amplifier is operating from a 50 c/s power supply the response time is likely to be excessive if the whole gain is obtained from one stage.

In such cases, therefore, multi-stage high-speed amplifiers similar to those discussed in Chapter 9 should be employed.

A two-stage three-phase magnetic amplifier has been designed

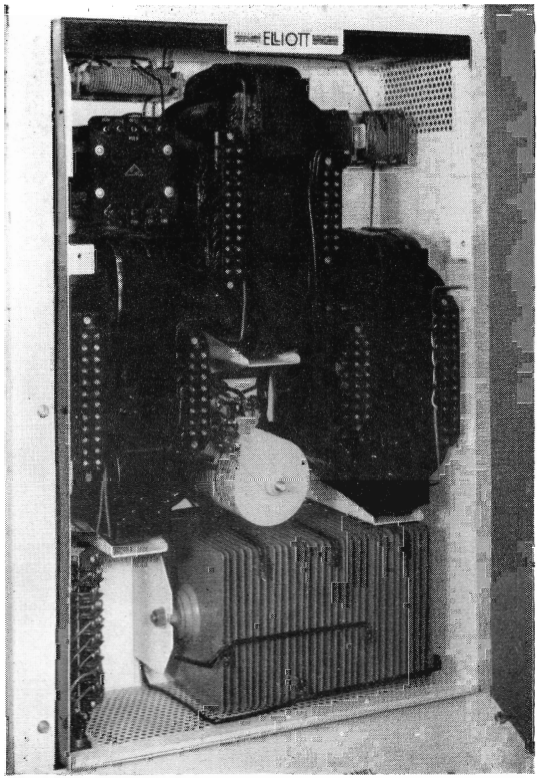

Figure 14.26. Magnetic amplifier speed control unit for a d.c. motor.

By courtesy Elliott Brothers (London) Ltd.

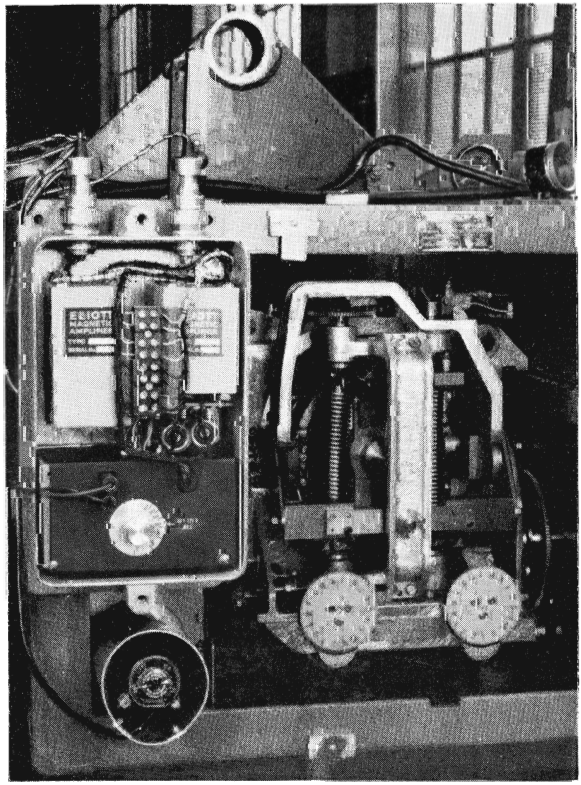

Figure 14.27. Magnetic amplifier for rate gyroscope speed control.

By courtesy Elliott Brothers (London) Ltd.

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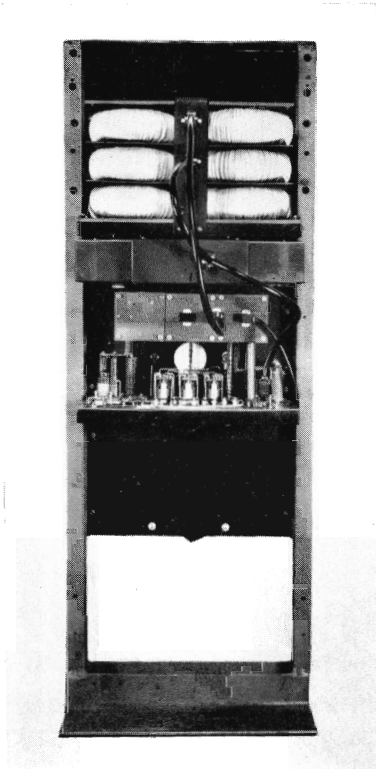

Figure 14.29. Fast response magnetic amplifier for motor speed control.

By courtesy de Havilland Propeller Co. Ltd.

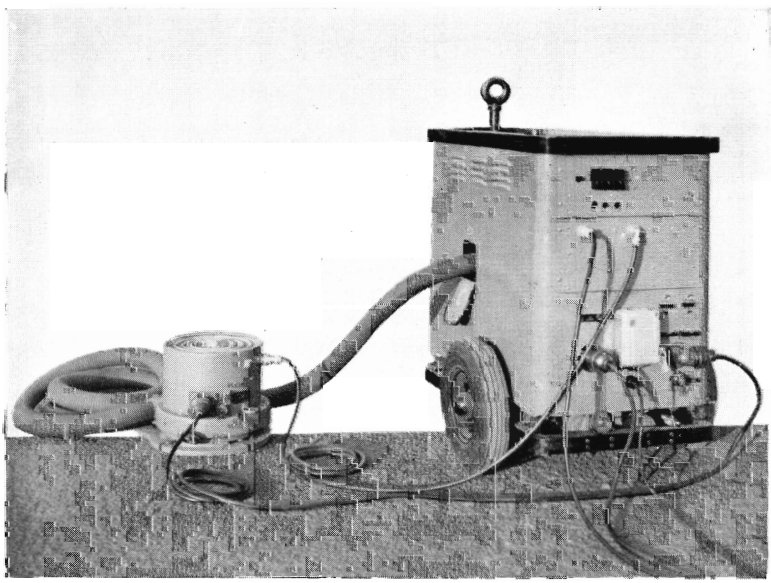

Figure 14.30. Equipment for aircraft vibration testing.

By courtesy de Havilland Propeller Co. Ltd.

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along these lines for the armature control of a $\frac{3}{4}$ h.p. d.c. motor having a speed range of 20 : 1.[14(f)]

The motor speed is measured by means of a tacho-generator, the output voltage from the tacho being compared with a constant voltage representing the speed reference. The resulting error signal is power amplified in a cathode follower which drives the first magnetic amplifier stage.

Both magnetic amplifiers employ three-phase circuits such as those described in Chapter 9.

The first stage employs six cores, each comprising $\frac{3}{8}$ in. stacks of H.C.R. laminations (No. 522). The turns ratio is $1\frac{1}{4}$: 1 and each core unit is of normal transformer construction.

The output stage, which is rated to supply a maximum output of 1 kW, consists of 6 H.C.R. toroids, each one being 6 in. diameter and having a sectional area of approximately 1·5 sq. in. B.T.H. germanium junction rectifiers are employed throughout.

The control characteristic of the two amplifiers in cascade is shown in *Figure* 14.28. When operating into a load of 200 Ω the overall power gain of the amplifiers is 16,000 and the response time approximately 1 period of the 50 c/s mains.

The overall loop gain of the system is 38 dB, and a photograph of the unit is shown in *Figure* 14.29.

14.9 500-W. Vibrator Magnetic Amplifier

This represents another application of the high-speed type of circuit in which an amplifier was required for energizing a moving coil vibrator for aircraft resonance testing. The vibrator in question has been designed for operation from a 500-W. electronic amplifier ; but the advent of larger aircraft provided a need for lower frequencies (down to 1 c/s) than the electronic amplifier would handle. Further requirements were extreme robustness and compactness for field work, good waveform with a minimum of spurious outputs, and full power from less than 1 c/s to more than 100 c/s.

These considerations led to the design of a push-pull magnetic amplifier driven from a motor alternator delivering 400 c/s three-phase.[14(g)] The moving coil of the vibrator is divided into two portions. These are connected to the two halves of the amplifier through a bridge circuit which removes the d.c. component of current from the vibrator, thus preventing overheating. The " push " and " pull " sides of the amplifier are quite separate electrically,

except for a common supply transformer. Each half comprises two stages and each stage is a full-wave three-phase circuit with the output voltage fed back in series with the input, as suggested by Scorgie,[9(d)] for single-phase amplifiers. The feedback enables the amplitude response of the amplifier to remain constant for all frequencies up to 400 c/s, although a choke introduced for ripple smoothing causes some attenuation above 200 c/s. In order to use the amplifier conveniently with other existing equipment, the input signal is derived from a very simple electronic device employing three miniature double triodes, This drives the two half amplifiers in such a way that " Class B " operation is obtained. Signals of the order of tens of millivolts are sufficient to load the system fully.

For convenience in handling, the magnetic amplifier with its frequency changer is mounted on a trolley. Leads to the trolley are for three-phase 50-cycle mains, input signals, and output to the vibrator. In normal use, up to four of these units with four vibrators are used for multi-point excitation of airframes. Accordingly the four electronic pre-amplifier units are placed on a common panel in the control cabin. Means are provided for producing any desired phase relationship between the four signals in order to suppress unwanted resonance modes. A photograph of the equipment is shown in *Figure* 14.30.

14.10 Line Voltage Regulator

The voltage regulation of a.c. supplies may be achieved by

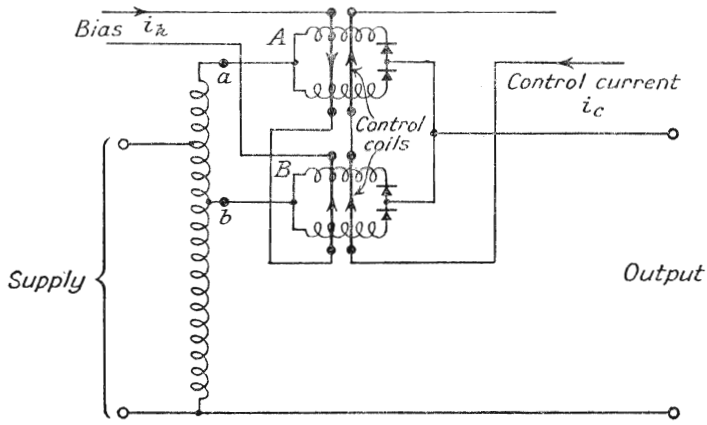

Figure 14.31. Circuit of line-voltage controller.

connecting a pair of transductors to the output of an auto-transformer as shown in *Figure* 14.31. This arrangement has been described by Hedstrom.[14(h)] When the control current is a maximum in one direction, the voltage drop across the transductor *A* is small and therefore the output is virtually connected to the maximum supply voltage at *a*. When the control current

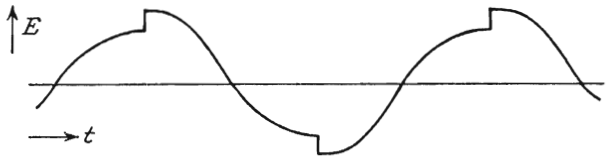

Figure 14.32. Waveform of output voltage from circuit of *Figure* 14.31.

is maximum in the reverse direction the biasing is such that transductor *B* has a low impedance and the output is effectively derived from *b*. Thus the transductors only have to control a small proportion of the total voltage, namely that required for regulation purposes. The advantage of this is that the waveform distortion resulting from such a controller is less pronounced than if the transductor were connected directly in series with the main supply line. A typical waveform obtained with this arrangement is shown in *Figure* 14.32.

14.11 Automatic Battery Charging

Lead-acid accumulators are in universal use, both as emergency supplies independent of the mains, or as regularly used d.c. supplies. The life of accumulators is dependent upon their being properly maintained. If they are left uncharged for long periods, sulphating occurs, and if they are overcharged excessively, mechanical strains are set up in the plates, which cause distortion and ultimate destruction of the battery. The battery should be maintained with a cell voltage between 2·15 and 2·18 volts and allowed to draw a small charging current sufficient to make up for losses due to normal action. The correct state of charge may be maintained automatically by charging the battery through a metal rectifier fed from the a.c. mains via a transductor, the current to the rectifier being controlled so as to maintain the voltage of the battery within the desired limits.

Referring first to *Figure* 14.33*a*, the transductor is placed in the a.c. line and the battery is charged through a transformer

and bridge rectifier as shown. If the charging current is i_a and the transformation ratio is unity, then this current will be proportional to the total control m.m.f. It will also depend to a small extent on the supply voltage E_a and the battery voltage V_b thus :

$$i_a = \frac{1}{N_a}[K_a E_a - K_b V_b + i_c N_c].$$

If no self-excitation is used the rectifier current will be independent of the load current drawn from the battery. If, however, the load current i_L were fed back, then an additional control m.m.f. $i_L N_L$ would be added to the constant term $i_c N_c$, and the charging current i_a would be increased with increased load current. Thus the rectifier could be made to supply all the load current by making N_L sufficiently large. The difficulty which now arises is that the feedback coil N_L would be in series with the load and therefore the voltage regulation would be poor due to the voltage drop in this coil. Furthermore, once the load current stops flowing there is no automatic method of ensuring rapid recharge of the battery.

A better arrangement is shown in *Figure* 14.33*b*. Normal

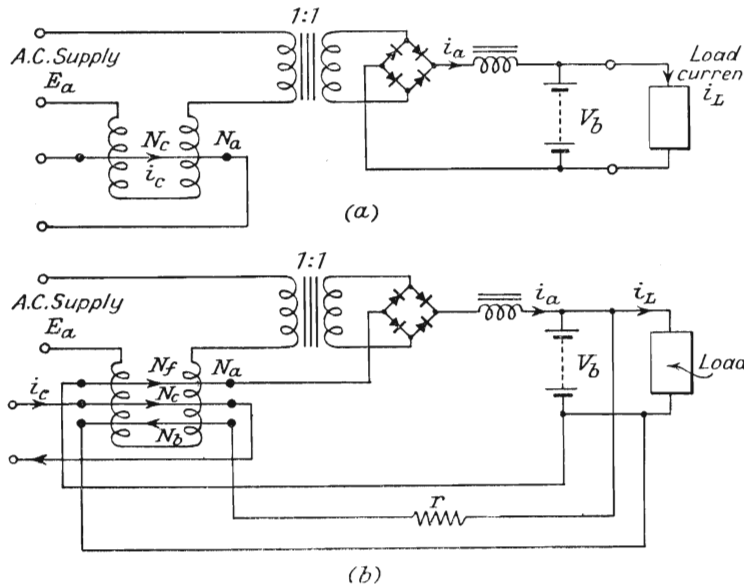

Figure 14.33. Automatic battery charger.
(*a*) Without feedback.
(*b*) With self-excitation and feedback.

self-excitation is applied to the transductor by feeding back the charging current. Thus the term N_a is now replaced by $N_a - N_f$ (section 3.11), and the control current i_c is adjusted to a constant value just sufficient to allow a trickle charge to take place when the battery voltage is at its correct value. Due to the self-excitation, the charging current is more sensitive to a fall in battery voltage than it would be if no self-excitation were applied.

Under these circumstances, the rectifier will supply load current only if the battery voltage V_b is reduced and if a heavy load current is drawn for a length of time appreciable discharge takes place. This can be improved by feeding back the battery voltage to provide an additional control current through a resistance r as shown in *Figure* 14.33*b*. The equation for the regulator is then given by

$$i_a = \frac{1}{N_a - N_f}\left[(K_a E_a + i_c N_c) - V_b\left(K_b + \frac{N_b}{r}\right)\right].$$

Thus the charging current is now made more sensitive to changes of V_b and reduces the effects of variation of supply voltage and rectifier characteristics. The system is in fact a self-excited transductor with overall negative voltage feedback, the latter ensuring a constant battery voltage. The sensitivity of this arrangement can be such that for a fall of about 2% in the battery voltage full charging current flows.

APPENDICES

APPENDIX 1

Basic Transductor Equations for Various States of Operation

ASSUMING a magnetization characteristic of the form shown in *Figure* 3.1b, the following results may be derived for the various states of operation :

(i) *Both cores unsaturated* (*Figure* 3.2*b*).

Equations for control circuit and a.c. circuit may be written down thus :

$$E_c = N_c\left[\frac{d\phi_A}{dt} + \frac{d\phi_B}{dt}\right] \quad . \quad . \quad . \quad (1)$$

$$E_a = N_a\left[\frac{d\phi_A}{dt} - \frac{d\phi_B}{dt}\right] \quad . \quad . \quad . \quad (2)$$

Also, since both cores are unsaturated, the m.m.f.s F_A and F_B must both be zero. Hence :

$$F_A = i_c N_c + i_a N_a = 0 \quad . \quad . \quad . \quad (3)$$

$$F_B = i_c N_c - i_a N_a = 0 \quad . \quad . \quad . \quad (4)$$

Therefore $i_a = 0$

and $i_c = 0.$

Dividing (1) by N_c, and (2) by N_a, adding and subtracting gives

$$\frac{E_c}{2N_c} + \frac{E_a}{2N_a} = \frac{d\phi_A}{dt} \quad . \quad . \quad . \quad (5)$$

and

$$\frac{E_c}{2N_c} - \frac{E_a}{2N_a} = \frac{d\phi_B}{dt} \quad . \quad . \quad . \quad (6)$$

Appendix 1

Assuming E_c is positive and considering a positive half-cycle of E_a then, provided $\frac{E_a}{N_a} > \frac{E_c}{N_c}$, ϕ_A will be increasing and ϕ_B will be decreasing. In the following half-cycle E_a changes sign, therefore the flux changes also reverse. Thus if ψ_1 is the rate of change of flux when increasing and ψ_2 the rate of change when decreasing, where $\psi = \frac{d\phi}{dt}$ we have in general for both cores unsaturated :

$$\psi_1 = \frac{E_c}{2N_c} + \frac{|E_a|}{2N_a} \quad . \quad . \quad . \quad (7)$$

$$\psi_2 = \frac{E_c}{2N_c} - \frac{|E_a|}{2N_a} \quad . \quad . \quad . \quad (8)$$

(ii) *One core saturated* (*Figures* 3.2*c* and *d*.

Considering *Figure* 3.2*c* :

$$E_c = R_c i_c + N_c \frac{d\phi_B}{dt} \quad . \quad . \quad . \quad (9)$$

$$E_a = R_a i_a - N_a \frac{d\phi_B}{dt} \quad . \quad . \quad . \quad (10)$$

also, since core B is unsaturated

$$F_B = 0 = i_c N_c - i_a N_a \quad . \quad . \quad . \quad (11)$$

Multiplying (9) by N_c/R_c and (10) by N_a/R_a, subtracting and collecting terms gives

$$\frac{E_c N_c}{R_c} - \frac{E_a N_a}{R_a} = \left[\frac{N_c^2}{R_c} + \frac{N_a^2}{R_a}\right]\frac{d\phi_B}{dt} \quad . \quad . \quad . \quad (12)$$

Multiplying throughout by R_a/N_a^2 and putting $\frac{N_c^2}{N_a^2} \cdot \frac{R_a}{R_c} = \gamma$ gives

$$\frac{d\phi_B}{dt} = \frac{E_c}{N_c}\frac{\gamma}{1+\gamma} - \frac{E_a}{N_a}\frac{1}{1+\gamma} \quad . \quad . \quad . \quad (13)$$

In the following half-cycle core B saturates leaving core A unsaturated, and it may similarly be shown for *Figure* 3.2*d* that

$$\frac{d\phi_A}{dt} = \frac{E_c}{N_c} \cdot \frac{\gamma}{1+\gamma} + \frac{E_a}{N_a}\frac{1}{1+\gamma} \quad . \quad . \quad . \quad (14)$$

Since in the last case, however, we are dealing with a negative half-cycle, the supply voltage has become numerically negative; we may therefore write a general equation for the flux change in the unsaturated core when the adjacent core is saturated thus :

$$\psi_3 = \frac{E_c}{N_c} \cdot \frac{\gamma}{1+\gamma} - \frac{|E_a|}{N_a} \cdot \frac{1}{1+\gamma} \quad . \quad . \quad . \quad (15)$$

Substituting equation (13) into (10) and (9) and collecting terms gives, for positive half-cycles,

$$i_a = \frac{\gamma}{1+\gamma}\left[\frac{E_c N_a}{N_c} + E_a\right]\frac{1}{R_a} \quad . \quad . \quad . \quad (16)$$

$$i_c = \frac{1}{1+\gamma}\left[E_c + \frac{E_a N_c}{N_a}\right]\frac{1}{R_c} \quad . \quad . \quad . \quad (17)$$

and similarly for negative half-cycles by considering *Figure* 3.2*d* :

$$i_a = \frac{\gamma}{1+\gamma}\left[-\frac{E_c N_a}{N_c} + \mathrm{E}_a\right]\frac{1}{R_a} \quad . \quad . \quad . \quad (18)$$

and

$$i_c = \frac{1}{1+\gamma}\left[E_c - \frac{E_a N_c}{N_a}\right]\frac{1}{R_c} \quad . \quad . \quad . \quad (19)$$

APPENDIX 2

Determination of Time-Constant of Transductor with a Square Wave Supply Voltage

If equations 3.17 and 3.18 are combined we get an expression relating two successive conducting periods β_1 and β_2 thus :

$$\beta_2 = \frac{\psi_1 + \psi_2}{\psi_1}\frac{T}{2} + \frac{\psi_3 - \psi_2}{\psi_1}\beta_1$$

putting $\dfrac{\psi_3 - \psi_2}{\psi_1} = a \quad . \quad . \quad . \quad (1)$

and $\dfrac{\psi_1 + \psi_2}{\psi_1}\dfrac{T}{2} = b \quad . \quad . \quad . \quad (2)$

gives $\beta_2 = a\beta_1 + b.$

Appendix 2

By considering the nth conducting period, a precisely similar relationship may be obtained :

$$\begin{aligned}\beta_n &= a\beta_{n-1} + b \\ &= a(a\beta_{n-2} + b) + b \\ &= a[a(a\beta_{n-3} + b) + b] + b. \\ &= a^3\beta_{n-3} + (a^2 + a + 1)b \\ &\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ &= a^r\beta_{n-r} + (a^{r-1} + a^{r-2} + \ldots + 1)b\end{aligned}$$

for positive integral values of $r < n$,
putting $r = n - 1$ gives

$$\beta_n = a^{n-1}\beta_1 + (a^{n-2} + a^{n-3} + \ldots + 1)b,$$

thus, by summing the geometrical series in brackets

$$\beta_n = a^{n-1}\beta_1 + \left[\frac{1 - a^{n-1}}{1 - a}\right]b \qquad . \quad . \quad . \quad (3)$$

This expresses the duration of the nth conducting period in terms of the first and the circuit parameters.
Now since ψ_2 and ψ_3 are numerically negative and

$$|\psi_1| > |\psi_2| > |\psi_3|$$

we have $0 < a < 1$.
Also the steady-state value of β is given by the value of β_n as n tends to infinity.
Therefore from (3) we have as $n \to \infty$

$$\beta = \frac{b}{1 - a} \qquad . \quad . \quad . \quad (4)$$

(i) *Value of steady-state mean current* $\bar{i}_a$

Now the mean value of the current i_a during the nth half-cycle is equal to the value of i_a during the nth conducting period multiplied by the fraction of the half-cycle over which this current flows, i.e.

$$\bar{i}_a = \frac{\beta_n}{(T/2)}\left[|\, i_a \,|\right].$$

In order to obtain the steady-state mean current we substitute for $|\, \bar{i}_a \,|$ from equation 3.12 and for β from equation 4 above giving

$$\bar{i}_a = \frac{1}{(T/2)} \cdot \frac{b}{(1 - a)}\left[\frac{\gamma}{1 + \gamma}\left(\frac{E_c N_a}{N_c} + |\, E_a |\right)\frac{1}{R_a}\right]$$

substituting for b and a from (1) and (2) gives

$$\left[\frac{\psi_1+\psi_2}{\psi_1+\psi_2-\psi_3}\right]\left[\frac{\gamma}{1+\gamma}\cdot\frac{2N_a}{R_a}\left(\frac{E_c}{2N_c}+\frac{|E_a|}{2N_a}\right)\right]$$

giving $$\bar{\imath}_a=\frac{\psi_1+\psi_2}{\psi_1+\psi_2-\psi_3}\,\psi_1\cdot\frac{\gamma}{1+\gamma}\,\frac{2N_a}{R_a}$$

substituting for ψ_1, ψ_2 and ψ_3 from Appendix 1 (equations (7), (8) and (15)) gives, for the steady-state,

$$\bar{\imath}_a=\frac{E_c}{R_c}\cdot\frac{N_c}{N_a} \qquad . \quad . \quad . \quad (5)$$

(ii) *Time-constant*

The time-constant is defined by the time required for the output to reach a fraction, $\left(1-\frac{1}{e}\right)$, of its steady-state value. Thus putting $\beta_n=\left(1-\frac{1}{e}\right)\beta$

we have from (3) and (4)

$$\left(1-\frac{1}{e}\right)\frac{b}{1-a}=a^{n-1}\beta_1+\left[\frac{1-a^{n-1}}{1-a}\right]b \quad . \quad . \quad . \quad (6)$$

If this equation is solved for n, the value found for n will be the number of half-cycles which occur before the output has reached $\left(1-\frac{1}{e}\right)$ of its final value.

Firstly it is necessary to determine the value of β_1. Now β_1 is the duration of the conducting period in the first half-cycle. Assuming that the initial flux in core A in the first half-cycle is at negative saturation, then if E_c had not been switched on (*Figure* 3.4) the flux would have increased at a rate equal to $\left|\frac{E_a}{2N_a}\right|$ and would just reach the positive peak value at $T/2$; therefore we could write the total change of flux in the first half cycle thus :

$$\frac{|E_a|}{2N_a}\cdot\frac{T}{2}=\psi_1\left(\frac{T}{2}-\beta_1\right)$$

but since $\frac{|E_a|}{2N_a}=\frac{1}{2}(\psi_1-\psi_2)$ (Appendix 1, equations 7 and 8)

we have $$\beta_1=\frac{\frac{1}{2}(\psi_1+\psi_2)}{\psi_1}\,\frac{T}{2}$$

$$=\frac{b}{2}.$$

Therefore substituting in (6) and multiplying out gives

$$2\left(1 - \frac{1}{e}\right) = a^{n-1}(1 - a) + 2(1 - a^{n-1})$$

so that, collecting terms :

$$\frac{2}{e} = a^{n-1}(1 + a)$$

and

$$n = 1 + \frac{\log \dfrac{2}{e(1 + a)}}{\log a} \qquad . \quad . \quad . \quad (7)$$

It may easily be shown by substituting that

$$a = \frac{\gamma - 1}{\gamma + 1} \text{ and } 1 + a = \frac{2\gamma}{1 + \gamma}$$

giving

$$n = 1 + \frac{\log \dfrac{\gamma + 1}{\gamma e}}{\log \dfrac{\gamma - 1}{\gamma + 1}} \qquad . \quad . \quad . \quad (8)$$

Now since n is the number of half-cycles corresponding to the time-constant and the duration of each half-cycle is $\frac{1}{2f}$, we have

$$\tau = \frac{n}{2f}$$

substituting from (8) and interchanging numerators and denominators in logarithmic terms, gives

$$\tau = \frac{1}{2f}\left[1 + \frac{\log \dfrac{\gamma e}{\gamma + 1}}{\log \dfrac{\gamma + 1}{\gamma - 1}}\right] \qquad . \quad . \quad . \quad (9)$$

APPENDIX 3

Flux and Current Waveforms with Sinusoidal Excitation

The following analysis indicates the method of establishing the cyclic flux and current variations in a transductor which is

excited from a sinusoidal supply voltage $\hat{E}_a \sin \omega t$, and to which is applied a direct voltage E_c.

Referring to *Figure* 3.5, it is first necessary to determine the time at which a core comes out of saturation. This occurs when the core m.m.f. decreases to zero, which in turn is governed by the condition $i_a \rightarrow 0$.

The current i_a during conduction is given by equations 16 and and 18 in Appendix 1 and, putting $E_a = \hat{E}_a \sin \omega t$, by equating these to zero it is seen that the current decreases to zero after a positive half-cycle

when $$\hat{E}_a \sin \omega t = -\frac{E_c N_a}{N_c} \text{ (from 16)}$$

and after a negative half-cycle

when $$\hat{E}_a \sin \omega t = \frac{E_c N_a}{N_c} \text{ (from 18).}$$

Thus if two horizontal lines are drawn each displaced from the zero by equal amounts $\frac{E_c N_a}{N_c}$ in opposite directions as in *Figure* 3.5*a*, the times of desaturation t_{Q1}, t_{Q2}, t_{Q3}, etc., are established where these lines cut the sine wave as shown in the figure.

Let us now concentrate on the desaturation which occurs at a time t' after the end of any one particular half-cycle of the supply. It is clear from the geometry of *Figure* 3.5*a* that t' has the same value in relation to all half-cycles.

Let us therefore concentrate on the flux and current changes which commence at time t' in the nth half-cycle say. Suppose the flux in core A is increasing towards saturation, integration of equation 3.5 gives the total flux change from t' to t as

$$\Delta\phi_A = \frac{E_c}{2N_c}\int_{t'}^{t} dt + \frac{\hat{E}_a}{2N_a}\int_{t'}^{t} \sin \omega t \, dt.$$

$$= \frac{E_c}{2N_c}(t - t') + \frac{\hat{E}_a}{2\omega N_a}(\cos \omega t' - \cos \omega t) \quad . \quad (1)$$

In order to get the actual flux value it is necessary to know the initial flux $\phi_{Q(n-1)}$. Also, since the voltage across each a.c. coil is half the supply and, furthermore, $\frac{\pi}{2}\bar{E}_a = \hat{E}_a$, we have from equation 1.23, $\frac{\frac{1}{2}\bar{E}_a}{2fN_a} = 2\Phi_s$

so that $$\Phi_s = \frac{\hat{E}_a}{2\omega N_a} \text{ (putting } \omega = 2\pi f).$$

Thus the value of flux at any time t is given by equation 1 as

$$\phi_A = \phi_{Q(n-1)} + \frac{E_c}{2N_c}(t - t') + \Phi_s(\cos \omega t' - \cos \omega t) \quad . \quad . \quad . \quad (2)$$

Thus assuming $\phi_{Q(n-1)}$ is known, ϕ_A may be plotted and the saturation time t_{qn} will be given by putting $\phi_A = \Phi_s$ in equation (2). Meanwhile the flux in core B has been departing from saturation and at any instant after t' it is given by integrating equation 3.6 and adding the initial flux Φ_s thus :

$$\phi_B = \Phi_s + \frac{E_c}{2N_c}(t - t') - \Phi_s(\cos \omega t' - \cos \omega t) \quad . \quad (3)$$

The flux ϕ_B follows equation (3) until core A saturates at the time t_{qn} already determined from (2) and at time t_{qn}

$$\phi_B = \phi_{qn} = \Phi_s + \frac{E_c}{2N_c}(t_{qn} - t') - \Phi_s(\cos \omega t' - \cos \omega t_{qn}) \quad . \quad (4)$$

During the time following t_{qn} while core A is saturated, the flux ϕ_B now changes according to equation (3.16) from the initial conditions $\phi_B = \phi_{qn}$ and $t = t_{qn}$. Thus integrating 3.16 and putting in the initial conditions we now have

$$\phi_B = \phi_{qn} + \frac{\gamma}{1+\gamma} \cdot \frac{E_c}{N_c}(t - t_{qn}) - \frac{2}{1+\gamma} \cdot \Phi_s[\cos \omega t_{qn} - \cos \omega t] \quad (5)$$

The flux ϕ_B will continue to change in this way until core A desaturates at a time $t_{Qn} = \frac{T}{2} + t'$; the final value of ϕ_B is given by

$$\phi_B = \phi_{Qn} = \phi_{qn} + \frac{\gamma}{1+\gamma} \cdot \frac{E_c}{N_c}(t_{Qn} - t_{qn}) - \frac{2}{1+\gamma}\Phi_s[\cos \omega t_{qn} - \cos \omega t_{Qn}] \quad (6)$$

and since $t_{Qn} = \frac{T}{2} + t'$

$$\phi_{Qn} = \phi_{qn} + \frac{\gamma}{1+\gamma} \cdot \frac{E_c}{N_c}\left[\frac{T}{2} + t' - t_{qn}\right] - \frac{2}{1+\gamma}\Phi_s[\cos \omega t_{qn} + \cos \omega t'] \quad (7)$$

In the following $(n+1)^{\text{th}}$ half-cycle the flux in core B proceeds towards saturation repeating exactly the same process (equation 2) but inserting the newly found initial flux ϕ_{Qn}. At the same time core A departs from saturation from the initial value Φ_s according to equation 3. Thus in each successive half-cycle the

fluxes ϕ_A and ϕ_B exchange their operation, and therefore knowing the initial flux in the unsaturated core in any half-cycle (e.g. $\phi_{Q(n-1)}$), the flux waveforms for all succeeding half-cycles may be traced.

Now the initial flux corresponding to ϕ_{Qn} at the beginning of the first half-cycle when E_c is applied is known to be equal to $-\Phi_s$ and therefore the complete transient can be traced from the first half-cycle as above. Having determined the fluxes, the currents i_a and i_c are easily found since the times of conduction are now known. The current waveforms are found from equations 3.9 and 3.10—replacing E_a by $\hat{E}_a \sin \omega t$ and inserting the appropriate values of t as fixed by the saturation of the core fluxes.

APPENDIX 4

Determination of Transductor Gain Assuming Core of Finite Permeability

The following equations are used to derive the transductor voltage gain :

$$\bar{\imath}_c N_c + \lambda S \Phi_s = \bar{\imath}_a (N_a - N_f) + (1 + \lambda) S \phi_0 \text{ (equation 4.12)} \quad (1)$$

$$\phi_0 = \Phi_0 + \phi_c \text{ (see } \textit{Figure } 3.10) \quad . \quad . \quad . \quad (2)$$

$$\phi_c = \frac{\bar{e}_a}{8 f N_a} \text{ (equation 3.39)} \quad . \quad . \quad . \quad (3)$$

$$\bar{\imath}_a = \frac{\bar{e}_a}{\bar{R}_a} \text{ (by definition section 4.3)} \quad . \quad . \quad . \quad (4)$$

$$\bar{\imath}_c = \frac{E_c}{R_c} \quad . \quad . \quad . \quad (5)$$

Supply voltage factor

$$K = \frac{\bar{E}_a}{\bar{E}_{as}} = \frac{\Phi_s - \Phi_0}{\Phi_s} \text{ (equation 3.38)} \quad . \quad . \quad . \quad (6)$$

It is required to find a value for the transductor output voltage $\bar{e}_a$ in terms of the control voltage E_c.

All the necessary equations are available (1–6) and obtaining the result is merely a matter of substituting.

Appendix 4

Considering equation (1), substituting for ϕ_0 from (2) collecting terms and eliminating Φ_0 from (6) gives

$$\bar{\imath}_c N_c + S\Phi_s[K(1+\lambda) - 1] = \bar{\imath}_a(N_a - N_f) + (1+\lambda)S\phi_c$$

substituting for ϕ_c, $\bar{\imath}_a$ and $\bar{\imath}_c$ from equations (3), (4) and (5) respectively gives

$$\bar{e}_a\left[\frac{N_a - N_f}{\bar{R}_a} + \frac{S(1+\lambda)}{8fN_a}\right] = \frac{E_c N_c}{R_c} + S\Phi_s[K(1+\lambda) - 1]$$

multiplying by $\frac{R_c}{N_c}$ and putting $\Phi_s = \frac{\bar{E}_{as}}{8fN_a}$

$$\bar{e}_a\left[\frac{N_a - N_f}{\bar{R}_a}\cdot\frac{R_c}{N_c} + \frac{(1+\lambda)R_c}{(8fN_aN_c)/S}\right] = E_c + \frac{\bar{E}_{as}R_c}{(8fN_aN_c)/S}[K(1+\lambda) - 1].$$

Now the ratio $\frac{N_aN_c}{S}$ is equal to the mutual inductance M_{ca} between a control winding and an a.c. winding

also putting $$h_v = \frac{N_a - N_f}{N_c}\frac{R_c}{\bar{R}_a}$$

and $$\omega = 2\pi f$$

gives

$$\bar{e}_a\left[h_v + \frac{\pi}{4}\frac{(1+\lambda)R_c}{\omega M_{ca}}\right] = E_c + \frac{\pi}{4}\cdot\frac{R_c}{\omega M_{ca}}\cdot\bar{E}_{as}[K(1+\lambda) - 1]$$

putting $\frac{4}{\pi(1+\lambda)}\frac{\omega M_{ca}}{R_c} = g_v$ and multiplying through by g_v gives

$$\bar{e}_a[g_vh_v + 1] = g_vE_c + \frac{\bar{E}_{as}}{1+\lambda}[K(1+\lambda) - 1]$$

i.e. $$\bar{e}_a = \frac{g_v}{1+g_vh_v}E_c + \frac{\bar{E}_{as}}{1+g_vh_v}\left[\frac{K(1+\lambda) - 1}{1+\lambda}\right].$$

This gives the voltage gain of the transductor as

$$\frac{\delta\bar{e}_a}{\delta E_c} = \frac{g_v}{1+g_vh_v},$$

from which it follows that the current gain is given by

$$\frac{\delta\bar{\imath}_L}{\delta\bar{\imath}_c} = \frac{g_v}{1+g_vh_v}\cdot\frac{R_c}{\bar{R}_{aL}}$$ (page 107)

which may be written

$$G_i = \frac{g_i}{1+g_ih_i}$$

where
$$g_i = \frac{4}{\pi\,(1+\lambda)}\,\frac{\omega M_{ca}}{\bar{R}_{aL}}$$
$$h_i = \frac{N_a - N_f}{N_c}\,\frac{\bar{R}_{aL}}{\bar{R}_a}$$

and for resistive load since $\bar{\imath}_a = \bar{\imath}_L$ and $\bar{R}_a = \bar{R}_{aL} = R_a$

$$g_i = \frac{4}{\pi\,(1+\lambda)}\,\frac{\omega M_{ca}}{R_a}$$
$$h_i = \frac{N_a - N_f}{N_c}.$$

For resistive load the ampere-turns ratio is

$$\frac{\delta(\bar{\imath}_a N_a)}{\delta(\bar{\imath}_c N_c)} = G_m = \frac{g_i}{1 + g_i h_i}\,\frac{N_a}{N_c}$$

now
$$g_i\,\frac{N_a}{N_c} = \frac{4}{\pi(1+\lambda)}\,\frac{\omega L_{aw}}{R_a}$$

where $L_{aw} = \dfrac{N_a^2}{S}$ and is the self-inductance of one a.c. coil assuming 100% coupling.

Thus the ampere-turns ratio G_m may be written $\dfrac{g_m}{1 + g_m h_m}$

where
$$g_m = \frac{4}{\pi(1+\lambda)}\,\frac{\omega L_{aw}}{R_a}$$
$$h_m = \frac{N_a - N_f}{N_a}.$$

For smoothed control current $\lambda = 1$

and
$$g_m = \frac{2}{\pi}\,\frac{\omega L_{aw}}{R_a}.$$

APPENDIX 5

Similarity Theorem for Self-Excited and Auto-Excited Transductor

The object of this proof is to establish an equivalence between self-excited and auto-excited transductors and in particular for a

given auto-excited (parallel circuit) to determine the circuit constants which must be applied to a self-excited transductor in order to make the two circuits electrically identical.

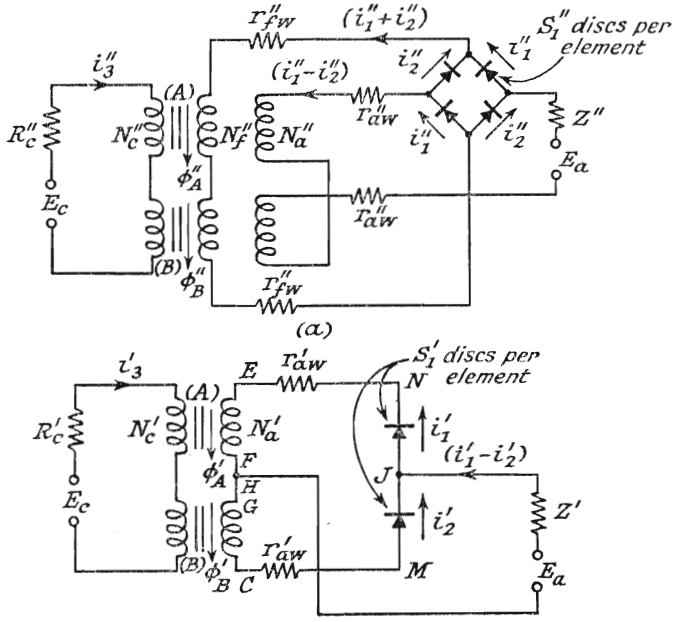

Figure A.

The auto-excited circuit is shown in *Figure A* (*b*) and the following known circuit constants apply :

Control circuit resistance	$= R'_c$
Load circuit impedance	$= Z'$
Control turns per coil	$= N'_c$
A.C. turns per coil	$= N'_a$
Resistance per a.c. coil	$= r'_{aw}$
Number of discs per rectifier element	$= S'_1$

We now wish to know the corresponding *numerical* values of turns, resistance, etc., in the *equivalent* self-excited circuit for which the corresponding unknown constants are respectively R''_c, Z'', N''_c, N''_a, r''_{aw} and S''_1 ; in addition the turns on each self-exciting coil are N''_f and the resistance of each one is r''_{fw}.

It will be assumed that the cores are all identical and have magnetization curves defined by the function

$$\phi = \psi(F)$$

and that the voltage drop across a rectifier element is $S_1 f(i)$

where S_1 is the number of discs in series and i the current in the element. Considering the auto-excited circuit, *Figure A* (*b*),

$$\phi'_A = \psi(N'_c i'_3 + N'_a i'_1) \quad . \quad . \quad . \quad (1)$$

$$\phi'_B = \psi(N'_c i'_3 + N'_a i'_2) \quad . \quad . \quad . \quad (2)$$

$$E_c = R'_c i'_3 + N'_c\left[\frac{d\phi'_A}{dt} + \frac{d\phi'_B}{dt}\right] \quad . \quad . \quad . \quad (3)$$

For the a.c. circuit through $JNEH$

$$E_a = Z'(i'_1 - i'_2) + r'_{aw} i'_1 + N'_a \frac{d\phi'_A}{dt} + S'_1 f(i'_1)$$

and for the a.c. circuit through $JMCH$

$$E_a = Z'(i'_1 - i'_2) - r'_{aw} i'_2 - N'_a \frac{d\phi'_B}{dt} - S'_1 f(i'_2).$$

Adding and dividing by 2

$$E_a = Z'(i'_1 - i'_2) + \frac{r'_{aw}}{2}(i'_1 - i'_2) + \frac{N'_a}{2}\left[\frac{d\phi'_A}{dt} - \frac{d\phi'_B}{dt}\right] + \frac{S'_1}{2}[f(i'_1) - f(i'_2)] \quad . \quad . \quad . \quad (4)$$

Also through the closed circuit $MNEHC$:

$$0 = r'_{aw}(i'_1 + i'_2) + N'_a\left[\frac{d\phi'_A}{dt} + \frac{d\phi'_B}{dt}\right] + S'_1 f(i'_1) + S'_1 f(i'_2).$$

Dividing by 2

$$0 = \frac{r'_{aw}}{2}(i'_1 + i'_2) + \frac{N'_a}{2}\left[\frac{d\phi'_A}{dt} + \frac{d\phi'_B}{dt}\right] + \frac{S'_1}{2}[f(i'_1) + f(i'_2)] \quad . \quad . \quad . \quad (5)$$

Now considering the self-excited circuit *Figure A* (*a*) :

$$\phi''_A = \psi[N''_c i''_3 + N''_f(i''_1 + i''_2) + N''_a(i''_1 - i''_2)]$$

making $N_f'' = N_a''$

$$\phi_A'' = \psi[N_c''i_3'' + 2N_a''i_1''] \quad . \quad . \quad . \quad (6)$$

Similarly

$$\phi_B'' = \psi[N_c''i_3'' + 2N_a''i_2''] \quad . \quad . \quad . \quad (7)$$

Also

$$E_c = R_c''i_3'' + N_c''\left[\frac{d\phi_A''}{dt} + \frac{d\phi_B''}{dt}\right] \quad . \quad . \quad . \quad (8)$$

For the a.c. circuit

$$E_a = Z''(i_1'' - i_2'') + 2r_{aw}''(i_1'' - i_2'') + N_a''\left[\frac{d\phi_A''}{dt} - \frac{d\phi_B''}{dt}\right] + S_1''[f(i_1'') - f(i_2'')] \quad . \quad . \quad . \quad (9)$$

For the self-excitation circuit with $N_f'' = N_a''$

$$0 = 2r_{fw}''(i_1'' + i_2'') + N_a''\left[\frac{d\phi_A''}{dt} + \frac{d\phi_B''}{dt}\right] + S_1''[f(i_1'') + f(i_2'')] \quad . \quad . \quad . \quad (10)$$

Considering equations (1) to (5) : here we have five equations with five variables, namely

$$\phi_A', \phi_B', i_1', i_2' \text{ and } i_3',$$

and assuming the functions representing rectifiers and core materials are single-valued ones, a unique value can be assigned to each variable in terms of the applied voltages and circuit constants.

Turning now to equations (6) to (10) for the self-excited transductors, we have five unknowns and five equations. If equations (1), (2), (3), (4) and (5) are compared with equations (6), (7), (8), (9) and (10) respectively, it is seen that these equations are of precisely the same form and, provided the circuit constants are given appropriate values, the solutions for the fluxes and currents must be the same.

Thus, examining these constants in each equation, it follows that all the equations are identical if the following conditions hold :

$$N_c'' = N_c'$$

$$N_a'' = N_f'' = \frac{N_a'}{2}$$

$$R_c'' = R_c'$$

$$Z'' = Z'$$

$$r''_{aw} = r''_{fw} = \frac{r'_{aw}}{4}$$

$$S''_1 = \frac{S'_1}{2}.$$

If the equations are identical, the circuits must be identical so far as the five unknowns are concerned and since the latter completely specify the circuit this gives the conditions for which the self-excited transductor is identical to the auto-excited transductor.

It should be noted that in the self-excited circuit the self-exciting current $i_f = i''_1 + i''_2$

and the load current $i_a = i''_1 - i''_2$.

Thus $$i''_1 = \frac{i_f + i_a}{2}$$

and $$i''_2 = \frac{i_f - i_a}{2}.$$

Similarly the load current i_a in the auto-excited circuit is $i'_1 - i'_2$. Thus the currents i'_1 and i'_2 flowing in the a.c. windings A and B respectively may be written as $\frac{i_f + i_a}{2}$ and $\frac{i_f - i_a}{2}$ where the component $\frac{i_f}{2}$ is regarded as a circulating current as shown in *Figure* 5.1.

APPENDIX 6

Evaluation of the Ratios $\frac{\bar{e}_a}{\bar{i}_a} = \bar{R}_a$ and $\frac{\bar{e}_a}{\bar{i}_L} = \bar{R}_{aL}$ for a D.C. Load with Parallel Capacitance and Square Wave Excitation

Considering the waveforms in *Figure B* (*a*) and the circuit (*b*), the conducting period commences at t_q and ends at t_Q. Over this period the supply voltage is ungated and applied to the load. Due to the routing action of the load rectifier the transductor voltage

e_a always acts in the same direction in the d.c. circuit and is opposed by the load voltage for both negative and positive conducting periods (see *Figure B (a)*). The load voltage is

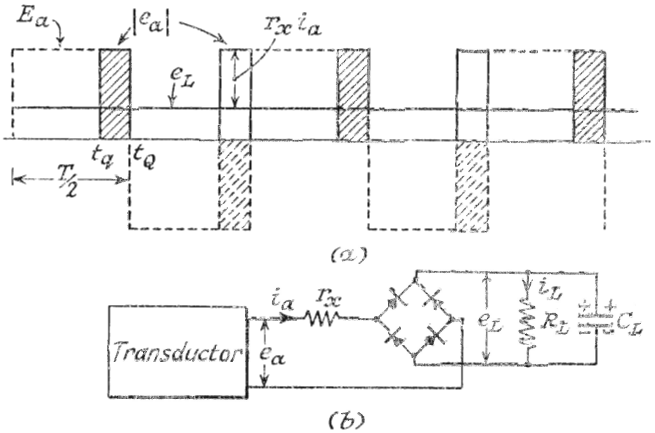

Figure B.

assumed to remain constant throughout the cycle due to the smoothing action of the capacitor. The transductor current i_a only flows during conducting periods but, as shown in Chapter 6, equation 6.16, we have

$$\bar{i}_a = \bar{i}_L$$

also

$$\bar{e}_L = R_L \bar{i}_L.$$

From *Figure B* over the conducting period we have

$$|e_a| - e_L = |r_x i_a|.$$

Integrating throughout from t_q to t_Q gives

$$\int_{t_q}^{t_Q} |e_a| dt - \int_{t_q}^{t_Q} e_L dt = r_x \int_{t_q}^{t_Q} |i_a| dt.$$

Now, in any half-cycle, the output e_a only appears during the interval t_q to t_Q, therefore the first integral on the left is the total voltage-time area due to e_a over a half-cycle and must therefore be $\frac{T}{2} \bar{e}_a$.

Similarly, the integral on the right is $\frac{T}{2} r_x \bar{i}_a = \frac{T}{2} r_x \bar{i}_L$ from (6.16).

Finally, e_L is constant throughout the cycle and it may therefore be written

$$\int_{t_q}^{t_Q} \bar{e}_L dt = \bar{e}_L \beta = \beta R_L \bar{i}_L$$

thus
$$\frac{T}{2}\bar{e}_a - \beta R_L \bar{i}_L = \frac{T}{2} r_x \bar{i}_L$$

therefore
$$\frac{\bar{e}_a}{\bar{i}_L} = r_x\left[1 + (2\beta/T)\frac{R_L}{r_x}\right] = \frac{\bar{e}_a}{\bar{i}_a}$$

thus
$$\bar{R}_a = \bar{R}_{aL} = r_x\left[1 + (2\beta/T)\frac{R_L}{r_x}\right].$$

APPENDIX 7

Effect of Smoothing Capacitor with Sinusoidal Supply

The improvement obtained in the maximum output by using a smoothing capacitor across a d.c. resistive load will now be

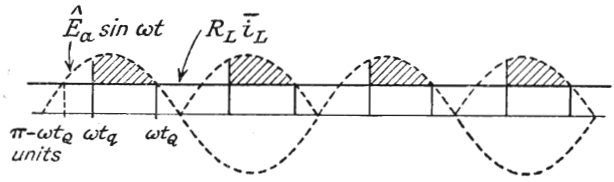

Figure C.

considered for the case of a transductor with a sine-wave supply voltage.

Referring to *Figure C*, the supply voltage is ungated and applied to the load at time t_q in each half-cycle. Also, as discussed in section 6.5, the conducting period ends when the transductor current i_a decreases to zero. The e.m.f. which is available to drive current through the external transductor circuit and its own internal resistance is given by the difference between the

output voltage e_a and the smooth load voltage $\bar{e}_L = \bar{\imath}_L R_L$, this difference being given by the shaded area. Therefore the end of conduction t_Q is defined by the condition

$$\hat{E}_a \sin \omega t_Q = R_L \bar{\imath}_L \qquad . \quad . \quad . \quad (1)$$

Let us first consider the voltage developed across the load in the *absence of smoothing* for two cases :

(*a*) At maximum possible output :

in this case $$\bar{\imath}_L = \hat{E}_a \cdot \frac{2}{\pi} \frac{1}{R_L + r_x}$$

i.e. $$R_L \bar{\imath}_L = \frac{2\hat{E}_a}{\pi} \frac{1}{1 + (r_x/R_L)} \qquad . \quad . \quad . \quad (2)$$

(*b*) At a firing angle of 90°.

This case is relevant because at this stage the transductor is often approaching its limit of linearity. Since there is no smoothing, conduction ceases at approximately the end of the half-cycle, i.e. $t_Q = T/2$ in this case ; the mean output voltage is clearly one-half that of the previous case

and $$R_L \bar{\imath}_L = \frac{\hat{E}_a}{\pi} \frac{1}{1 + (r_x/R_L)} \qquad . \quad . \quad . \quad (3)$$

Now, in the presence of smoothing, the mean output voltage is given by equation (1). Hence the improvement obtained by using smoothing can be found either at maximum output or for a firing angle $t_q = 90°$, by combining equation (1) with either (2) or (3) respectively.

Thus, combining (1) and (2), the factor of improvement at maximum possible output is

$$\frac{\pi}{2}\left(1 + \frac{r_x}{R_L}\right) \sin \omega t_Q \qquad . \quad . \quad . \quad (4)$$

and at a firing angle of $\frac{\pi}{2}$ the corresponding factor is

$$\pi\left(1 + \frac{r_x}{R_L}\right) \sin \omega t_Q \qquad . \quad . \quad . \quad (5)$$

In order to evaluate these it is necessary to know the value of t_Q in terms of the circuit constants.

Determination of t_Q.

Referring to the waveform in *Figure C* and the circuit in *Figure B* (*b*), we have :

Over the conducting period from t_q to t_Q

$$r_x i_a = \hat{E}_a \sin \omega t - R_L \bar{i}_L.$$

Integrating between the limits t_q and t_Q gives

$$r_x \int_{t_q}^{t_Q} i_a dt = \hat{E}_a \int_{t_q}^{t_Q} \sin \omega t \,.\, dt - R_L \int_{t_q}^{t_Q} \bar{i}_L dt.$$

Since the current i_a only appears during conducting periods, the left-hand integral must be equal to the total current-time over the half-cycle,

i.e. $$\frac{T}{2} r_x \bar{i}_a = -\frac{\hat{E}_a}{\omega} [\cos \omega t_Q - \cos \omega t_q] - R_L \bar{i}_L [t_Q - t_q].$$

Also $$\hat{E}_a = (R_L \bar{i}_L)/\sin \omega t_Q$$

and $$\bar{i}_a = \bar{i}_L \text{ (equation 6.16)},$$

therefore substituting for $\hat{E}_a$, multiplying out by ω and collecting terms gives

$$\frac{\pi r_x}{R_L} = \left[\frac{\cos \omega t_q}{\sin \omega t_Q} - \cot \omega t_Q\right] + \omega(t_q - t_Q) \quad . \quad . \quad . \quad (6)$$

Case (*a*) *at maximum possible output.*

In this case we have conduction commencing as soon as the supply voltage exceeds the load voltage,

i.e. when $\omega t_q = \pi - \omega t_Q$ (see *Figure C*).

Substituting this condition in (6) gives

$$\cot \omega t_Q + \omega t_Q = \frac{\pi}{2}\left(1 - \frac{r_x}{R_L}\right) \quad . \quad . \quad . \quad (7)$$

Case (*b*) *for a firing angle* $\omega t_q = \pi/2$.

Substituting $\omega t_q = \pi/2$ in equation (6) gives

$$\cot \omega t_Q + \omega t_Q = \frac{\pi}{2}\left(1 - \frac{2r_x}{R_L}\right).$$

Thus the time at which conduction ends, t_Q, is determined by the time at which it commences, t_q, and the ratio of the internal resistance to load resistance. In the table below the value of ωt_Q is

tabulated against the ratio r_x/R_L for the two conditions shown. The factor of improvement for both conditions is also shown.

$\frac{r_x}{R_L}$	*Case (a)* ωt_Q for max. output.	*Case (b)* ωt_Q for firing at $\frac{\pi}{2}$	Factor of improvement $= \frac{\pi}{2}\left(1+\frac{r_x}{R_L}\right)\sin \omega t_Q$ Case (*a*)	Factor of improvement $= \pi\left(1+\frac{r_x}{R_L}\right)\sin \omega t_Q$ Case (*b*)
0	90°	90° *	1·57	3·15
0·05	125°	131°	1·36	2·5
0·1	131°	140°	1·29	2·2
0·2	140°	149°	1·21	1·93
0·3	145°	154°	1·17	1·78
0·4	149°	157·5°	1·12	1·68
0·5	151·5°	160·5°	1·11	1·56
0·6	154	162·5°	1·10	1·51
0·7	156	164·0°	1·09	1·47
0·8	157·5	165·3°	1·08	1·44
0·9	159·0	166·5°	1·06	1·39
1·0	160·5	167·5°	1·05	1·36

* In this particular case it appears that since both ωt_Q and ωt_q are equal to 90° there should be no output whatever. It should be remembered, however, that r_x also is zero, for which condition the capacitor stores a large amount of energy in a very short time. Thus this case is clearly a limiting condition for which the mean output is given by $\hat{E}_a$.

APPENDIX 8

Time-Constant of Transductor with Several Isolated Control Windings

When several control windings are coupled together, the flux time-constant of the transductor is equal to the sum of the individual time-constants. This will now be proved on the following assumption :

(*a*) That the a.c. supply voltage is sufficient to cause the core fluxes to vary from positive to negative saturation in the absence of a polarizing flux.

(*b*) That the whole core flux links each winding.

(*c*) That the magnetization curve is linear in the unsaturated region and that hysteresis effects are negligible.

The following argument applies in particular to an auto-

excited transductor and since the supply voltage is equal to $\bar{E}_{as}$ any mean flux, however small, will cause saturation during a part of the cycle. Thus

$$\phi_0 = \phi_c \text{ (section 3.13)} \quad . \quad . \quad . \quad (1)$$

Furthermore, as shown in section 5.1, the core fluxes reach a minimum value ϕ_Q during the cycle and this flux is determined solely by the control m.m.f. thus :

$$F_c = S\phi_Q \quad . \quad . \quad . \quad (2)$$

From *Figure* 3.10 it is seen that

$$\phi_Q = 2\phi_0 - \Phi_s = 2\phi_c - \Phi_s \text{ (from condition } (a) \text{ above)} \quad . \quad (3)$$

Considering the n control circuits in *Figure* 8.1 and writing the equation for each circuit

$$E_{c_1} = R_{c_1} i_{c_1} + N_{c_1} \frac{d(\phi_A + \phi_B)}{dt}$$

multiplying by $\frac{N_{c_1}}{R_{c_1}}$ gives

$$\frac{E_{c_1} N_{c_1}}{R_{c_1}} = N_{c_1} i_{c_1} + \frac{N_{c_1}^2}{R_{c_1}} \frac{d}{dt} (\phi_A + \phi_B)$$

also

$$\frac{E_{c_2} N_{c_2}}{R_{c_2}} = N_{c_2} i_{c_2} + \frac{N_{c_2}^2}{R_{c_2}} \frac{d}{dt} (\phi_A + \phi_B)$$

. . . . . .

$$\frac{E_{c_n} N_{c_n}}{R_{c_n}} = N_{c_n} i_{c_n} + \frac{N_{c_n}^2}{R_{c_n}} \frac{d}{dt} (\phi_A + \phi_B)$$

where any of the control voltages may of course be zero.

Adding and putting $\frac{1}{2}(\phi_A + \phi_B) = \phi_c$ gives

$$\sum_1^n \frac{E_{c_n} N_{c_n}}{R_{c_n}} = \sum_1^n N_{c_n} i_{c_n} + 2 \sum_1^n \frac{N_{c_n}^2}{R_{c_n}} \left[\frac{d\phi_c}{dt}\right] \quad . \quad . \quad . \quad (4)$$

The term on the left-hand side is the final value of the control m.m.f. after the steady-state is reached. It therefore defines the final value of the minimum flux ϕ_Q' where

$$S\phi_Q' = \sum_1^n \frac{E_{c_n} N_{c_n}}{R_{c_n}}$$

also

$$S\phi_Q = \sum_1^n N_{c_n} i_{c_n}.$$

Therefore substituting for ϕ_Q and ϕ'_Q from (3) gives

$$\phi'_c = \phi_c + \left[\frac{N_{c_1}^2}{SR_{c_1}} + \frac{N_{c_2}^2}{SR_{c_2}} + \ldots + \frac{N_{c_n}^2}{SR_{c_n}}\right]\frac{d\phi_c}{dt}$$

thus
$$\phi'_c = \phi_c + \left[\frac{L_{c_1}}{R_{c_1}} + \frac{L_{c_2}}{R_{c_2}} + \ldots + \frac{L_{c_n}}{R_{c_n}}\right]\frac{d\phi_c}{dt} \quad . \quad . \quad . \quad (5)$$

$$= \phi_c + \tau_t \frac{d\phi_c}{dt} \quad . \quad . \quad . \quad (6)$$

Equation (6) is the equation for the variation of control flux ; its solution is a single exponential rising to a steady-state value ϕ'_c with a time-constant τ_t where

$$\tau_t = \frac{L_{c_1}}{R_{c_1}} + \frac{L_{c_2}}{R_{c_2}} + \ldots + \frac{L_{c_n}}{R_{c_n}}.$$

Thus the flux time-constant is equal to the sum of the individual time-constants due to each control circuit. It should also be noted that each time-constant is one-half that applicable to the control circuit operating under d.c. conditions, the latter being equal to $\frac{2L_c}{R_c}$. In the foregoing, the total contribution has been assumed to be due entirely to extra control windings. Any closed coils magnetically coupled to the control windings will give this effect, however, *e.g.*, eddy current paths in the core.

APPENDIX 9

Investigation of Time-Constant Applicable to Rectifier-fed D.C. Loads with Inductive or Capacitive Smoothing

It has been shown that during conducting periods the load current increases and during quiescent periods it decreases (*Figure D* (*a*)). Also that the load current reaches a steady-state value when the increase is precisely equal to the decrease.

We will therefore consider the general relations applicable to the current i (*Figure D*) during one half-cycle between the

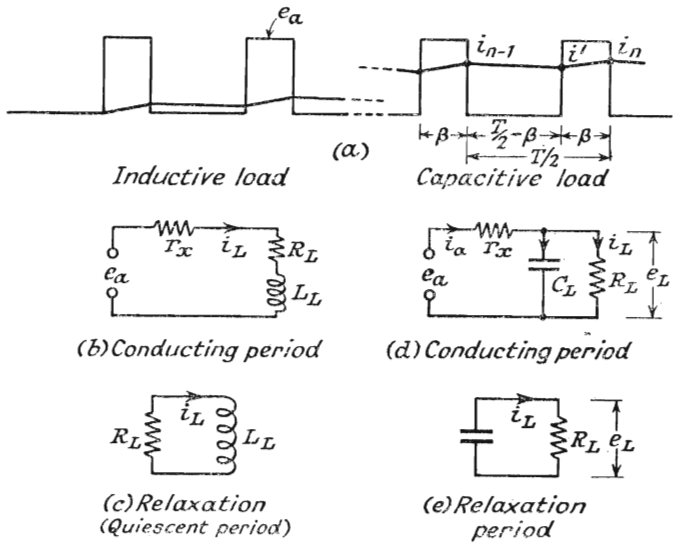

Figure D.

$(n-1)^{\text{th}}$ and the n^{th} conducting period. At the beginning of the n^{th} quiescent period the current starts to decrease according to an equation of the form

$$K_1 \frac{di}{dt} + K_2 i = 0 \qquad . \quad . \quad . \quad (1)$$

from the initial conditions $i = i_{n-1}$ when $t = 0$

to the final conditions $\quad i = i'$ when $\quad t = \dfrac{T}{2} - \beta.$

Solution of this equation by standard methods gives

$$i' = i_{n-1}\, e^{-(K_2/K_1)(T/2-\beta)} \qquad . \quad . \quad . \quad (2)$$

In the following conducting period the current is forced to increase by an applied voltage according to an equation of the form

$$K_3 \frac{di}{dt} + K_4 i = e_a \qquad . \quad . \quad . \quad (3)$$

and assuming both operations commence from *zero time,* this latter change takes place from the initial condition

$$i = i' \text{ when } t = 0$$

to the final condition $\quad i = i_n$ when $t = \beta.$

The solution of equation (3) gives

$$i_n = i' \, e^{-(K_2/K_3)\beta} + \frac{e_a}{K_4}[1 - e^{-(K_4/K_3)\beta}] \quad . \quad . \quad . \quad (4)$$

Combining (2) and (4)

$$i_n = i_{n-1} \, e^{-(K_2/K_1)(T/2-\beta)} . \quad e^{-(K_4/K_3)\beta} + \frac{e_a}{K_4}[1 - e^{-(K_4/K_3)\beta}]$$

$$= i_{n-1} \, e^{-[(K_2/K_1)(T/2-\beta)+(K_4/K_3)\beta]} + \frac{e_a}{K_4}[1 - e^{-(K_4/K_3)\beta}]$$

which may be written

$i_n = a \, i_{n-1} + b$ where $0 < a < 1$

where $a = e^{-[(K_2/K_1)(T/2-\beta)+(K_4/K_3)\beta]}$ . . . (5)

and $b = \frac{e_a}{K_4}[1 - e^{-(K_4/K_3)\beta}]$ . . . (6)

Referring to Appendix 2, this may be formed into a sequence giving

$$i_n = a^{n-1} \, i_1 + \left[\frac{1 - a^{n-1}}{1 - a}\right] b \quad \text{cf. equation (3), Appendix 2}$$

the steady-state value, i_{ss} of i_n is found by making $n \to \infty$, thus since $0 < a < 1$

$$i_{ss} = \frac{b}{1 - a}$$

and therefore, if the time-constant is defined as the time required for the current to reach the fraction $\left(1 - \frac{1}{e}\right)$ of i_{ss}, we have, putting $i_n = \left(1 - \frac{1}{e}\right) i_{ss}$

$$\left(1 - \frac{1}{e}\right) \frac{b}{1 - a} = a^{n-1} \, i_1 + \left[\frac{1 - a^{n-1}}{1 - a}\right] b \quad . \quad . \quad . \quad (7)$$

Now i_1 is the current at the end of the first conducting period for which the initial conditions were $i' = 0$.
It therefore follows from (4) and (6) that

$$i_1 = \frac{e_a}{K_4}[1 - e^{-(K_4/K_3)\beta}] = b \quad . \quad . \quad . \quad (8)$$

Thus from (7) and (8)

$$\frac{1}{e} = a^n \quad \text{and hence}$$

$$n \log a = \log \frac{1}{e}$$

thus $$n = \frac{-1}{\log a}$$

and $$\tau = \frac{nT}{2} = \frac{-T/2}{\log a}.$$

Thus substituting from (5)

$$\tau = \frac{T/2}{\frac{K_2}{K_1}\left(\frac{T}{2} - \beta\right) + \frac{K_4}{K_3}\beta} \quad . \quad . \quad . \quad (9)$$

It only remains to determine the constants K for the particular cases with which we are concerned.

(i) *Inductive load*

(*a*) During relaxation (quiescent period) from *Figure D* (*c*)

$$L_L \frac{di_L}{dt} + R_L i_L = 0 \quad . \quad . \quad . \quad (10)$$

(*b*) During conduction from *Figure D* (*b*)

$$L_L \frac{di_L}{dt} + (R_L + r_x)\, i_L = e_a \quad . \quad . \quad . \quad (11)$$

comparing (10) and (11) with (1) and (3) gives

$$K_1 = L_L$$
$$K_2 = R_L$$
$$K_3 = L_L$$
$$K_4 = R_L + r_x.$$

Substituting these in the general expression for the time-constant gives

$$\tau_L = \frac{L_L}{R_L\left(1 + (2\beta/T)\,\frac{r_x}{R_L}\right)} = \frac{L_L}{\bar{R}_{aL}} \text{ (Chapter 6)} \quad . \quad . \quad . \quad (12)$$

(ii) *Capacitive load*

(*a*) During relaxation *Figure D* (*c*)

$$C_L \frac{de_L}{dt} + i_L = 0$$

thus $$R_L C_L \frac{di_L}{dt} + i_L = 0 \quad . \quad . \quad . \quad (13)$$

(*b*) During conduction
Figure D (*d*)

$$\frac{e_a - e_L}{r_x} = i_L + C_L \frac{de_L}{dt}$$

$$\frac{e_a - R_L i_L}{r_x} = i_L + R_L C_L \frac{di_L}{dt}$$

thus $$R_L C_L r_x \frac{di_L}{dt} + i_L(R_L + r_x) = e_a \quad . \quad . \quad . \quad (14)$$

Comparing (13) and (14) with (1) and (3) gives

$$K_1 = R_L C_L$$
$$K_2 = 1$$
$$K_3 = R_L C_L r_x$$
$$K_4 = R_L + r_x.$$

Substituting these values in the general expression for the time-constant gives, for a capacitive load,

$$\tau_L = \frac{R_L C_L}{1 + (2\beta/T)\frac{R_L}{r_x}} = \frac{R_L C_L r_x}{r_x\left(1 + (2\beta/T)\frac{R_L}{r_x}\right)} = \frac{R_L C_L r_x}{\bar{R}_{aL}} \quad . \quad . \quad (15)$$

Thus, having estimated the time-constant governing changes of load current, the overall transfer functions for inductive and capacitive loads may be written down :

(*a*) *Inductive load*

$$\frac{\bar{\imath}_L}{\bar{e}_a} = \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}} = \frac{1}{\bar{R}_{aL}}\left[\frac{1}{1 + (L_L/\bar{R}_{aL})D}\right] \quad . \quad . \quad (16)$$

Considering the transductor current $\bar{\imath}_a$, this is always a definite fraction of the load current and therefore it has the same response characteristics but a different steady-state value,

i.e. $\bar{\imath}_a = (2\beta/T)\,\bar{\imath}_L$ and :

$$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{\overline{Y}_a(D)}{\bar{R}_a} = \frac{1}{\bar{R}_a}\left[\frac{1}{1 + (L_L/\bar{R}_{aL})D}\right] \quad . \quad . \quad . \quad (17)$$

where $\bar{R}_a = \dfrac{R_L}{2\beta/T}\left(1 + (2\beta/T)\dfrac{r_x}{R_L}\right)$ (for inductive load).

(*b*) *Capacitive load*

$$\frac{\bar{\imath}_L}{\bar{e}_a} = \frac{\overline{Y}_{aL}(D)}{\bar{R}_{aL}} = \frac{1}{\bar{R}_{aL}}\left[\frac{1}{1+\left(\frac{C_L R_L r_x}{\bar{R}_{aL}}\right)D}\right] \quad . \quad . \quad . \quad (18)$$

In order to determine the transients for the mean transductor current $\bar{\imath}_a$ we refer to *Figure D* (*d*).

The transductor current only flows during conducting periods during which time the transductor output voltage appears and we have

$$e_a - e_L = i_a r_x.$$

Integrating over the conducting period and multiplying throughout by $\frac{2}{T}$ gives

$$\frac{2}{T}\int_{(\frac{T}{2}-\beta)}^{\frac{T}{2}} e_a dt - \frac{2}{T}\int_{(\frac{T}{2}-\beta)}^{\frac{T}{2}} e_L dt = \frac{2}{T}\int_{(\frac{T}{2}-\beta)}^{\frac{T}{2}} i_a r_x$$

thus, since the load voltage e_L is smooth, $e_L = \bar{e}_L = \bar{\imath}_L R_L$, and since the output voltage and transductor current are confined to conducting periods we have

$$\bar{e}_a - \frac{2\beta}{T} R_L \bar{\imath}_L = \bar{\imath}_a r_x$$

i.e.
$$\bar{\imath}_a = \frac{\bar{e}_a}{r_x}\left[1 - (2\beta/T) R_L \frac{\bar{\imath}_L}{\bar{e}_a}\right]$$

therefore
$$\bar{\imath}_a = \frac{\bar{e}_a}{r_x}\left[1 - (2\beta/T)\frac{R_L}{\bar{R}_{aL}}\overline{Y}_{aL}(D)\right]$$

substituting for $\overline{Y}_{aL}(D)$ from (18) gives

$$\frac{\bar{\imath}_a}{\bar{e}_a} = \frac{\overline{Y}_a(D)}{\bar{R}_a} = \frac{1}{\bar{R}_a}\left[\frac{1 + C_L R_L D}{1+\left(\frac{C_L R_L r_x}{\bar{R}_{aL}}\right)D}\right] \quad . \quad (19)$$

where
$$\bar{R}_a = \bar{R}_{aL} = r_x\left(1 + (2\beta/T) \cdot \frac{R_L}{r_x}\right).$$

APPENDIX 10

Voltage Gain Transfer Function for Transductor with Any Type of Load

It will be assumed, as previously, that the supply voltage is at its optimum value $\bar{E}_{as}$ so that any finite mean flux, however small, causes saturation and

$$\phi_0 = \phi_c.$$

The general ampere-turns relationship (equation 4.12) assuming a smooth control current ($\lambda = 1$) is thus :

$$\bar{F}_c + S\Phi_s = \bar{\imath}_a (N_a - N_f) + 2S\phi_c \quad . \quad . \quad . \quad (1)$$

When a control voltage E_c is applied to a control winding we have

$$\frac{E_c N_c}{R_c} = \bar{F}_c + 2S \sum_1^n \frac{N_{cn}^2}{SR_{cn}} \left[\frac{d\phi_c}{dt}\right]$$

where $\sum_1^n \frac{N_{cn}^2}{SR_{cn}} = \tau_t$ (Appendix 8).

Thus

$$\frac{E_c N_c}{R_c} = \bar{F}_c + 2S\tau_t \frac{d\phi_c}{dt} \quad . \quad . \quad . \quad (2)$$

Combining (1) and (2) and multiplying by $\frac{R_c}{N_c}$

$$E_c + \frac{SR_c}{N_c}\Phi_s = \bar{\imath}_a R_c \left[\frac{N_a - N_f}{N_c}\right] + 2S\frac{R_c}{N_c}\phi_c (1 + \tau_t D) \quad (3)$$

Now $\phi_c = \frac{\bar{e}_a}{8fN_a}$ (equation 3.39)

and $\bar{\imath}_a = \frac{\bar{e}_a}{\bar{R}_a} \cdot \bar{Y}_a(D)$ (by definition Chapter 8).

Therefore substituting in (3) and considering changes of voltage only, so that the constant term disappears,

$$\delta E_c = \delta\bar{e}_a \left[\frac{R_c}{\bar{R}_a} \cdot \bar{Y}_a(D) \cdot \frac{N_a - N_f}{N_c} + \frac{SR_c}{4fN_a N_c}(1 + \tau_t D)\right]$$

now $\frac{R_c}{\bar{R}_a}\left(\frac{N_a - N_f}{N_c}\right) = h_v$

$$\frac{2}{\pi}\frac{\omega M_{ca}}{R_c} = g_v$$

and $$M_{ca} = \frac{N_a N_c}{S}$$

thus

$$\delta E_c = \delta \bar{e}_a \left[\overline{Y}_a(D) h_v + \frac{(1 + \tau_t D)}{g_v} \right]$$

$$\frac{\delta e_a}{\delta E_c} = G_v \overline{Y}_v(D) = \frac{g_v}{(1 + \tau_t D) + g_v h_v \overline{Y}_a(D)} \quad . \quad (4)$$

APPENDIX 11

General Input/Output Relations for Even-Harmonic Transductors

Considering the basic circuit of *Figure* 11.2, at a certain time t' it will be assumed that the numerical value of the alternating current i_a is equal to i'_a, thereby giving m.m.f.s in the two cores as follows :

$$F_A = (i'_a N_a + i_c N_c) \quad . \quad . \quad . \quad (1)$$

$$F_B = (i'_a N_a - i_c N_c) \quad . \quad . \quad . \quad (2)$$

If the two cores are perfectly matched, their m.m.f./flux relationships will be identical and may be represented by the general equation :

$$\phi = \psi(F) \quad . \quad . \quad . \quad (3)$$

The voltages e_t and e_d may also be expressed in terms of the core fluxes ϕ_A and ϕ_B thus :

$$e_t = N_a \left[\frac{d\phi_A}{dt} + \frac{d\phi_B}{dt} \right] \quad . \quad . \quad . \quad (4)$$

and $$e_d = N_d \left[\frac{d\phi_A}{dt} - \frac{d\phi_B}{dt} \right] \quad . \quad . \quad . \quad (5)$$

General expressions for the fluxes ϕ_A and ϕ_B may be obtained from (1), (2) and (3) and hence substituting in (4) and (5) gives the instantaneous values of e_t and e_d at time t'.

$$e_t(t') = N_a \frac{d}{dt} [\psi(i'_a N_a + i_c N_c) + \psi(i'_a N_a - i_c N_c)] \quad . \quad (6)$$

and $$e_d(t') = N_d \frac{d}{dt}[\psi(i'_a N_a + i_c N_c) - \psi(i'_a N_a - i_c N_c)] \quad . \quad (7)$$

From symmetry considerations the a.c. coils will appear to have the same " impedance " to the generator during positive and negative half-cycles of the supply voltage $\hat{E}_a \sin \omega t$. It follows that at exactly one half-period after t' the current i_a will be equal to $-i'_a$. This gives the following result for the instantaneous values of the voltages e_t and e_d by replacing i'_a by $-i'_a$ in equations (6) and (7).

$$e_t(t' + T/2) = N_a \frac{d}{dt}[\psi(-i'_a N_a + i_c N_c) + \psi(-i'_a N_a - i_c N_c)] \quad (8)$$

and

$$e_d(t' + T/2) = N_d \frac{d}{dt}[\psi(-i'_a N_a + i_c N_c) - \psi(-i'_a N_a - i_c N_c)] \quad (9)$$

The magnetization curve for magnetic materials is symmetrical and therefore we have

$$\psi(-F) = -\psi(F) \quad . \quad . \quad . \quad (10)$$

Considering this fact in conjunction with equations (6), (7), (8) and (9) shows that

$$e_t(t') = -e_t(t' + T/2) \quad . \quad . \quad . \quad (11)$$

$$e_d(t') = e_d(t' + T/2) \quad . \quad . \quad . \quad (12)$$

Now since the time t' and the current i'_a have been chosen arbitrarily, any of the above equations apply at all times during the cycle.

(1) *Effect of zero control current*

Putting $i_c = 0$ in equation (6) shows that

$$e_t = 2N_a \frac{d}{dt}[\psi(N_a i_a)] \quad . \quad . \quad . \quad (13)$$

and from equation (7)

$$e_d = 0 \quad . \quad . \quad . \quad (14)$$

(2) *Effect of reversing the sign of the control current*

In order to investigate the effect of reversing the sign of the signal, the current i_c is replaced by $-i_c$. If this is done it is seen from equation (6) that the sign of the voltage e_t at a given instant (t') remains unchanged. From equation (7), however, it also

follows that the sign of the voltage e_d reverses when the control current reverses.

APPENDIX 12

Analysis of Even-Harmonic Transductor

(*a*) *Zero load capacitance*

Referring to *Figure E* (*a*), this represents the effective circuit over the conducting period from $t_q(A)$ to $t_q(B)$ when one core is saturated.

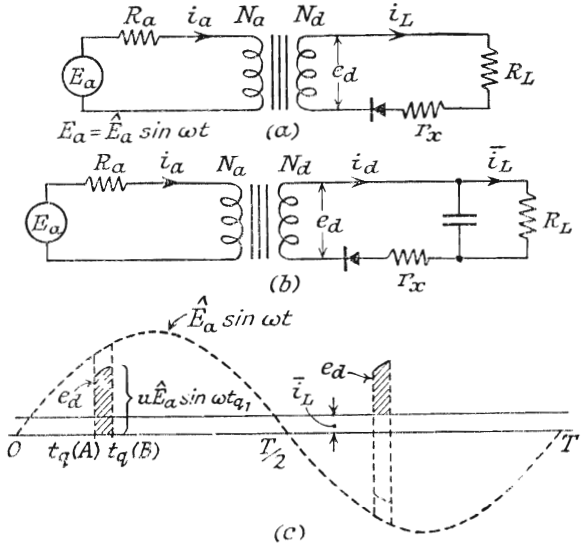

Figure E.

During this time the voltage e_d appears across the pick-off windings and a current i_L flows where

$$e_d = (R_L + r_x)i_L.$$

Integrating from $t_q(A)$ to $t_q(B)$ gives

$$\int_{t_q(A)}^{t_q(B)} e_d dt = (R_L + r_x) \int_{t_q(A)}^{t_q(B)} i_L dt \quad . \quad . \quad . \quad (1)$$

Now the integral on the left is the voltage-time area over the interval $\beta = (t_q(B) - t_q(A))$ and is therefore proportional to the flux difference in cores (A) and (B), i.e. :

$$\int_{t_q(A)}^{t_q(B)} e_d dt = N_d \Delta\phi = \frac{2N_d N_c \bar{\imath}_c}{S} \quad . \quad . \quad . \quad (2)$$

(See section 11.5.)

Thus from (1)

$$\frac{2N_d N_c}{S} \bar{\imath}_c = (R_L + r_x) \frac{T}{2} \bar{\imath}_L$$

putting $$T = \frac{1}{f}$$

and $$\frac{N_d N_c}{S} = M_{cd}$$

gives $$\bar{\imath}_L = \frac{2}{\pi} \frac{\omega M_{cd}}{(R_L + r_x)} \bar{\imath}_c \quad . \quad . \quad . \quad (3)$$

(*b*) *Large load capacitance*

The effect of a large load capacitance may be seen by reference to *Figures E* (*b*) and (*c*).

During conducting periods the capacitor is charged up from a low impedance source of voltage e_d. If the capacitance is sufficiently large the resulting load current $\bar{\imath}_L$ will be substantially constant and it will be assumed that these conditions apply. Therefore from *Figure E* (*b*) over the interval $t_q(A)$ to $t_q(B)$

$$e_d - \bar{\imath}_L R_L = i_d r_x \quad . \quad . \quad . \quad (4)$$

Integrating over this conducting period gives

$$\int_{t_q(A)}^{t_q(B)} e_d dt - R_L \int_{t_q(A)}^{t_q(B)} \bar{\imath}_L dt = r_x \int_{t_q(A)}^{t_q(B)} i_d dt \quad . \quad . \quad . \quad (5)$$

Substituting from (2) for the voltage-time integral and integrating the currents $\bar{\imath}_L$ and i_d gives

$$\frac{2N_d N_c \bar{\imath}_c}{S} = R_L \beta \bar{\imath}_L + r_x \frac{T}{2} \bar{\imath}_d.$$

In the steady state the total charge flowing into the capacitor in a half-cycle must be equal to that discharged into the load over the same period, thus

$$\bar{\imath}_d = \bar{\imath}_L$$

which gives

$$\bar{\imath}_L = \frac{2}{\pi} \frac{\omega M_{cd}}{r_x \left[1 + (2\beta/T) \frac{R_L}{r_x} \right]} \bar{\imath}_c \quad . \quad . \quad . \quad (6)$$

(c) *Evaluation of $2\beta/T$ in terms of control current*

The above expression for the current amplification involves the term $2\beta/T$, and since the conducting period is dependent upon the control current, this factor introduces a non-linearity in the control characteristic which will now be investigated.

It will be assumed that the conducting period is of very short duration and during this period the load capacitor appears as an effective short circuit across the load. Thus the effective resistance in the pick-off circuit will be r_x.

In these circumstances the total effective impedance in the a.c. circuit will be $R_a + r_x \frac{N_a^2}{N_d^2}$ and the voltage across the a.c. windings will be

$$u \hat{E}_a \sin \omega t$$

where

$$u = \frac{r_x \frac{N_a^2}{N_d^2}}{R_a + r_x \frac{N_a^2}{N_d^2}} \quad . \quad . \quad . \quad (7)$$

Provided the conducting period is small, the total voltage-time integral over the period β from $t_q(A)$ to $t_q(B)$ is $\beta u \hat{E}_a \sin \omega t_{qA}$, and since this is equal to the total flux-change $\times$ a.c. turns we have

$$\frac{\beta u \hat{E}_a}{N_a} \sin \omega t_{qA} = \frac{2N_c i_c}{S}$$ (see p. 318).

Therefore

$$\beta = \frac{2N_c N_a \bar{\imath}_c}{S \,.\, u \,.\, \hat{E}_a \sin \omega t_q(A)} \quad . \quad . \quad . \quad (8)$$

It now remains to evaluate $\hat{E}_a \sin \omega t_q(A)$.

During the period from $t = 0$ to $t = t_q(A)$ the fluxes in the two cores increase from $-\Phi_s$ to $+\Phi_s$ since the cores are driven well into saturation and $t_q(A)$ is hardly affected by the control current, hence

$$\frac{1}{2N_a} \int_0^{t_q(A)} \hat{E}_a \sin \omega t \, dt = 2\Phi_s$$

giving $$2\Phi_s = \frac{\hat{E}_a}{2\omega N_a}[1 - \cos \omega t_q(A)]$$

$$= \frac{\bar{E}_a}{8fN_a}[1 - \cos\ \omega t_q(A)]$$

$$(\text{since } \hat{E}_a = \frac{\pi}{2}\bar{E}_a)$$

but also

$$2\Phi_s = \frac{\bar{E}_{as}}{4fN_a} \quad \text{where } \frac{\bar{E}_a}{\bar{E}_{as}} \text{ is the supply voltage factor } K.$$

Thus $$2\bar{E}_{as} = \bar{E}_a(1 - \cos \omega t_q(A))$$

and putting $$\cos \omega t_q(A) = \sqrt{1 - \sin^2 \omega t_q(A)}$$

gives $$\bar{E}_a - 2\bar{E}_{as} = \bar{E}_a \sqrt{1 - \sin^2 \omega t_q(A)}$$

squaring, collecting terms and taking the square root gives :

$$\bar{E}_a \sin \omega t_q(A) = 2\bar{E}_{as}\sqrt{\frac{\bar{E}_a}{\bar{E}_{as}} - 1}$$

or $$\hat{E}_a \sin \omega t_q(A) = \pi\bar{E}_{as}\sqrt{K - 1}$$

$$= 8\pi\Phi_s f N_a \sqrt{K - 1}$$

now $2\Phi_s$ is the maximum possible flux difference and corresponds to the control current $\bar{i}_{cs}$ at maximum output such that

$$2\Phi_s = \frac{2N_c\bar{i}_{cs}}{S}$$

therefore :

$$\hat{E}_a \sin \omega t_q(A) = \frac{8\pi f N_a N_c \bar{i}_{cs}}{S}\sqrt{K - 1} \quad . \quad . \quad . \quad (9)$$

Combining (8) and (9) gives :

$$\beta = \frac{\bar{i}_c}{4\pi f u \bar{i}_{cs}\sqrt{K - 1}} \quad . \quad . \quad . \quad (10)$$

Thus $$\frac{2\beta}{T} = 2f\beta = \frac{\bar{i}_c}{2\pi\bar{i}_{cs} u\sqrt{K - 1}}$$

and substituting this in equation (6) :

$$\bar{i}_L = \frac{2}{\pi} \cdot \frac{\omega M_{cd}}{r_x\left[1 + \dfrac{\bar{i}_c}{2\pi\bar{i}_{cs} u\sqrt{K - 1}}\left(\dfrac{R_L}{r_x}\right)\right]}\bar{i}_c. \quad . \quad . \quad (11)$$

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